DERIVATION OF UNCERTAINTY IMPORTANCE MEASURE AND ITS APPLICATION

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ABSTRACT

The uncertainty quantification process in probabilistic Risk Assessment usually involves a specification of the uncertainty in the input data and the propagation of this uncertainty to the final risk results. The distributional sensitivity analysis is to study the impact of the various assumptions made during the quantification of input parameter uncertainties on the final output uncertainty. The uncertainty importance of input parameters, in this case, should reflect the degree of changes in the whole output distribution and not just in a point estimate value. A measure of the uncertainty importance is proposed in the present paper. The measure is called the distributional sensitivity measure (DSM) and explicitly derived from the definition of the Kullback's discrimination information. The DSM is applied to three typical cases of input distributional changes: 1) Uncertainty is completely eliminated, 2) Uncertainty range is increased by a factor of 10, and 3) Type of distribution is changed. For all three cases of application, the DSM-based importance ranking agrees very well with the observed changes of output distribution while other statistical parameters are shown to be insensitive.

1. INTRODUCTION

In Probabilistic Risk Assessment (PRA), the uncertainties associated with the traditional results (e.g., core melt frequency, and various health risk indices) are as important as the point estimate values. The uncertainty quantification process in PRA usually involves a specification of the uncertainty in the input data and the propagation of this uncertainty to the final risk results. It is also often the case that subjective assessment is used to specify the uncertainties. Thus, it is expected that the output distributions change depending on the different assumptions made in the quantification of input uncertainties. The degree of changes, on the other hand, may depend on the importance of each input parameter.

A study analyzing the variation of output distribution can be called the distributional sensitivity analysis in analogy to the traditional point estimate sensitivity analysis. The importance of the input parameters, in this case, should reflect the degree of changes in the whole output

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distribution and not just in a point estimate value, as there could be a case where the shape of the output distribution is changed without much changes in the uncertainty ranges or in the point estimate value.

The present paper proposes a measure for assessing the impact of different assumptions used in the input uncertainty quantification on the output uncertainties. The measure is called the distributional sensitivity measure (DSM) and is derived from the definition of Kullback's discrimination information [1].

In Section 2, the distributional sensitivity analysis is briefly introduced. In Section 3, the distributional sensitivity measure is derived from Kullback's definition for a situation that the output distribution is assumed to be a two-parameter Weibull distribution. This measure is applied to a specific distributional sensitivity analysis and the results are discussed in Section 4. Finally conclusions are given in Section 5.

2. DISTRIBUTIONAL SENSITIVITY ANALYSIS

The distributional sensitivity analysis studies the impact on the output distributions of different assumptions made during the quantification of input parameter distributions. It is different from the traditional sensitivity analysis where the quantity of interest is the change in the point estimate value.

A typical example of point estimate sensitivity analysis is the evaluation of the partial derivative of the output variable with respect to one of the input variables [2]. The distributional sensitivity analysis, on the other hand, is defined in terms of the changes in the probability density function (PDF) of the output variable. The changes in the output PDF may be caused by various assumptions made during the input uncertainty quantification. These assumptions include the types of input distributions or the uncertainty ranges of input parameters. Moreover, for certain physical phenomena with which the current understanding is very limited, subjective judgment on the input uncertainty by various experts may be very much different from each other and the aggregate distribution could become multimodal. Since the choice of one distribution over the other may lead to conflicting results, the distributional sensitivity analysis may, in such cases, be useful for studying the impact of different choices.

Since the analytical propagation of input uncertainties is practically impossible, the quantification of output PDF is usually performed through a Monte Carlo type computer simulation. Within such a computer simulation framework, the distributional sensitivity analysis can be performed through the following steps:

Step 1. Quantification of an initial set of input distributions.

Step 2. Propagation of input samples through the computer model to estimate the output distribution.
Step 3. Requantification of input distributions.
Step 4. Repetition of Step 2 to estimate output distribution based on the requantified input uncertainties.
Step 5. Repetition of Steps 3 and 4 for other input assumptions.
Step 6. Finally, analysis of all output distributions thus obtained.

Steps 1 and 3 above involve selection of the measure of the uncertainty and its quantification. The measure of the uncertainty in the current paper is the probability density function (PDF). There are several approaches to the PDF quantification. For example, if enough statistical samples are available, either classical or Bayesian approach can be used to estimate the PDF. On the other hand, in case of rare events, the Bayesian method with expert opinion as a prior might be the preferred approach.

The uncertainties of input variables or parameters quantified in the previous step are propagated (Steps 2 and 4 above) through physical or logical models utilized by the computer code. Various methods for the propagation of input uncertainties are available and a partial list of those methods includes: analytical method, discrete probability distribution (DPD) method, moments method, and Monte Carlo method. One way or another, these methods have their own advantages and disadvantages.

The analytical approach to the propagation of input uncertainties is unlikely to find extensive use in any uncertainty studies due to its analytical complexity. The use of DPD method in the propagation step needs a considerable care in combining the distributions; otherwise, the number of estimated points in the propagated distributions can become unmanageable. The method using moments are limited to quantities that can be expressed as relatively simple models (e.g., first order Taylor Series). For complex propagation models like in the Level 2 or 3 PRA studies, this method has the potential to become unmanageable too. Monte Carlo techniques appear to offer effective ways to propagate distributions through either physical or logical models provided the simulation costs do not become excessive. Latin Hypercube Sampling (LHS) rather than random sampling can be used as a possible way to improve the efficiency of such propagation [3].

Typical outputs from these methods include estimated distribution functions of output variables and corresponding statistical parameters. However, these statistical characteristics of the output distributions depend upon various assumptions made during the quantification of input distributions. These assumptions may include the type of the input distributions, estimated uncertainty ranges expressed in terms of percentiles or error factors, and the subjective judgement on the current understanding of various physical phenomena involved. Therefore, in order to study the robustness of the conclusion based upon one assumption over the other, it is necessary to repeat the steps for the quantification of input uncertainties and propagation of these uncertainties for different assumptions (Step 5). Step 6 above involves definition of a proper measure of distributional changes in the output variables and its calculation.

In general, the sensitivity measure can be defined as:

$$S = \Delta PDF\,(Y_j,\, \beta_j^l \,:\, X_i,\, \alpha_i^k)$$

In other words, the sensitivity measure S is defined as the perturbation of type β on the PDF of output variable Yj due to the perturbation of α on the PDF of input variable Xi. The current paper proposes a collective measure of various types of the output distributional perturbation. The proposed collective measure estimates the closeness of one output PDF based on one set of input distribution assumptions to the other, and then compares the magnitude of this closeness to identify the importance of the input assumption.

3. DISTRIBUTIONAL SENSITIVITY MEASURE

There are a number of possible measures for closeness between two probability density functions. The present paper proposes to use, as a measure of probabilistic distance, the Kullback-Leibler discriminator, I(1:2). Given two probability distributions, f1(x) and f2(x), I(1:2) is defined as [1]:

$$I\,(1:2) = \int f_1(x)\,\ln\left[\frac{f_1(x)}{f_2(x)}\right] dx$$

I(1:2) can be interpreted as the mean information for discrimination in favor of one hypothesis against the other. In other words, I(1:2) is the expected difference in the information conveyed by two distributions f1(x) and f2(x). Assuming that f1(x) is an output distribution based on the initial set of input distributions reflecting one analyst's state of knowledge, f2(x) may be another output distribution from some other part of the spectrum of analyst. The magnitude of I(1:2) is then an index of the robustness of conclusions based on f1(x) against any changes in the assessment of the state of knowledge reflected in the other output distribution f2(x).

The approach taken by the current paper for the calculation of Eq.(2) is the use of two analytical output PDFs derived by approximation of the simulated distributions through a least-squares curve fitting. The output distributions usually cover a wide range of parameter space and may have a variety of distributional shapes. Thus, a suitable choice of analytical distribution to fit the simulated output distributions must be flexible and versatile. The two-parameter Weibull distribution satisfies these requirements and can be completely quantified by specifying the shape factor and the scale factor. These two parameters are defined in the cumulative Weibull distribution as follows:

$$F(x) = 1 - \mathrm{EXP}\left[-\left(\frac{x}{\lambda}\right)^{\beta}\right]$$

Where is the Weibull scacle factor, and β the Weibull shape factor.

With the assumption of Weibull distribution, Kullback's definition of the discrimination information can be analytically integrated to give the measure of distributional sensitivity proposed in the current paper:

$$DSM(o:i) = -1 + \ln\left(\frac{\beta_o}{\beta_i}\right) + \left(\frac{\beta_i}{\beta_o} - 1\right) C$$

$$-\beta_i \ln\left(\frac{\lambda_o}{\lambda_i}\right) + \left(\frac{\lambda_o}{\lambda_i}\right)^{\beta_i} \Gamma\left(\frac{\beta_o + \beta_i}{\beta_o}\right)$$

Where C is a constant equal to 0.577216, and the subscript o refers to the base case and i to the sensitivity case in which i-th input distribution is changed. (x) denotes a gamma function.

Figure 1 shows the variation of the distributional sensitivity measure (DSM) due to the variations of two ratios, and $\beta i/\beta o$. If both ratios approach unity, the two distributions become identical and DSM goes to zero. When each ratio moves away from 1, DSM becomes larger. A large DSM thus reflects the situation that two distributions are different in their respective shapes as well as in their ranges of variation.

4. APPLICATIONS

In the previous section, the DSM has been derived explicitly for the Weibull distributions. The DSM is a collective measure of the changes of the output distributions introduced by a distributional change in the input parameter. Applications are made for three typical cases of input distributional changes. For each selected input parameter;

Case 1 : Uncertainty is completely eliminated.
Case 2 : Uncertainty range is increased by a factor of 10.
Case 3 : Type of distribution is changed.

The first two cases are analogous to the risk reduction and risk increase measures in the point estimate importance analysis introduced by Vesely [2]. In Case 1, a complete knowledge of the particular input parameter is assumed by replacing the probability distribution with its nominal value. Case 2 reflects the other extreme case where not much is known about the input, hence a wide distribution is assigned. In Case 3, the impact of the changes in the input distribution type is studied. For each of the above three cases, a distributional sensitivity analysis is performed. The DSM is then calculated and input parameters are ranked accordingly. A personal computer-based fast running code, SMART, is selected for the present applications [4]. SMART is a simplified nuclear accident consequence calculation code. It was benchmarked against more sophisticated codes and showed excellent agreement. Total of 21 input variables are selected among which 13 variables are source term parameters and the remainder consequence parameters. The base case input distribution types and ranges are shown in Table 1 along with a brief description of each input variable. The lower and upper bounds of each range are calculated by dividing or multiplying the nominal value by a factor of 3, respectively.

The output variable under study is the probability of one person being killed at the 2 miles from the nuclear power plant, should an accident occur with the specified source term characteristics (input variables 1 through 13) and weather and site-specific conditions (input variables 14 through 21).

The Latin Hypercube Sampling (LHS) Code is used for estimating output distributions. The number of samples is 1000. The output distributions are fitted to the two-parameter Weibull distribution. The distributional sensitivity measures(DSMs) are calculated using Eq.(4) and the uncertainty importance of each input parameter is ranked accordingly. The results are discussed in more detail in the following paragraphs.

Case 1: Zero Input Uncertainty

This case is in analogy to the risk reduction measure introduced by Vesely. For each input, the uncertainty is assumed to be zero and the input value remains at the nominal value throughout the calculations. The DSM for the zero input uncertainty is different from the risk reduction measure. While the latter assumes zero point estimate value, the former assumes zero uncertainty and not zero point estimate value. Instead, it takes the nominal value.

The calculated DSM for each sensitivity case is the probabilistic distance between the base case and the changed output distribution under the no uncertainty assumption in the input variable. A larger DSM means longer probabilistic distance and the change in the output distribution is greater. Therefore, the input variable which has the largest DSM is the most important in terms of its impact on the output distribution.

The DSM results are compared with other statistical parameter variations. Table 2 shows the changes in the output statistical parameters when each input uncertainty is assumed to be zero one at a time. Also shown in Table 2 is the variation of the error factors. The error factor is defined as the square root of the ratio between the 95th percentile and the 5th percentile. Figure 2 shows graphically an example of the changes of cumulative output distributions: One from the source term parameter group(input variable #3), and the other from the consequence parameter group (input variable #20). The selection of most important input variables by DSM agrees well with our engineering intuition and the degree of changes in output distributions shown, for example, in Figure 2. Variations of DSM along with other possible measures, such as the 5th, 50th, and 95th percentiles, are shown in Figure 3. In Figure 3, the importance of each input parameter is shown in such a way that the calculated DSMs are in the descending order. For example, the input parameter, #20 has the largest calculated DSM value, and therefore, is ranked 1. #14 has the second largest DSM and ranked 2, and so on. The other possible measures such as the percentiles in Figure 3 do not appear to be as sensitive as the DSM, even in such cases where major portions of output distribution have changed, as shown in Figure 2.

Case 2: Increased Input Uncertainty

In this case, each input's base case uncertainty range is increased by a factor of 10, or a factor of 3.3 for the lower and upper bounds, respectively. This case corresponds to a situation where not much is known about the particular input parameter and thus very wide uncertainty range is assumed. This is analogous to the risk increase measure by Vesely. The difference is that, in this case, a complete ignorance is assumed rather than complete failure of system or component. The uncertainty importance of each input parameter is shown in Figure 4 in a similar fashion as in Figure 3 for the zero uncertainty importance. A tabular form summary is given in Table 3. Selection of the most important input variables on the basis of the DSM shows similar results obtained in Case 1 and confirms the validity and usefulness of DSM.

Case 3: Changed Input Distribution Type

Initially it is assumed that all input variables follow uniform distribution. By assuming uniform distribution, the whole range of uncertainty is made sure to be sampled with equal probability. Each input distribution type is, in Case 3, changed to the normal distribution one at a time and its impact on the output distribution is analyzed. The changes in the statistical parameters and DSM are shown in Table 4. Figure 5 displays the ranks of input parameters for this case. A similar set of input variables is identified to be important.

5. CONCLUSIONS

The distributional sensitivity analysis is briefly described. The uncertainty importance measure based on the information theory is derived explicitly for the case that the simulated output distributions are approximated by the two-parameter Weibull distribution. Each input variable's importance on the output uncertainty is ranked according to the values of uncertainty importance measure (DSM) for the following three cases: (1) Complete knowledge is assumed, (2) Complete ignorance is assumed, and (3) Different distributional type is assumed. For all three cases, the DSM-based ranking agrees very well with the observed changes of output distributions while other statistical parameters are shown to be insensitive.

The proposed distributional sensitivity measure (DSM) can be used as a tool for exploring the sensitivity of a risk profile to certain changes including the effect of incorporating beliefs from some other part of the spectrum of analysts. It is an indicator of the robustness of conclusions based on the overall probability densities against changes in the assessment of the state of knowledge reflected in the output distribution.

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Table 1 Base Case Input Parameters*

Variable Number	Parameter	Range
1	Release fraction: Noble Gases	.288 - 1.
2	" " I_2	.0261 -.2352
3	" " Cs	.0553 -.498
4	" " Te	7.57×10^{-3} - 0.0681
5	" " S_2	1.4×10^{-3} - 1.27×10^{-2}
6	" " Ru	2.74×10^{-7} - 2.46×10^{-6}
7	" " La	1.74×10^{-4} - 1.56×10^{-3}
8	" " Ce	3.93×10^{-5} - 3.54×10^{-4}
9	" " Ba	5.83×10^{-3} - 0.0525
10	Time of release (hours)	.67 - 6.
11	Duration of release (hours)	3.12 - 28.1
12	Height of release (in.)	5. - 45.
13	Energy of release (watts)	3.3×10^{6} - $3. \times 10^{7}$
14**	Weather stability class	C, D, E
15	Average wind speed (m/s)	.77 - 6.94
16	Dry deposition velocity (m/s)	3.3×10^{-3} - 0.3
17	Breathing rate (m^3/s)	8.9×10^{-5} - 7.98×10^{-4}
18	Rainfall duration (s)	35.3 - 318.
19	Cloud - shine shielding factor	.25 - 1
20	Ground - shine shielding factor	.11 - .99
21	Rain depletion coefficient	1.33×10^{-4} - 1.2×10^{-3}

*All distributions are uniform.
**Site-specific propability obtained from MACCS database.

Table 2 Sensitivity/Importance Analysis for Complete Knowledge (Case 1)

CASE	5-th	50-th	95-th	Mean	Sigma*	EF Ratio+	DSM†	Rank
BASE	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	NA	NA	NA
1-	1.575E-04	1.623E-02	9.989E-01	1.716E-01	3.037E-01	9.918E-01	3.469E-05	17
2-	1.074E-04	9.463E-03	9.279E-01	1.259E-01	2.638E-01	1.158E+00	2.919E-02	5
3-	1.246E-04	1.238E-02	9.844E-01	1.348E-01	2.720E-01	1.107E+00	1.406E-02	6
4-	1.229E-04	1.260E-02	9.630E-01	1.490E-01	2.845E-01	1.103E+00	5.218E-03	11
5-	1.493E-04	1.571E-02	9.984E-01	1.688E-01	3.015E-01	1.019E+00	1.431E-05	19
6-	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	1.000E+00	< 1.E-07	21
7-	1.482E-04	1.554E-02	9.984E-01	1.685E-01	3.014E-01	1.022E+00	3.207E-05	18
8-	1.529E-04	1.582E-02	9.985E-01	1.695E-01	3.023E-01	1.007E+00	5.722E-06	20
9-	1.407E-04	1.321E-02	9.861E-01	1.491E-01	2.821E-01	1.043E+00	4.142E-03	12
10-	1.771E-04	1.820E-02	9.998E-01	1.791E-01	3.100E-01	9.359E-01	9.264E-04	14
11-	2.183E-04	2.373E-02	9.999E-01	2.008E-01	3.271E-01	8.431E-01	8.311E-03	7
12-	1.551E-04	1.788E-02	9.995E-01	1.801E-01	3.095E-01	1.000E+00	1.048E-03	13
13-	1.515E-04	1.510E-02	9.985E-01	1.632E-01	2.994E-01	1.011E+00	8.612E-04	15
14-	9.544E-04	3.225E-02	9.740E-01	1.699E-01	2.835E-01	3.979E-01	6.263E-02	3
15-	4.101E-04	4.071E-02	9.999E-01	2.313E-01	3.409E-01	6.151E-01	4.014E-02	4
16-	1.439E-04	1.123E-02	9.862E-01	1.421E-01	2.796E-01	1.031E+00	1.001E-01	2
17-	1.097E-04	1.158E-02	9.966E-01	1.522E-01	2.916E-01	1.187E+00	6.101E-03	8
18-	1.199E-04	1.194E-02	9.938E-01	1.529E-01	2.923E-01	1.134E+00	5.694E-03	10
19-	1.638E-04	1.632E-02	9.988E-01	1.720E-01	3.042E-01	9.728E-01	4.137E-05	16
20-	1.291E-04	6.187E-03	3.183E-01	5.327E-02	1.313E-01	6.184E-01	3.108E-01	1
21-	1.325E-04	1.105E-02	9.955E-01	1.529E-01	2.917E-01	1.080E+00	5.985E-03	9

*Standard Deviation.
†Uncertainty Importance Measure.
+EF ratio = EF of base case/EF of case i.
NA = Not Applicable.

Table 3 Sensitivity/Importance Analysis for Complete Ignorance (Case 2)

CASE	5-th	50-th	95-th	Mean	Sigma*	EF ratio⁺	DSM†	Rank
BASE	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	NA	NA	NA
1+	1.519E-04	1.577E-02	9.984E-01	1.692E-01	3.019E-01	1.010E+00	6.795E-06	20
2+	4.913E-04	1.138E-01	1.000E+00	3.547E-01	4.087E-01	5.620E-01	1.569E-01	2
3+	2.663E-04	3.158E-02	1.000E+00	2.450E-01	3.609E-01	7.633E-01	3.114E-02	8
4+	3.481E-04	4.384E-02	1.000E+00	2.667E-01	3.704E-01	6.676E-00	5.391E-02	5
5+	1.626E-04	1.697E-02	9.992E-01	1.753E-01	3.069E-01	9.764E-01	3.009E-04	16
6+	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	9.289E-01	1.192E-07	21
7+	1.796E-04	1.814E-02	9.991E-01	1.763E-01	3.073E-01	9.289E-01	5.834E-04	15
8+	1.605E-04	1.690E-02	9.986E-01	1.718E-01	3.035E-01	9.824E-01	1.023E-04	18
9+	2.859E-04	4.169E-02	1.000E+00	2.552E-01	3.663E-01	7.367E-01	4.432E-02	6
10+	1.210E-04	1.144E-02	9.873E-01	1.440E-01	2.798E-01	1.125E+00	8.986E-03	13
11+	9.787E-05	9.455E-03	9.839E-01	1.322E-01	2.702E-01	1.249E+00	2.010E-02	10
12+	1.366E-04	1.137E-02	9.860E-01	1.403E-01	2.792E-01	1.058E+00	1.273E-02	12
13+	1.549E-04	1.782E-02	9.985E-01	1.744E-01	3.050E-01	1.000E+00	2.460E-04	17
14+	1.180E-05	4.692E-03	1.000E+00	1.721E-01	3.242E-01	3.626E+00	9.332E-02	3
15+	3.502E-05	2.832E-03	8.105E-01	8.870E-02	2.336E-01	1.895E+00	2.167E-01	1
16+	2.173E-04	2.731E-02	9.995E-01	2.118E-01	3.321E-01	8.447E-01	1.624E-02	11
17+	4.567E-04	4.900E-02	1.000E+00	2.524E-01	3.543E-01	5.828E-01	5.888E-02	4
18+	3.167E-04	3.642E-02	1.000E+00	2.236E-01	3.382E-01	6.999E-01	2.899E-02	9
19+	1.480E-04	1.573E-02	9.983E-01	1.686E-01	3.014E-01	1.023E+00	1.919E-05	19
20+	9.788E-05	1.327E-02	9.976E-01	1.595E-01	2.968E-01	1.258E+00	3.357E-03	14
21+	3.372E-04	3.884E-02	1.000E+00	2.291E-01	3.403E-01	6.784E-01	3.399E-02	7

*Standard Deviation.
†Uncertainty Importance Measure.
⁺EF ratio = EF of base case/EF of case i.
NA = Not Applicable.

Table 4 Sensitivity/Importance Analysis for Changes of Distributional Shape (Case 3)

CASE	5-th	50-th	95-th	Mean	Sigma*	EF ratio⁺	DSM†	Rank
BASE	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	NA	NA	NA
1N	1.540E-04	1.598E-02	9.985E-01	1.699E-01	3.024E-01	1.003E+00	1.192E-07	16
2N	1.798E-04	1.745E-02	9.977E-01	1.699E-01	3.017E-01	9.278E-01	2.443E-04	7
3N	1.568E-04	1.680E-02	9.990E-01	1.674E-01	2.997E-01	9.944E-01	8.261E-05	11
4N	1.551E-04	1.597E-02	9.959E-01	1.701E-01	3.031E-01	9.981E-01	7.629E-06	14
5N	1.546E-04	1.590E-02	9.984E-01	1.699E-01	3.025E-01	1.001E+00	< 1.E-07	18
6N	1.549E-04	1.596E-02	9.985E-01	1.700E-01	3.025E-01	1.000E+00	1.192E-07	17
7N	1.539E-04	1.587E-02	9.986E-01	1.700E-01	3.025E-01	1.003E+00	< 1.E-07	19
8N	1.554E-04	1.596E-02	9.985E-01	1.699E-01	3.025E-01	9.984E-01	< 1.E-07	20
9N	1.613E-04	1.691E-02	9.979E-01	1.693E-01	3.009E-01	9.796E-01	5.245E-06	15
10N	1.513E-04	1.605E-02	9.989E-01	1.687E-01	3.015E-01	1.012E+00	1.442E-05	13
11N	1.433E-04	1.430E-02	9.958E-01	1.612E-01	2.949E-01	1.038E+00	8.132E-04	5
12N	1.549E-04	1.609E-02	9.990E-01	1.721E-01	3.041E-01	1.000E+00	5.019E-05	12
13N	1.549E-04	1.627E-02	9.985E-01	1.718E-01	3.039E-01	1.000E+00	2.789E-01	1
14N	3.403E-05	6.183E-02	9.750E-01	1.282E-01	2.764E-01	2.108E+00	6.052E-02	2
15N	1.523E-04	1.563E-02	9.824E-01	1.520E-01	2.829E-01	1.000E+00	2.181E-03	4
16N	1.839E-04	1.664E-02	9.991E-01	1.751E-01	3.074E-01	9.182E-01	2.514E-04	6
17N	1.711E-04	1.616E-02	9.988E-01	1.687E-01	3.016E-01	9.517E-01	1.423E-04	8
18N	1.670E-04	1.610E-02	9.990E-01	1.706E-01	3.023E-01	9.633E-01	8.744E-05	10
19N	1.582E-04	1.606E-02	9.985E-01	1.700E-01	3.026E-01	9.895E-01	< 1.E-07	21
20N	2.859E-04	1.922E-02	9.659E-01	1.629E-01	2.854E-01	7.240E-01	4.242E-03	3
21N	1.972E-04	1.576E-02	9.987E-01	1.705E-01	3.032E-01	8.846E-01	1.212E-04	9

*Standard Deviation.
†Uncertainty Importance Measure.
⁺EF ratio = EF of base case/EF of case i.
NA = Not Applicable.

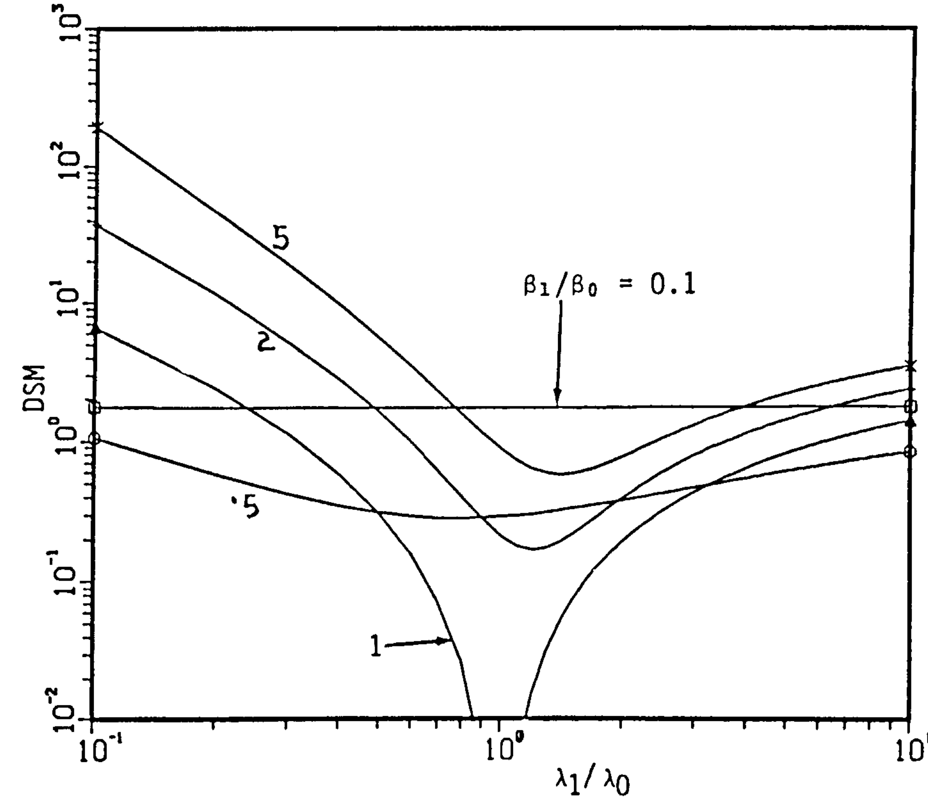

Figure 1 DSM as a Function of the Weibull Parameters

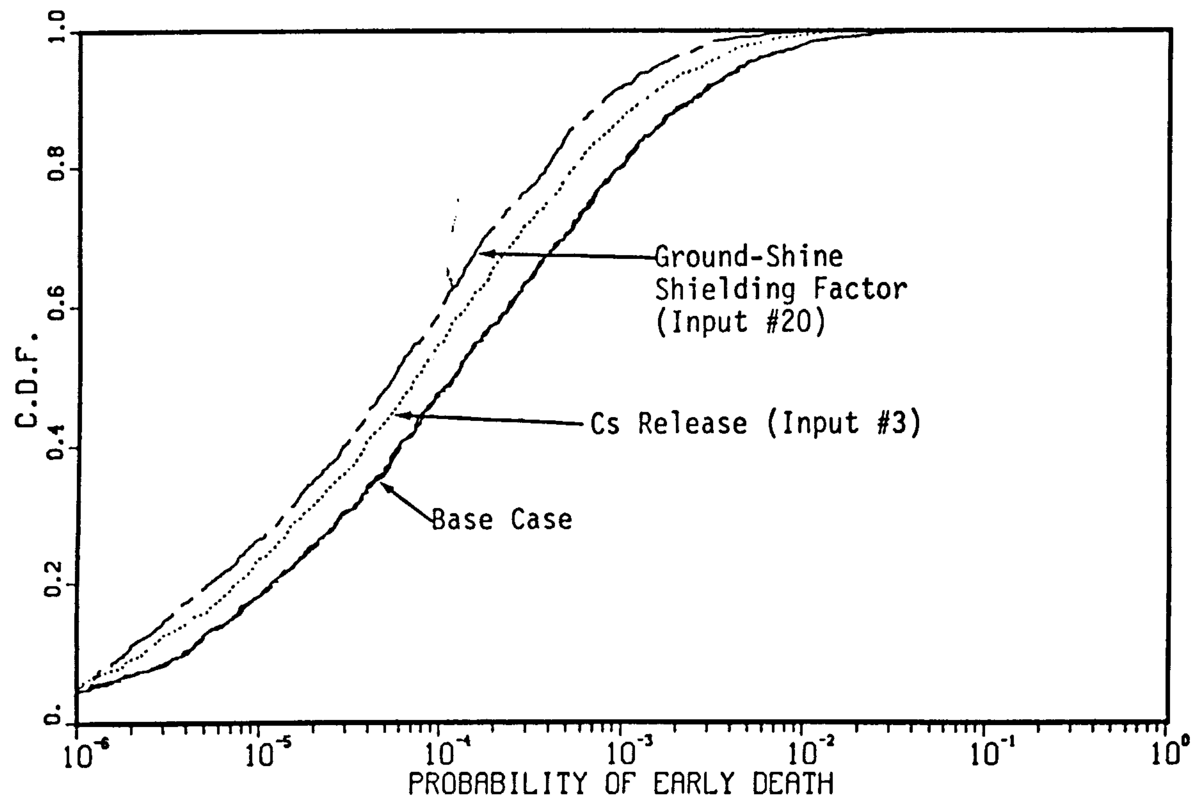

Figure 2 Example of CDF Changes for Zero Uncertainty Case (Case 1)

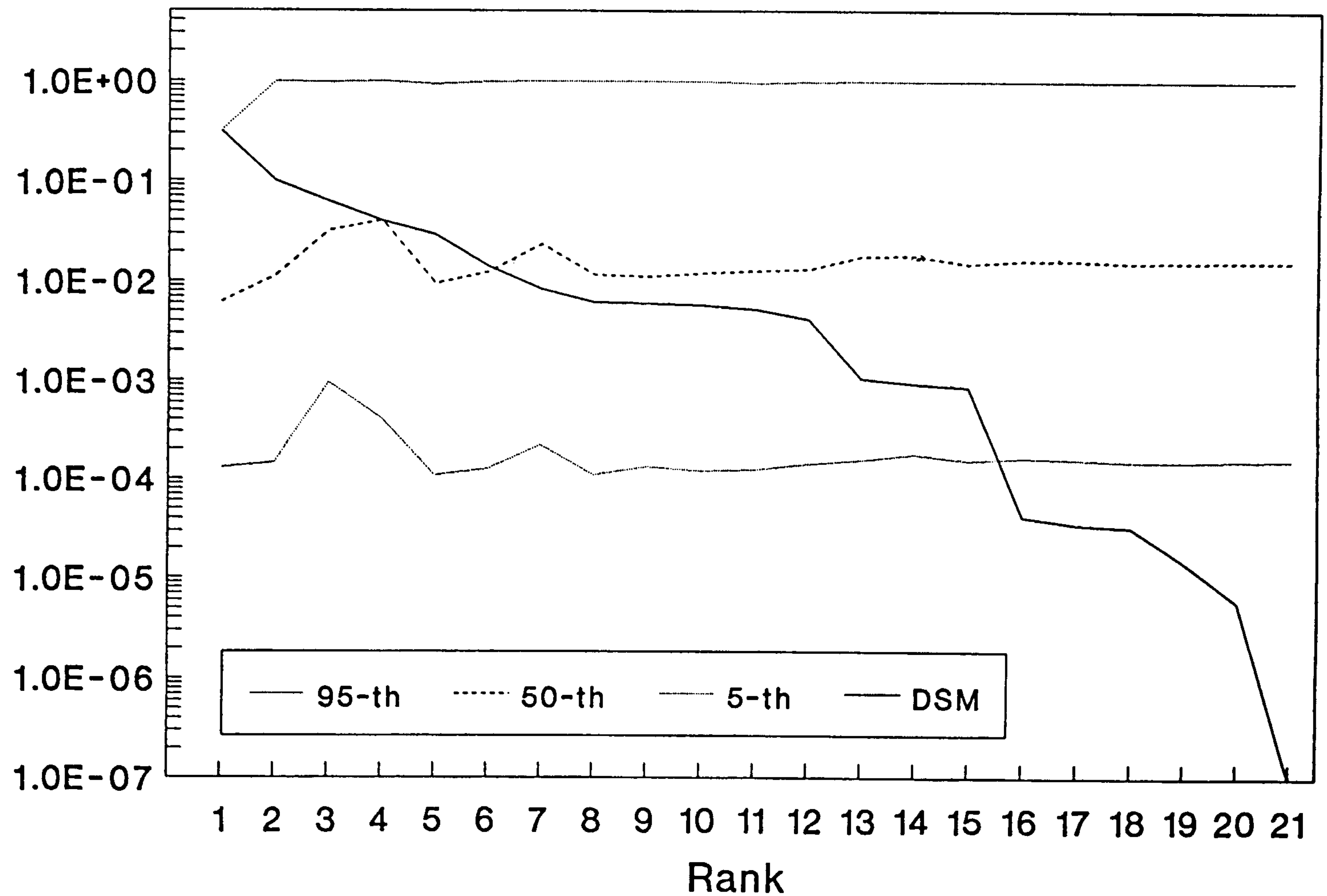

Figure 3 Importance Analysis for Case 1

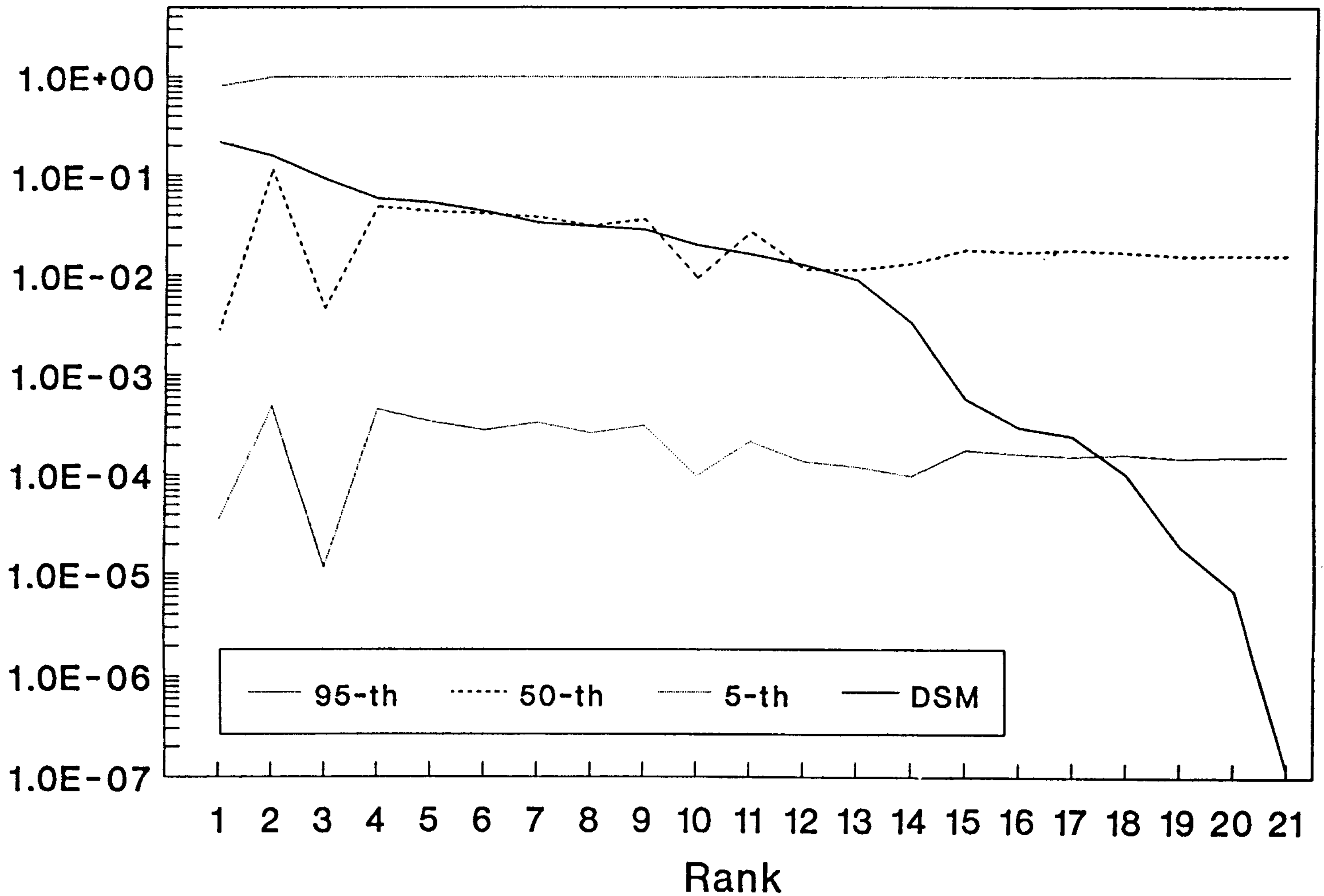

Figure 4 Importance Analysis for Case 2

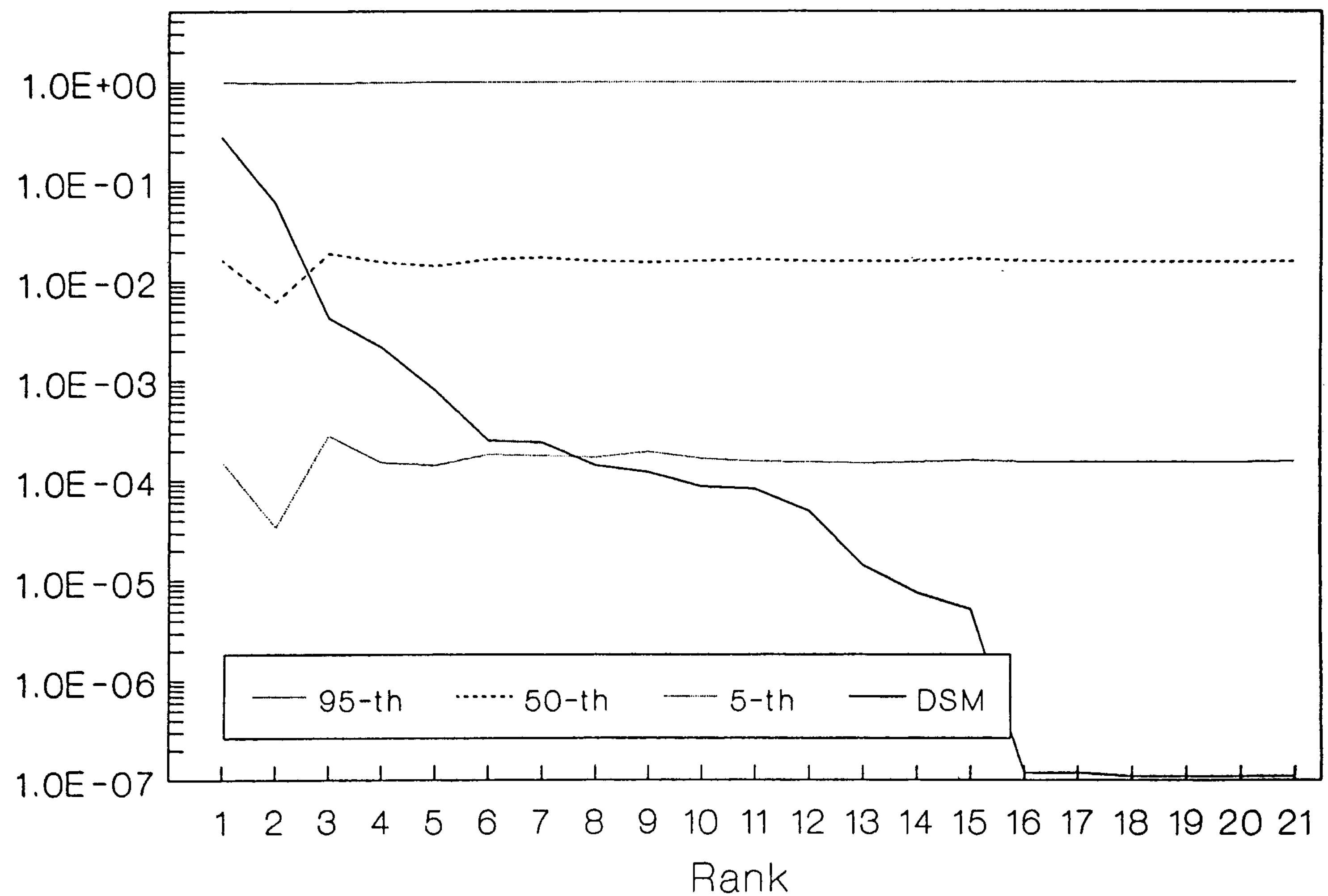

Figure 5 Importance Analysis for Case 3