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IDENTIFICATION OF MODAL PARAMETERS BY SEQUENTIAL PREDICTION ERROR METHOD

이 창근 윤 정방
Lee, Chang-Guen* Yun, Chung-Bang**

ABSTRACT

The modal parameter estimations of linear multi-degree-of-freedom structural dynamic systems are carried out in time domain. For this purpose, the equation of motion is transformed into the auto-regressive and moving average model with auxiliary stochastic input (ARMAX) model. The parameters of the ARMAX model are estimated by using the sequential prediction error method. Then, the modal parameters of the system are obtained thereafter. Experimental results are given for a 3-story building model subject to ground exitations.

1. INTRODUCTION

In this paper, a method for the time domain identification of the modal parameters of a linear multi-degree-of-freedom structural dynamic system are studied. For the time domain identification, it is very important to obtain a reasonable model for the sampled data system, since the observation data are commonly measured at discrete time instances and contaminated by the measurement noises [1,2]. The auto-regressive and moving average (ARMAX) model is used for this purpose.

Moreover, in actual experiments, the response of a system may be measured in terms of displacement, velocity or acceleration depending on sensor types. Hence, different discrete time models depending on the measured response components are derived. Then, the sequential prediction error method is applied to the model to estimate its parameters. The modal parameters are obtained thereafter. Experimental results are given for a 3-story building model. It has been found that the present method utilizing the ARMAX model and the sequential prediction error method yields good estimates of the modal parameters.

* Graduate Research Assistant,
Department of Civil Engineering, KAIST

** Professor,
Department of Civil Engineering, KAIST

2. MATHEMATICAL MODEL

2.1 Experimental modal analysis

Consider a structural system that can be modeled as an n-degrees of freedom linear system. Then, the dynamics of the system can be described by the equation of motion as

$$M_0\ddot{\xi}(t) + C_0\dot{\xi}(t) + K_0\xi(t) = L_0 u(t) \quad (1)$$

where $\xi(t)$, $\dot{\xi}(t)$ and $\ddot{\xi}(t)$ are n-dimensional response vectors; $u(t)$ is an m-dimensional input vector; M_0, C_0 and K_0 are mass, damping and stiffness matrices, respectively; L_0 is the $n \times m$ input coefficient matrix; and $(\cdot)$ denotes differentiation with respect to time.

In actual experiments, the responses can be measured in terms of acceleration, velocity or displacement depending on the sensor type. Therefore, it is necessary to give a different expression of the system by using the measured response component. Also , in general, the responses cannot be measured at all the nodes of the equation of motion. Hence, it is analytically convenient that the coupled equation of motion is transformed into a set of uncoupled equations by using the modal decomposition. After taking the Laplace transform, the transfer functions corresponding to the response components can be written as [3]

for $d(t) = \xi(t)$;

$$D(s) = \sum_{i=1}^{n} \left[\frac{\psi_i \psi_i^T}{s - \lambda_i} + \frac{\psi_i^* \psi_i^{*T}}{s - \lambda_i^*} \right] L_0 U(s) \quad (2)$$

for $v(t) = \dot{\xi}(t)$;

$$V(s) = \sum_{i=1}^{n} [\frac{\lambda_i \psi_i \psi_i^T}{s - \lambda_i} + \frac{\lambda_i \psi_i^* \psi_i^{*T}}{s - \lambda_i^*}] L_0 U(s) \quad (3)$$

for $a(t) = \ddot{\xi}(t)$;

$$A(s) = \sum_{i=1}^{n} [\frac{\lambda_i^2 \psi_i \psi_i^T}{s - \lambda_i} + \frac{\lambda_i^{*2} \psi_i^* \psi_i^{*T}}{s - \lambda_i^*}] L_0 U(s) + M_0^{-1} L_0 U(s) \quad (4)$$

where $D(s)$, $V(s)$, $A(s)$ and $U(s)$ are the Laplace transforms of $d(t)$, $v(t)$, $a(t)$ and $u(t)$, respectively and ψ_i is the complex mode shape which is purely imaginary when the damping matrix is proportional; and λ_i is the system pole which is related to the natural frequency and damping ratio associated with the mode shape; and asterisk (*) denotes the complex conjugate.

In this study, the response of the system is assumed to be measured in terms of acceleration. Under the assumption of zero-order hold reconstruction [4], the discrete time equations corresponding to Eq. 4 can be simply obtained as

$$a(k) = \sum_{i=1}^{n} [\frac{\gamma_i \psi_i \psi_i^T}{z - \mu_i} + \frac{\gamma_i^* \psi_i^* \psi_i^{*T}}{z - \mu_i^*}] L_0 u(k) + M_0^{-1} L_0 u(k) \quad (5)$$

where z can be regarded as one time ahead operator for the present purpose and

$$\mu_i = \exp(\lambda_i \Delta t), \quad \gamma_i = (\mu_i - 1)\lambda_i \quad (5.a)$$

Reducing the fractions to a common denominator, Eq. 5 can be rewritten as

$$a(k) = \frac{Q(z)}{p(z)} u(k) + M_0^{-1} L_0 u(k) \quad (6)$$

where

$$p(z) = (z - \mu_1)(z - \mu_1^*)(z - \mu_2) \cdots (z - \mu_n^*)$$
$$= z^{2n} + p_1 z^{2n-1} + p_2 z^{2n-2} + \cdots + p_{2n} \quad (6.a)$$

$$Q(z) = \sum_{i=1}^{n} \left[\frac{\gamma_i p(z) \psi_i \psi_i^T}{z - \mu_i} + \frac{\gamma_i^* p(z) \psi_i^* \psi_i^{*T}}{z - \mu_{i*}} \right] L_0$$
$$= Q_1 z^{2n-1} + Q_2 z^{2n-2} + \cdots + Q_{2n} \quad (6.b)$$

It is apparent that the polynomial $p(z)$ and the polynomial matrix $Q(z)$ have real coefficients. It is noted from Eq. 6 that the coupled equation of motion is decomposed into a set of ARMA models for the individual output.

From the estimates of $p(z)$ and $Q(z)$, the modal parameters can be obtained as described below :

The system poles μ_i's may be obtained from

$$p(z) |_{z=\mu_i} = 0 \quad (7)$$

Then, from Eq. 5.a,

$$\lambda_i = \frac{1}{\Delta t} \ln(\mu_i), \quad \gamma_i = (\mu_i - 1)\lambda_i \quad (8)$$

where the value of $\ln(\mu_i)$ are chosen as its principal values.

It is well known that λ_i is related to the natural frequency ω_i and damping ratio ζ_i by

$$\lambda_i = -\zeta_i \omega_i + j \omega_i \sqrt{1 - \zeta_i^2} \quad (9)$$

where $j = \sqrt{-1}$. Hence, ω_i and ζ_i are given as

$$\omega_i = |\lambda_i|, \quad \zeta_i = -\frac{\mathrm{Re}(\lambda_i)}{|\lambda_i|} \quad (10)$$

Finally, it can be easily shown from Eq. 6.b that the mode shape can be obtained as

$$\psi_i \psi_i^T L_0 = \frac{(z - \mu_i) Q(z)}{\gamma_i p(z)} \bigg|_{z=\mu_i} \quad (11)$$

It is noted that for a multi-input case the resultant matrix from Eq. 11 consists of m-column vectors which are proportional to the i-th mode shape but with different proportionality constants, where m is the number of inputs.

2.2 Earthquake loading

When the input force is given by ground acceleration the input term can be written as

$$L_0 u(t) = -M_0 \{ 1 \} \ddot{u}_g(t) \quad (12)$$

where $\ddot{u}_g(t)$ is the base acceleration and $\{1\}$ is a column vector of which the elements consist of unities.

Hence, the corresponding discrete time equation for acceleration measurement can be written as

$$a(k) = \frac{Q(z)}{p(z)} \ddot{u}_g(k) - \{ 1 \} \ddot{u}_g(k) \quad (13)$$

In Eq. 13, $a(k)$ denotes the floor acceleration relative to the base defined by

$$a(k) = a'(k) - \{1\} \ddot{u}_g(k) \quad (14)$$

where $a'(k)$ is the absolute acceleration. Therefore, Eq. 13 can be rewritten as

$$a'(k) = \frac{Q(z)}{p(z)} \ddot{u}_g(k) \quad (15)$$

It is interesting to note from Eq. 15 that for a structure subjected to a ground acceleration, the ARMA model can be constructed in terms of the absolute acceleration responses, but without the concurrent input term.

2.3 Noise consideration (ARMAX Model)

Defining $y(k)$ and $u(k)$ as the measured acceleration at a node and the measured ground acceleration of Eq. 15, they can be written as

$$y(k) = a_j^t(k) + \eta(k) \qquad (16)$$

$$u(k) = \ddot{u}_g(k) + \omega(k) \qquad (17)$$

where $a_j^t(k)$ denotes the total acceleration at *jth* node and $\eta(k)$ and $\omega(k)$ are measurement noises which are generally assumed as white processes with finite variances.

Based on the Kalman's innovations theory [5], Eq. 15 can be asymtotically represented by using the measured data $y(k)$ and $u(k)$ and innovations sequences $e(k)$ as

$$p(z)y(k) = q(z)u(k) + c(z)e(k) \qquad (18)$$

where $q(z)$ is a polynomial related to the measured point of $n\times 1$ polynomial matrix $Q(z)$ and has the following form :

$$q(z) = q_1 z^{2n-1} + q_2 z^{2n-2} + \cdots + q_{2n} \qquad (18.a)$$

and $c(z)$ is a polynomial to describe the innovations process and can be written as

$$c(z) = z^{2n} + c_1 z^{2n-1} + \cdots + c_{2n} \qquad (18.b)$$

3. SEQUENTIAL PREDICTION ERROR METHOD

Eq. 18 can be equivalently rewirtten as

$$y(k) = \phi^T(k-1)\theta + e(k) \qquad (19)$$

where

$$\phi^T(k-1) = < y(k-1), y(k-2), \cdots, y(k-2n), u(k-1), u(k-2), \cdots, u(k-2n), e(k-1), e(k-2), \cdots, e(k-2n) > \qquad (19.a)$$

$$\theta^T = < -p_1, -p_2, \cdots, -p_{2n}, q_1, q_2, \cdots, q_{2n}, c_1, c_2, \cdots, c_{2n} > \qquad (19.b)$$

It may be impossible in the stochastic situation to obtain the value of θ for which the prediction error $e(k,\theta)$ is equal to zero at every instances of time. Instead, one can only estimates the value of θ which minimizes the prediction errors. The following criterion is generally taken as the mesure of the quality of approximation :

$$V_N(\theta) = \frac{1}{N}\sum_{i=1}^{N} e^2(k,\theta) \qquad (20)$$

where N is the number of data points. It is not possible to minimize the criterion with respect to θ by analytical means, since $e(k,\theta)$ is depent on θ implicitly as well as explicitly. The numerical search procedure can be approximately implemented in the sequential form. The algorithm of the sequential prediction error method is summarized as follows [6,7] :

$$\hat{\theta}(k) = \hat{\theta}(k-1) + F(k-1)\phi_f(k-1)(y(k) - \hat{\phi}(k-1)\hat{\theta}(k-1)) \qquad (21)$$

$$F^{-1}(k) = F^{-1}(k-1) + \phi_f(k-1)\phi_f^T(k-1) \qquad (22)$$

$$\phi_f(k-1) = \hat{c}^{-1}(z, k-1)\hat{\phi}(k-1) \qquad (23)$$

where

$$\hat{\phi}^T(k-1) = < y(k-1), y(k-2), \cdots, y(k-2n), u(k-1), u(k-2), \cdots, u(k-2n), \hat{e}(k-1), \hat{e}(k-2), \cdots, \hat{e}(k-2n) > \qquad (24)$$

and

$$\hat{e}(k-1) = y(k-1) - \hat{\phi}(k-2)\hat{\theta}(k-1) \qquad (24.a)$$

As shown in Eq. 23, the gradient vector $\phi_f(k-1)$ is obtained by passing the regression vector $\hat{\phi}(k-1)$, through a time varying filter $\hat{c}^{-1}(z, k-1)$ which is estimated at time $k-1$. In order to keep away from the divergence problem, the projection technique can be employed to make the filter remain in the stability region.

4. NUMERICAL EXAMPLES AND DISCUSSIONS

The present estimation method is applied to a three-story building model to evaluate the modal parameters. As shown in Fig. 1, the input to the model is given by the base acceleration. The accelerations are measured at the base and three floors. The measurement records of two different experiments are shown in Fig. 2 and Fig. 3. One experiment is for the case with a long duration input and the other is for an impulse. For the present purpose, the structural model is approximately regarded as a system with three degrees of freedom.

Estimations of the modal parameters are carried out for three different sets of experiments. In Table 1, the results are compared with those obtained by the structural analysis, in which the structure used in the experiment is idealized as a shear building model. The natural frequencies and the damping ratios are evaluated redundantly from the acceleration records at each floor. In this study, the natural frequencies and damping ratios are obtained by taking the average of the estimates. The estimated natural frequencies and the first mode shapes from three different experiments are found to be very consistent with each other. As for the second and third mode shapes, however, a little discrepancies can be observed between the results from different experiments. It has been found that the estimated values for the damping ratios are not quite consistent. It may be caused by the fact that the damping values are so small (i.e. less than 0.01) that the properties of damping cannot be observed well. Compared with the estimated natural frequencies, those calculated by the eigenvalue analysis on the shear building model are found to be slightly large. It may be caused by the fact that the fixity conditions at the ends of the vertical members might be idealized more rigidly in the shear building model than the realities of the actual model used in the experiment. The responses are re-evaluated based on the estimated parameters and compared with the measured records in Figs. 4 and 5. It can be seen that the formers are almost identical to the latters, except that the measurement noises are filtered out.

5. CONCLUSIONS

In this paper, methods for the estimation of the modal parameters of linear multi-degrees-of-freedom structural dynamic systems are presented. The following is the summary of this study:

(1) The auto-regressive and moving-average models with the auxiliary stochastic input (ARMAX model) are derived for three different types of the measured response components; i.e. displacement, velocity and acceleration.

(2) Estimated modal parameters of a building model from different sets of experimental data by the present method are found to be very consistent, particularly for the natural frequencies and the first two mode shapes. The estimated responses based on the identified parameters show good agreements with the measurements.

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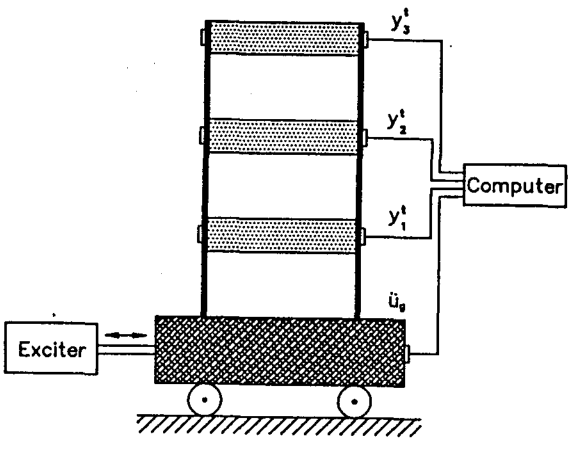

Fig. 1. Experimental Set-up of a Building Model

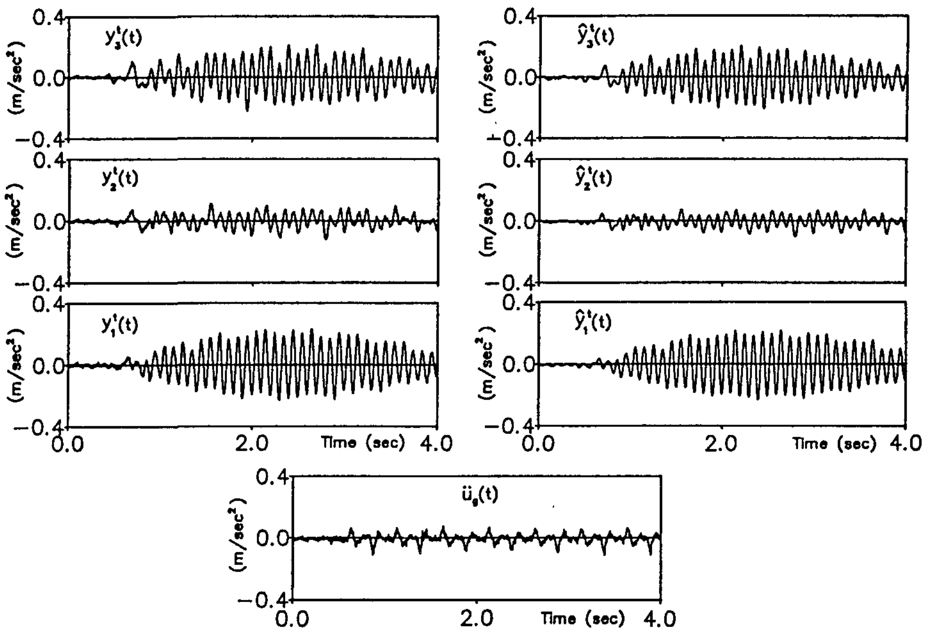

Fig. 2. Time Histories of Measured and Estimated Acceleration Responses (Experiment 1)

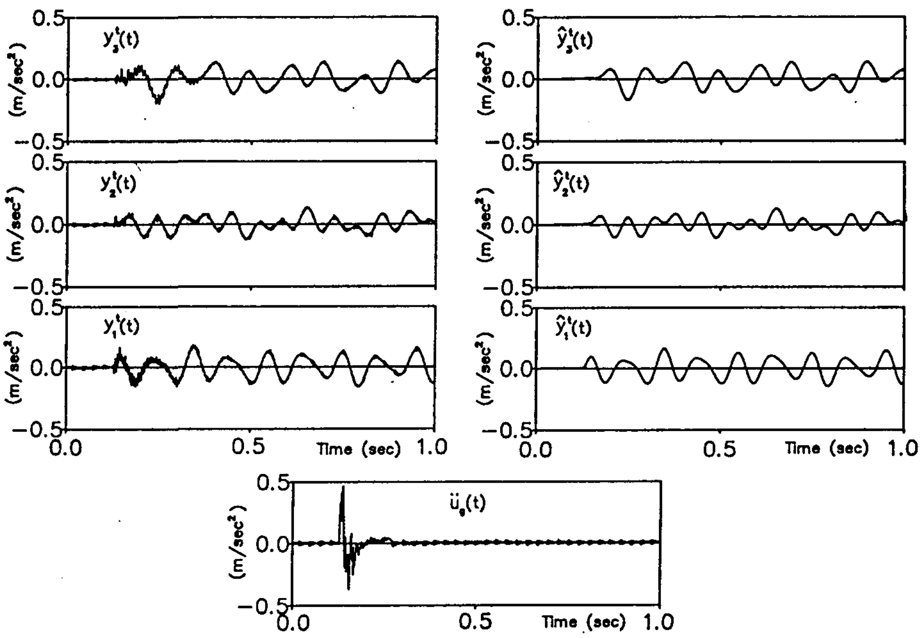

Fig. 3. Time Histories of Measured and Estimated Acceleration Responses (Experiment 2)

Table 1. Estimated Modal Parameters

Modal properties \ Cases		Experiment 1		Experiment 2		Experiment 3		Structural Analysis	
ω_1 , ζ_1		22.6 ,	.0037	22.9 ,	.0076	22.8 ,	.0070	25.5 ,	-
ω_2 , ζ_2		63.9 ,	.0015	62.8 ,	.0013	62.8 ,	.0030	71.1 ,	-
ω_3 , ζ_3		94.4 ,	.0075	92.9 ,	.0032	92.8 ,	.0029	103.6 ,	-
ψ_1	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	.809	(2)	.790	(-1)	.816	(-1)	.802	(0)
	1F	.440	(-1)	.444	(-1)	.447	(-1)	.445	(0)
ψ_2	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	.307	(175)	.400	(175)	.556	(175)	.554	(180)
	1F	1.106	(177)	1.152	(178)	1.103	(176)	1.250	(180)
ψ_3	3F	1.	(0)	1.	(0)	1.	(0)	1.	(0)
	2F	3.307	(-177)	2.226	(-174)	1.824	(-168)	2.242	(180)
	1F	1.641	(-8)	1.481	(4)	1.402	(-6)	1.801	(0)
Input Types		Long duration		Impulsive		Impulsive			

Note : 1. The unit of ω_i is in rad/sec.

2. Values in parentheses denote the phase angles of the complex mode shapes in degrees.