BLDCM의 코깅토크와 역기전력에 대한 스큐 영향의 해석

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Analysis of Skew Effects on Cogging Torque and BEMF for BLDCM

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Abstract

Air gap flux density is obtained strictly from air gap permeance and MMF distribution using the design parameter of BLDCM when the stator slot is considered. Then the theoretical equations representing the cogging torque and BEMF are proposed. The effect of skew is described analytically, and skew effect on air gap permeance and MMF is studied. Consequently we verified analytically that skew of slots and magnets influenced on the reduction of the cogging torque and BEMF ripple, and the shape of BEMF.

I. Key Point of the Work

(1) Exact air gap flux density distribution is obtained from the air gap permeance and MMF.

(2) Energy stored in the air gap is calculated using air gap flux density, and the equation of cogging torque is induced from it.

(3) Flux linkage in the coil of the stator is computed, and BEMF induced in that coil is determined.

(4) The effect of skew is expressed by the function of the space angle, and the effect of slot skew on permeance and that of magnet skew on MMF is analyzed.

(5) The change of air gap flux density is investigated when the slots or magnets are skewed.

(6) Finally the effect of skew on cogging torque and BEMF is discussed.

II. Main Equations and Figures

(1) Let f be any function of θ in case of no skew. Then the function f_s which appears when the slots or magnets are skewed with skew ratio S_k is as follows[8]. $S_k = 1$ means the skew of 1 slot pitch. (Fig.1)

$$f_s(\theta) = \frac{1}{L_i}\int_{-L_i/2}^{L_i/2} f\left(\theta - \frac{2S_k\pi}{Q_s L_i} z\right) dz$$

$$= \frac{Q_s}{2S_k\pi}\int_{-S_k\pi/Q_s}^{S_k\pi/Q_s} f(\theta - \phi)\, d\phi$$

(2) Fourier series of air gap permeance per unit square (P_m) obtained from the relative permeance function w.r.t. 1 slot[5] and Fourier series of permeance with slot skew (P_{ms}) are given by Fig.2 and the following equations.

$$P_m(\theta_s) = \frac{\mu_0}{g}\left[G_0 + \sum_{k=1}^{\infty} G_n \cos(n\theta_s)\right], \qquad n = kQ_s$$

$$P_{ms}(\theta_s) = \frac{\mu_0}{g}\left[G_0 + \sum_{k=1}^{\infty} G_n \frac{\sin(nS_k\pi/Q_s)}{nS_k\pi/Q_s} \cos(n\theta_s)\right]$$

$$G_0 = 1 - \beta s/\tau_s, \qquad G_n = -\frac{2\beta}{k\pi}\frac{1}{1-(k\,s/\tau_s)^2}\sin\left[\frac{k\,s\,\pi}{\tau_s}\right]$$

θ_s: mechanical angle measured from the center of the slot

Q_s: number of the slots

(3) Air gap MMF F_m on the magnet surface is obtained from the flux density on the stator surface of slotless motor where the radially oriented magnet is attached to the rotor surface[2]. And F_{ms} is MMF with magnet skew. (Fig.3,4)

$$F_m(\theta_s) = \sum_{m=0}^{\infty} \frac{g}{\mu_0} K_n \cos(n\theta_s), \qquad n = (2m+1)P$$

K_n means F_{anR} in reference [2].

$$F_{ms}(\theta_s) = \sum_{m=0}^{\infty} \frac{g}{\mu_0} K_n \frac{\sin(nS_k\pi/Q_s)}{nS_k\pi/Q_s}\cos(n\theta_s)$$

(4) Air gap flux density B_m is defined as air gap MMF multiplied by air gap permeance per unit square and expressed by the following equaiton. B_{ms} and B_{mr} are flux densities in case of slot skew and magnet skew respectively. (Fig.5,6)

$$B_m(\theta_s) = P_m(\theta_s)\, F_m(\theta_s)$$

$$= \sum_{m=0}^{\infty} G_0 K_{nm} \cos(n_m\theta_s) + \sum_{m=0}^{\infty}\sum_{k=1}^{\infty} G_{nk}K_{nm}\cos(n_k\theta_s)\cos(n_m\theta_s)$$

$n_k = kQ_s$, $n_m = (2m+1)P$

$$B_{ms}(\theta_s) = \sum_{m=0}^{\infty} G_0 K_{nm} \cos(n_m\theta_s) + \sum_{m=0}^{\infty}\sum_{k=1}^{\infty} G_{nk}K_{nm}\frac{\sin(n_kS_k\pi/Q_s)}{n_kS_k\pi/Q_s}\cos(n_k\theta_s)\cos(n_m\theta_s)$$

$$B_{mr}(\theta_s) = \sum_{m=0}^{\infty} G_0 K_{nm}\frac{\sin(n_mS_k\pi/Q_s)}{n_mS_k\pi/Q_s} \cos(n_m\theta_s) + \sum_{m=0}^{\infty}\sum_{k=1}^{\infty} G_{nk}K_{nm}\frac{\sin(n_mS_k\pi/Q_s)}{n_mS_k\pi/Q_s}\cos(n_k\theta_s)\cos(n_m\theta_s)$$

(5) Cogging torque in the motor is the rate of change of energy stored in the air gap to the rotor angular position when the stator circuit is opened, and it is obtained from the flux density considering the rotation of the rotor. When the slots or magnets are skewed, energy W_s and W_r are obtained considering Fig.7,8 respectively. In conclusion they are led to the same result. Therefore cogging torques in both case are same and are expressed by T_s and Fig.9.

$$W = \int_0^{2\pi}\int_{-L_i/2}^{L_i/2}\int_{r_m}^{r_m+g} \frac{1}{2\mu_0} B_m^2(\theta_s - \theta_r)\; r\, dr\, dz\, d\theta_s$$

$$T = \partial W / \partial\theta_r$$

$$= -\sum n_k\pi\, C_w\, G_0G_{nk}K_{nm}K_{nl}\sin(n_k\theta_r)^{*} - \sum (n\pi/4)\, C_w\, G_{nj}G_{nk}K_{nm}K_{nl}\sin(n\theta_r)^{**}$$

$$C_w = (1/2\mu_0)\, g\, L_i\, (r_m + g/2)$$

$n_m = (2m+1)P$, $n_l = (2l+1)P$, $n_k = kQ_s$, $n_j = jQ_s$

Above summation contains the terms which satisfy the following.

$$* : n_m + n_l = n_k, \quad |n_m - n_l| = n_k$$

$$** : n_m + n_l = n_j + n_k = n, \quad n_m + n_l = |n_j - n_k| = n, \quad |n_m - n_l| = n_j + n_k = n, \quad |n_m - n_l| = |n_j - n_k| = n$$

$$W_s = \int_{r_m}^{r_m+g}\int_{-L_i/2}^{L_i/2}\int_{\phi}^{2\pi+\phi} \frac{1}{2\mu_0} (P_m(\theta_s - \phi)\ F_m(\theta_s - \theta_r))^2\ r\ d\theta_s\ dz\ dr$$

$$= C_w \frac{Q_s}{2S_k\pi} \int_{-S_k\pi/Q_s}^{S_k\pi/Q_s} \int_{\phi}^{2\pi+\phi} (P_m(\theta_s - \phi)\ F_m(\theta_s - \theta_r))^2\ d\theta_s\ d\phi$$

$$W_r = \int_{r_m}^{r_m+g}\int_{-L_i/2}^{L_i/2}\int_{0}^{2\pi} \frac{1}{2\mu_0}(P_m(\theta_s)\ F_m(\theta_s - \phi - \theta_r))^2\ r\ d\theta_s\ dz\ dr$$

$$= C_w \frac{Q_s}{2S_k\pi} \int_{-S_k\pi/Q_s}^{S_k\pi/Q_s}\int_{0}^{2\pi} (P_m(\theta_s)\ F_m(\theta_s - \phi - \theta_r))^2\ d\theta_s\ d\phi$$

$$T_s = -\Sigma\ n_k\pi C_w\ G_0G_{nk}K_{nm}K_{nl} \frac{\sin(n_kS_k\pi/Q_s)}{n_kS_k\pi/Q_s} \sin(n_k\theta_r)^* - \Sigma\ (n\pi/4)C_w\ G_{nj}G_{nk}K_{nm}K_{nl} \frac{\sin(nS_k\pi/Q_s)}{nS_k\pi/Q_s} \sin(n\theta_r)^{**}$$

(6) BEMF e is the time rate of the change of flux linkage λ of the coil. Considering the coil distribution of the tested motor(Fig.10), it is as follows. Skewed slots and magnets have the same effect on BEMF as in the case of cogging torque. (Fig.11)

$$\lambda = N\ P\ r_s\ L_i \left[\int_{0}^{7\pi/18} B_m(\theta_s - \theta_r)\ d\theta_s + \int_{\pi/18}^{\pi/3} B_m(\theta_s - \theta_r)\ d\theta_s\right]$$

$$e = d\lambda / dt = \omega\ (d\lambda / d\theta_r)$$

$$= -C_e\left[\sum_{m=0}^{\infty} 4G_0K_{nm} \sin\left(\frac{n_m\pi}{6}\right) \cos\left(\frac{n_m\pi}{36}\right) \sin(n_m\theta_r)\right.$$

$$+ \sum_{n_k=n_m} G_{nk}K_{nm}\left[\sin\left(\frac{n_m\pi}{3}\right) \cos\left(\frac{n_m\pi}{18}\right) \sin(n_m\theta_r) + \frac{n_m\pi}{3} \sin(n_m\theta_r)\right]$$

$$\left.+ \sum_{n_k\neq n_m} \frac{2n_m\ G_{nk}K_{nm}}{n} \sin\left(\frac{n\pi}{6}\right) \cos\left(\frac{n\pi}{36}\right) \sin(n_m\theta_r)\right]$$

N : Number of turns in the winding per pole and phase

ω : rotor angular velocity

$C_e = \omega\ N\ P\ r_s\ L_i \qquad n = n_k \pm n_m$

$$\lambda_s = NPr_s\left[\int_{-L_i/2}^{L_i/2}\int_{\phi}^{7\pi/18+\phi} P_m(\theta_s - \phi)\ F_{mt}(\theta_s - \theta_r)\ d\theta_s\ dz\right.$$

$$\left.+ \int_{-L_i/2}^{L_i/2}\int_{\pi/18+\phi}^{\pi/3+\phi} P_m(\theta_s - \phi)\ F_{mt}(\theta_s - \theta_r)\ d\theta_s\ dz\right]$$

$$\lambda_r = NPr_s\left[\int_{-L_i/2}^{L_i/2}\int_{0}^{7\pi/18} P_m(\theta_s)\ F_m(\theta_s - \phi - \theta_r)\ d\theta_s\ dz\right.$$

$$\left.+ \int_{-L_i/2}^{L_i/2}\int_{\pi/18}^{\pi/3} P_m(\theta_s)\ F_m(\theta_s - \phi - \theta_r)\ d\theta_s\ dz\right]$$

$$e_s = -C_e\left[\sum_{m=0}^{\infty} 4G_0K_{nm} \sin\left(\frac{n_m\pi}{6}\right)\cos\left(\frac{n_m\pi}{36}\right) \frac{\sin(n_mS_k\pi/Q_s)}{n_mS_k\pi/Q_s} \sin(n_m\theta_r)\right.$$

$$+ \sum_{n_k=n_m} G_{nk}K_{nm}\left[\sin\left(\frac{n_m\pi}{3}\right)\cos\left(\frac{n_m\pi}{18}\right) \frac{\sin(n_mS_k\pi/Q_s)}{n_mS_k\pi/Q_s} \sin(n_m\theta_r) + \frac{n_m\pi\sin(n_mS_k\pi/Q_s)}{3n_mS_k\pi/Q_s} \sin(n_m\theta_r)\right]$$

$$+ \sum_{n_k \neq n_m} \frac{2 n_m G_{nk} K_{nm}}{n} \sin\left(\frac{n\pi}{6}\right) \cos\left(\frac{n\pi}{36}\right) \frac{\sin(n_m S_k \pi / Q_s)}{n_m S_k \pi / Q_s} \sin(n_m \theta_r) \Bigg)$$

V. Conclusion

In this paper we researched the change of air gap permeance and MMF in case of slot skew and magnet skew respectively. And it was proved more precisely than the previous papers with the accurate air gap flux density for a basis that cogging torque is the sum of the space harmonics which are caused by the slot. And we also verified analytically that the slot or magnet skewing affects the reduction of cogging torque and BEMF ripple and the formation of BEMF waveform in the same degree.

Appendix

List of physical values of tested motor

$P = 3$; $B_r = 0.98$; $r_r = 0.0495$; $\tau_s = 0.0097$; $\alpha = 57.88$ [Deg];

$Q_s = 36$; $\chi_m = 0.03$; $r_m = 0.0545$; $L_s = 0.0037$; $\omega = 3000$ [rpm];

$N = 23$; $g = 0.001$; $r_s = 0.0555$; $L_i = 0.0305$;

where SI system of units is used except for α, ω

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[3] H. R. Bolton et al., "Investigation into a Class of Brushless DC Motor with quasisquare voltages and currents, "IEE Proceedings, VOL. 133, Pt. B, NO. 2, March 1986

[4] Nady Boules and Jaime De La Ree, "Torque Production in Permanent - Magnet Synchronous Motors, "IEEE Conference on Industry Applications Systems, 1987

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[8] 大川光吉, 永久磁石回轉機 特性と設計, 總合電子出版社, pp.146 - 149

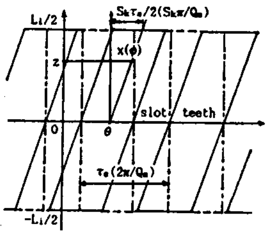

Fig.1 Distribution of skewed slots

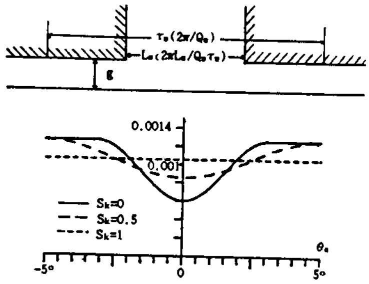

Fig.2 The stator slot and air gap permeance

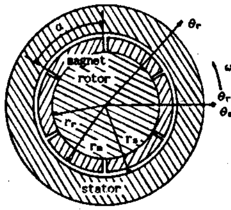

Fig.3 Arrangement and dimension of the stator, rotor, and magnet of slotless motor

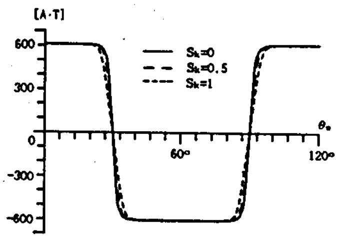

Fig.4 Air gap MMF

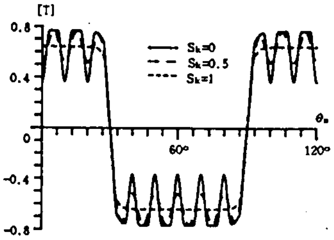

Fig.5 Air gap flux density (slot skew)

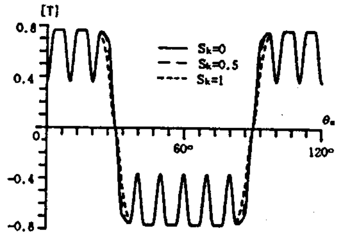

Fig.6 Air gap flux density (magnet skew)

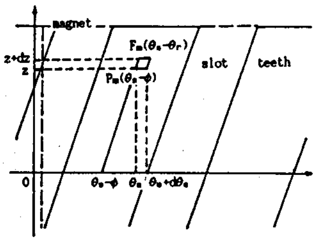

Fig.7 Distribution of slots and magnets in case of slot skewing

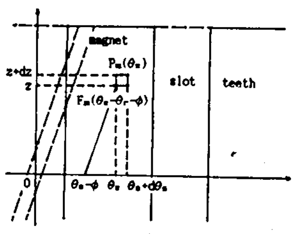

Fig.8 Distribution of slots and magnets in case of magnet skewing

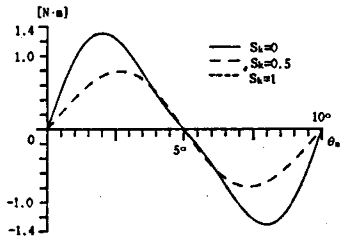

Fig.9 Cogging torque

Fig.10 Coil distribution of the tested motor

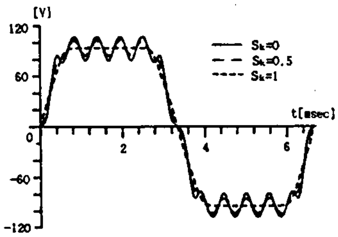

Fig.11 BEMF waveform