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TOOL SELECTION PROBLEM IN FLEXIBLE MANUFACTURING SYSTEMS

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ABSTRACT

This paper deals with a problem on the selection of alternative tools in a flexible manufacturing system (FMS) where a lot of tools are required to produce a large variety of product items. An approach using branch and bound method is proposed to minimize a total number of tools required through the optimal use of the alternative tools. In this approach, tools are initially divided into tool subgroups based on graph theory for the purpose of the effective search of the optimal solution. A small example is also presented to highlight the effectiveness of the proposed approach.

1. INTRODUCTION

In a flexible manufacturing system (FMS), variety of product items are produced by the computer numerically controlled machines with limited capacity tool magazine. To produce the product items with diverse spectrum,a large number of different tools are required.

In such manufacturing environment, tool management becomes a critical problem for the effective use of an FMS. Lack of attention to tool management results in a poor performance of the FMS [1-2].

This paper tries to reduce tooling cost by using alternatives among cutting tools. Reduction of both types and numbers of cutting tools brings not only the saving of tool cost but also the improvement of machine utilization through a reduction of tool-setup time.

The proposed method in this study is comprised of the following two stages. In the first stage, cutting tools are divided into tool subgroups having less alternative relationship with each other by using graph theory. In the second stage, total number of required tools are minimized by use of branch and bound method. Subdivision into tool subgroups in stage 1 can give the possibility of effective search of the optimal solution in the branch and bound method. A simple example is presented to show the effectiveness of the proposed method.

2. TOOL SELECTION PROBLEM

Given a production information on types, numbers, and machining processes of the parts, tool selection problem can be described as the following.

Let Np be a number of different parts to be produced, n_i a volume of part p_i, and Nc a number of different kind of tools to be used. Information on the alternative tools is given in the form of eq.(1)-(3),

$$C_{ij} = [c_{ij1},\dots,c_{ijk},\dots,c_{ijN_{ij}}] \quad (i=1,..,N_p;\ j=1,..,N_c) \quad \cdots\cdots (1)$$

$$t_{ij} = [t_{ij1},\dots,t_{ijk},\dots t_{ijN_{ij}}] \quad (i=1,..,N_p;\ j=1,..,N_c) \quad \cdots\cdots (2)$$

$$T_{ij} = [T_{ij1},\dots,T_{ijk},\dots,T_{ijN_{ij}}] \quad (i=1,..,N_p;\ j=1,..,N_c) \quad \cdots\cdots (3)$$

where c_{ijk} means a k-th alternative tool for primary tool c_j to be used in the machining process of part p_i. N_{ij} is a number of cutting tools for primary tool c_j in the machining process of part p_i. Tool c_{ij1} is a tool with first priority which means a primary tool c_j. In eq.(2), t_{ijk} means machining time of part p_i with tool c_{ijk}, which is arranged in the increasing order of t_{ijk}/T_{ijk} as shown in eq.(4).

$$t_{ijk}/T_{ijk} \leq t_{ij,k+1}/T_{ij,k+1} \quad \cdots\cdots (4)$$

In eq.(3), T_{ijk} is a tool life of c_{ijk} in the machining process of part p_i.

A load rate of tool c_{ijk} can be determined by eq.(5),

$$L_{ijk} = n_i \cdot t_{ijk}/T_{ijk} \quad (i=1,..,N_p;\ j=1,..,N_c;\ k=1,..,N_{ij}) \quad \cdots\cdots (5)$$

Then the tool selection problem can be formulated as the following integer program.

$$\min \quad V = \sum_{j=1}^{N_c} \left[\sum_{i=1}^{N_p} \sum_{k=1}^{N_{ij}} L_{ijk} \cdot x_{ijk} \right]^{+} \quad \cdots\cdots (6)$$

subject to

$$\sum_{k=1}^{N_{ij}} x_{ijk} = 1 \quad (i=1,..,N_p;\ j=1,..,N_c) \quad \cdots\cdots (7)$$

Here the notation $[u]^+$ gives the smallest integer value greater than or equal to u. The decision variable x_{ijk} takes the value 1 if the alternative tool c_{ijk} is selected to be used in the machining process of part p_i and takes 0 otherwise. Eq.(7) means that at each machining process of each part only one tool should be selected from the alternative tools.

3. OPTIMAL SELECTION OF ALTERNATIVE TOOLS

For the tool selection problem defined in eq.(6)-(7), a mathematical approach is proposed to give the optimal solution. This approach comprises of the following two phases. In the first phase, tools are divided into tool subgroups so as to minimize the interaction among tool subgroups with reference to the alternative load rate. In the second phase, the minimum number of tools required is calculated by the branch and bound method by using tool subgroups effectively.

3.1 GENERATION OF TOOL SUBGROUPS

The objective of the tool subgroup division is in the improvement of the efficiency of branch and bcund method succeeding this step. In this study, the subdivision of tools into tool subgroups are performed so as to minimize the interrelation of tool alternatives among tool subgroups.

Based on the information of alternative tools in the machining process, alternative load rate matrix M_A can be constructed as eq.(8),

$$M_A = \begin{bmatrix} a_{11} & \cdots & a_{1h} & \cdots & a_{1,Nc} \\ a_{j1} & \cdots & a_{jh} & \cdots & a_{j,Nc} \\ a_{Nc,1} & \cdots & a_{Nc,h} & \cdots & a_{Nc,Nc} \end{bmatrix} \quad \cdots\cdots (8)$$

here,

$$a_{jh} = \sum_{i=1}^{N_p} \sum_{k=1}^{N_{ij}} L_{ijl} \cdot \delta(C_{ijk} - C_h) \quad \cdots\cdots\cdots (9)$$

$$\delta(C_{ijk} - C_h) = \begin{cases} 1 : \text{if } C_{ijk} = C_h \\ 0 : \text{if } C_{ijk} \neq C_h \end{cases} \quad \cdots\cdots (10)$$

In eq.(9), an alternative load rate from tool c_j to c_h is counted into a_{jh}. Eq.(10) provides that $\delta(C_{ijk}-C_h)=1$ if tool c_j can be replaced by c_h as the alternative tool and $\delta(C_{ijk}-C_h)=0$ otherwise.

Matrix M_A in eq.(8) can be considered as a weighted incident matrix which represents the alternative relationship between tools, where a vertex corresponds to a tool, an edge corresponds to a relationship between tools, and a weight attached to an edge corresponds to an alternative load rate between tools.

The procedure for generating tool subgroups comprises of the following eight steps.

(step 1) Form the minimum spanning tree from the matrix M_A by using Kruscal's algorithm [3] in such a way that the total alternative load rate attached to the tree is to be minimized.

(step 2) Construct a fundamental tie-set matrix from the minimum spanning tree.

(step 3) Construct a fundamental cut-set matrix M_C from the fundamental tie-set matrix.

(step 4) Calculate all the possible cut-sets by combining fundamental cut-sets in M_C.

(step 5) Select the cut-set with minimum total value of the alternative load rate as a candidate cut-set. Then divide tools or the tool subgroup to form new tool subgroups by the candidate cut-set.

(step 6) Examine for each tool subgroup whether the minimum number of tools required is changed or not when the alternative tools are used. In case there is a change the candidate cut-set should be rejected and repeat the procedure from step 5, otherwise proceed to the next step.

(step 7) Accept the candidate cut-set as an actual cut-set to produce a tool subgroup. Apply the cut-set, then return to step 5. If no more cut-set remains, then go to step 8.

(step 8) Terminate the procedure.

The overall flow of the proposed algorithm to form tool subgroups is shown in Figure 1.

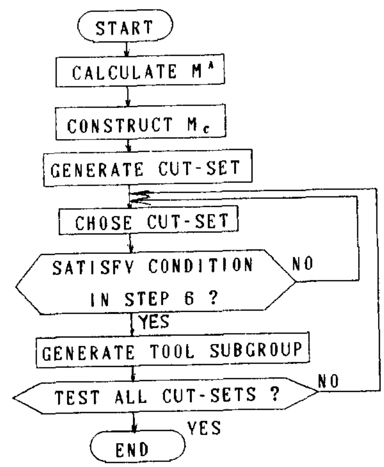

Figure 1 Flowchart of tool subgroup formation

3.2 DETERMINATION OF THE OPTIMAL ALTERNATIVE TOOLS BY USING BRANCH AND BOUND METHOD

The optimal solution of the tool selection problem can be obtained by using branch and bound method. The way of branching and bounding operations largely affects on the performance of this method, so these two operations should be carefully determined in consideration of the characteristics of the given problem.

In the proposed approach, the branching operation and bounding operation are performed in the following way.

[branching operation]

Select a node with the least lower bound among active nodes in the search tree, then branch the node. The number of nodes to be branched from the selected node is the same as the number of alternative tools in the corresponding machining process of the selected node.

[bounding operation]

Terminate nodes having greater lower bound than a minimum feasible solution obtained up to the point. The lower bound LB of the minimum number of tools required for each node can be calculated by eq.(11)-(12),

$$LB=\sum_{g=1}^{N_s} VL_g \qquad \cdots\cdots\cdots\cdots(11)$$

$$VL_g=\left[\sum_{i=1}^{N_p}\left(\sum_{(j,k)\in D_i} L_{ijk}+\sum_{j\in U_i} L_{ij1}\right)\right]^{+} \qquad \cdots\cdots(12)$$

where

$$D_i=\{(j,k)\mid x_{ijk}=1, C_{ijk}\in S_g\} \qquad \cdots\cdots(13)$$

$$U_i=\{j\mid C_j\in S_g, (i,j)\in R_i, \text{ for } \forall k\ C_{ijk}\in S_g\} \qquad \cdots\cdots\cdots\cdots(14)$$

In eq.(11), N_s is the number of tool subgroups constructed in the first phase. In eq.(12), VL_g means a lower bound of the number of tools required for tool subgroup S_g. R_i in eq.(14) means a set of operations for part p_i in which the alternative tool to be used is not still determined.

4. NUMERICAL EXAMPLE

In order to demonstrate the effectiveness of the proposed method, consider the information on the alternative tools given in Figure 2. It can be seen from this figure, for example, that the primary tool c1 in the machining process of part p1 can be replaced by the alternative tool c3. In this case, the machining time of this process increases from 0.2 to 0.3.

Based on the information of the alternative tools, the alternative load rate matrix M_A can be constructed as shown in Figure 3.

The weighted graph of the alternative relations among tools is shown in Figure 4. In this figure, vertex ○ repesents a tool, arrow → indicates an alternative relation between tools, and a value attached to an arrow represents an load rate of alternative tool. Each number attached to a vertex means a vertex number and a value brancketed in a vertex is a load rate of the tool for the whole product items.

(a) Tools

Parts						
p1	c1	c2	c3			
	c3	–	c1			
p2	c1	c2	c3	c4	c5	
	–	c5	–	–	c2	
p3	c1	c2	c3	c4	c5	c6
	–	–	–	–	c2	c1
p4	c1	c2	c3	c4	c5	c6
	c6	c1	–	c3	–	–
p5			c3	c4	c5	c6
			c4	c1	–	c1

(b) Load rate of tool

Parts						
p1	.2	.4	.2			
	.3	–	.3			
p2	.6	.3	.5	.4	.2	
	–	.4	–	–	.2	
p3	.5	.5	.8	.7	.6	.4
	–	–	–	–	.7	.5
p4	.1	.4	.6	.3	.8	.6
	.15	.5	–	.4	–	–
p5			.2	.3	.6	.2
			.25	.4	–	.3

(a) Alternative tools (b) Load rate of alternative tools

Figure 2 Information of altenative tools

$$M_A=\begin{array}{c} \begin{matrix} c1 & c2 & c3 & c4 & c5 & c6 \end{matrix} \\ \begin{bmatrix} * & - & .2 & - & - & .1 \\ .4 & * & - & - & .3 & - \\ .2 & - & * & .2 & - & - \\ .3 & - & .3 & * & - & - \\ - & .8 & - & - & * & - \\ .6 & - & - & - & - & * \end{bmatrix} \begin{matrix} c1 \\ c2 \\ c3 \\ c4 \\ c5 \\ c6 \end{matrix} \end{array}$$

Figure 3 Alternative load rate matrix

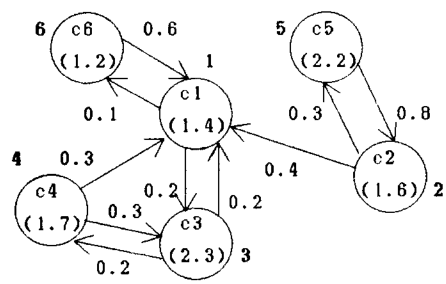

Figure 4 Graph for tool alternatives

By applying Kruscal's algorithm to the matrix M_A, the minimum spanning tree can be obtained as 12-13-14-16-25. Then the fundamental cut-set matrix Mc can be obtained as Figure 5 based on this minimum spanning tree. All the possible cut-set obtained from matrix Mc are also shown in this figure.

In the subsequent steps, cut-set is selected in the increasing order of the alternative load rate and test the condition described in step 6 to form a tool subgroup.

First, cut_1 which has the minimum alternative load rate is selected as a candidate for generating tool subgroups. Applying cut_1 to the graph shown in Fig.4 forms tool sub-group SG1={c2,c5} and SG2={c1,c3,c4,c6}.

For SG1, total load rate is 3.8 which can be reduced to 3.4 by using the alternative tool c1 in place of tool c2 in the machining process of part p4. In this case, the number of tools required for SG1 does not change even if the alternative tool c1 is used. At the same time, the number of tools required for SG2 is ascertained not to be

$$Mc=\begin{array}{c} \begin{matrix} 1\_2 & 1\_3 & 1\_4 & 1\_6 & 2\_5 & 3\_4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

cut_1: 1_2 (0.4)
cut_2: 1_3+3_4 (0.9)
cut_3: 1_4+3_4 (0.8)
cut_4: 1_6 (0.7)
cut_5: 2_5 (1.1)
cut_6: 1_3+1_4 (0.7)
[cut_2+cut_3]

(a) Fundamental cut-set matrix (b) Feasible cut-sets

Figure 5 Fundamental cut-set and feasible cut-sets changed because of the nonexistence of the alternative load rate from SG2 to SG1. Hence, the cut_1 is acceptable and tool subgroups SG1 and SG2 can be constructed.

Next, both cut_4 and cut_6 become the candidate cut-sets to form tool subgroups because they have the minimum alternative load rate in the remaining cut-sets. In case of cut_4, tool subgroup SG2 is divided into SG3={c6} and SG4={c1, c3,c4}. For SG3, the load rate is 1.2 and it can be reduced to 0.6 by the use of alternative tool c1 which causes the reduction of the number of tools required. So, this cut_4 cannot be accepted. On the other hand, in case of cut_6, tool subgroup SG2 is divided into SG3={c1,c6} and SG4={c3,c4}. In this case, it can be easily confirmed by the same way that the total number of tools required in each subgroup does not change if the alternative load rate among each subgroup is taken into account. Hence, the cut _6 is permitted to form tool subgroups SG3 and SG4.

At this stage, there is no other feasible cut-set in the remaining cut-sets to satisfy the condition in step 6. As a result, three subgroups of cutting tools are obtained as shown in Table 1.

Table 1 Constructed tool subgroups

cut_set	tool subgroup	alternative load rate
cut_1	SG1={c2,c5}	0.4 (SG1→SG3)
cut_6	SG3={c1,c6}	0.2 (SG3→SG4)
	SG4={c3,c4}	0.5 (SG4→SG3)

The next phase is to get the optimal selection of the alternative tools by using branch and bound method.

First, lower bound and upper bound of the total number of tools required to produce all the product items can be calculated as follows.

lower bound: $L_B = [2.2+1.6-0.4]^+ + [1.2+1.4-0.2]^+ + [1.7+2.3-0.5]^+ = 11$

upper bound: $U_B = [1.4]^+ + [1.6]^+ + [2.3]^+ + [1.7]^+ + [2.2]^+ + [1.2]^+ = 14$

These two values, L_B and U_B, indicate that there is a possibility to reduce the total number of tools by 3.

By using branch and bound method described in 3.2, the optimal solutions for the alternative tool selection are obtained as shown in Table 2. The total number of branching operations in the branch and bound method is 171 which is found to be considerably improved with efficiency of search in comparison with the enumeration method having 4096 times of branching operations.

5. DISCUSSION

The alternative tools should be chosen carefuly from various view points such as tool cost, load rate of tools, total number of tools required, and so on. In this paper, the number of required tools is taken as a primary objective and other items are omitted. These omitted items can only be taken into account when there exist plural optimal solutions. In case of the above example, the optimal solution No.1 would be chosen from the view point of load rate of tool c2.

Table 2 Optimal solutions for alternative tool selection

[solution 1]

part	p.tool	a.tool
p1	c3	c1
p2	c5	c2
p5	c3	c4
	c6	c1

p.tool=primary tool
a.tool=alternative tool

tool	n.tool	load
c1	2	2.00
c2	2	1.90
c3	2	1.90
c4	2	1.95
c5	2	2.00
c6	1	1.00

total number of tools = 11

[solution 2]

part	p.tool	a.tool
p1	c3	c1
p2	c2	c5
p3	c5	c2
p5	c3	c4
	c6	c1

n.tool=number of tools

tool	n.tool	load
c1	2	2.00
c2	2	2.00
c3	2	1.90
c4	2	1.95
c5	2	2.00
c6	1	1.00

total number of tools=11

6. CONCLUSION

Tool selection problem in an FMS is discussed and the mathematical approach is proposed to reduce the tooling cost by minimizing the total number of tools through the optimum use of the alternative tools in the machining processes. In the proposed approach, tools are divided into tool subgroups by using graph theory for the purpose of the efficient search of the solutions, and then branch and bound method is applied to the generated subgroups in order to find the optimal use of the alternative tools from the view point of minimization of the total number of tools required. There still remains other subjects to be considered in a tool selection problem such as tool cost, load rate of tools, increase of machining time and so on. In our approach, they can only be considered when there is a need to select reasonable solution from the plural ones.

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