

An Integrated Manufacturing and Distribution Model
for a Multi-Echelon Structure

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ABSTRACT

A multi-echelon structure of manufacturing and distribution system is considered, where the raw materials are transformed into a finished good through a number of manufacturing echelons and it is distributed to the lower echelons (retailers, or customers).

The raw material, work-in-process, finished good inventory and the distribution costs are unified into one model.

The objective is to determine the ordering policy of raw materials, manufacturing lot size, the number of sub-batch and the distribution policy of the finished good which minimize the annual total system cost. A computer program for a heuristic search technique is developed, by which a numerical example is examined.

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Introduction

This study considers an integrated manufacturing and distribution system which unifies the three steps of process; raw material procurement, manufacturing, and distribution process, into one model.

We assume that the raw materials are procured from the external source by appropriate lots, and through a number of manufacturing echelons these materials are transformed into finished good. During the manufacturing operations the movement of sub-batches allows an overlap between operations as Goyal [1] pointed out.

The work-in-process inventory starts to increase when the first manufacturing operation begins and it starts to decrease when the last manufacturing operation begins.

The finished good is to satisfy the retailers demand by a distribution policy. Figure 1 provides a schematic diagram illustrating the three steps of the system.

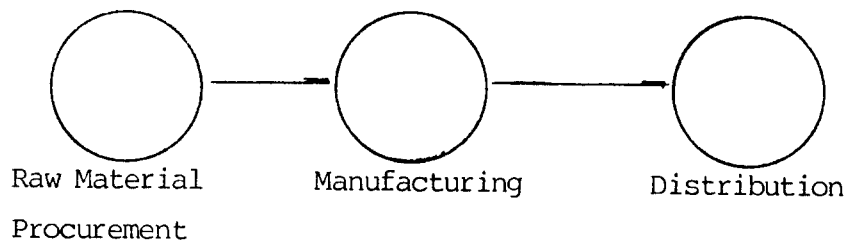


Figure 1. Three steps of the system

We are to develop a procedure for determining the raw material order quantities, the manufacturing lot size, and distribution policy of the finished good which minimizes the total system cost.

While a number of researchers have investigated the multi-echelon structure of manufacturing and the inventory system using a variety of techniques, the functional relationship among raw material procurement, manufacturing and distribution has received relatively little attention.

Goyal [1] developed an integrated inventory model which unifies the inventory problem of raw materials and finished product for one-stage production system. The system cost consists of ordering costs, set-up cost, and inventory carrying costs. The objective is to determine the optimal policies for procurement of raw materials and the optimal production lot size. The solution method suggested is an approximation technique based on an iterative search algorithm.

Szendrovits [7] presents a model for determining the manufacturing cycle time and establishing the relationship among production lot sizes, manufacturing cycle time, and the work-in-process inventory in a serial multi-stage production system. In his model he assumes that a constant and uniform lot size is manufactured through several operations, with only one set-up at each stage, and that transportation of sub-batches allows an overlap between operations to reduce the manufacturing time.

The system cost is the sum of the fixed costs per lot and the

inventory holding costs of both the work-in-process and finished good inventory. The optimization technique employed is differential calculus.

The model Goyal [1] does not incorporate in-process inventory and distribution process in his model and Szendrovits [7] provides no policies on the procurement of raw materials and also does not incorporate the distribution process.

Most other recently published articles, [2], [4], [6], do not incorporate all the three functions; raw material procurement, manufacturing, and distribution functions.

Thus, in this research we consider a model incorporating all the three functions and develop a solution algorithm. Figure 2 provides a schematic diagram illustrating an integrated model.

The Mathematical Model

The following assumptions are made in this study:

1. Demand rate for finished good is uniform and constant over time.
2. The inventory cost (ordering, holding) for raw material, work-in-process and finished good are known and constant over time.
3. Lead times for both procuring the raw material and finished good in distribution are zero.
4. No shortage of raw materials or the finished product is allowed.
5. For the distribution process, the single cycle policy [5] is

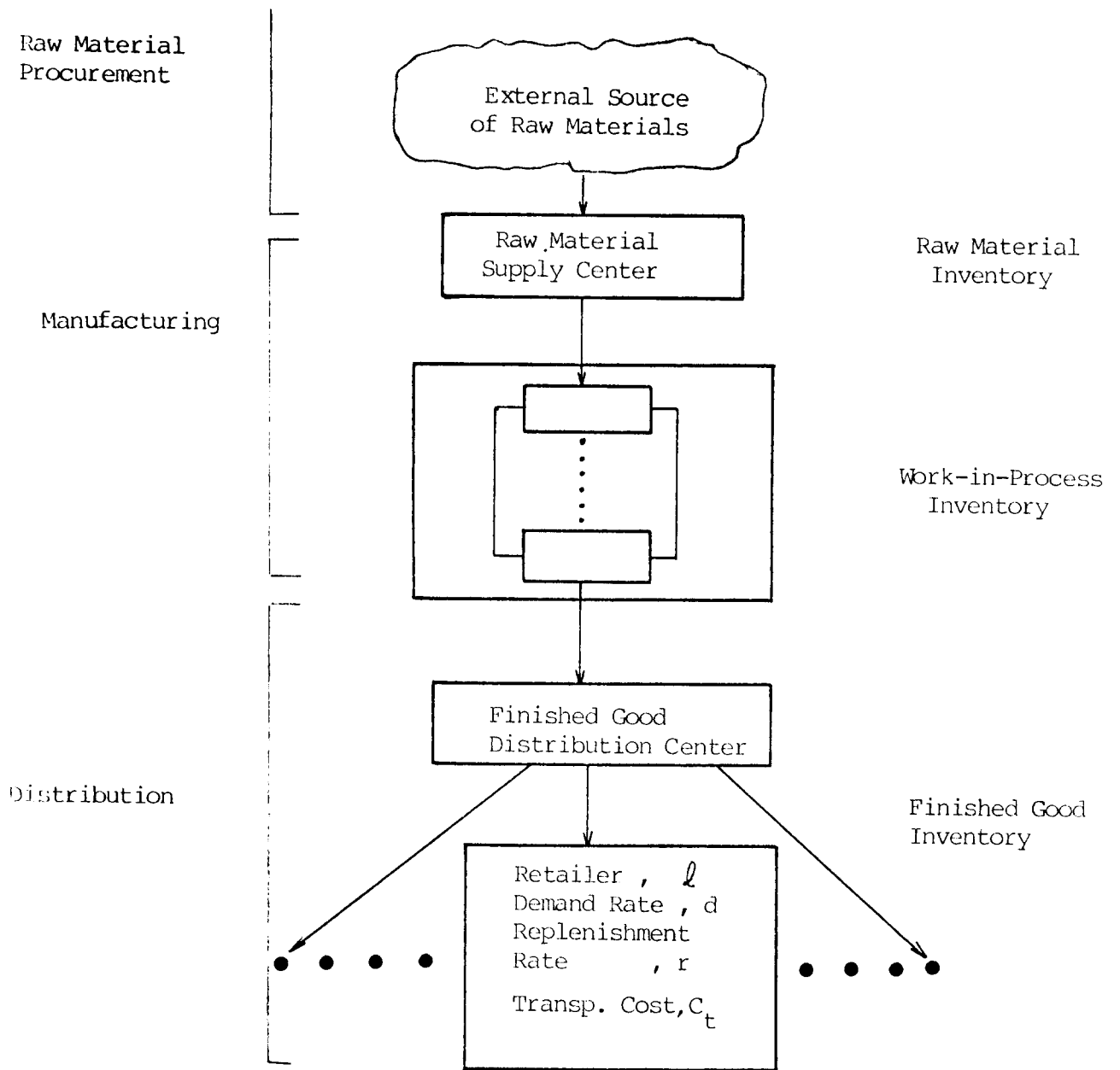


Figure 2, An Integrated manufacturing and Distribution Model

adopted.

6. The manufacturing rate is larger than demand rate of finished good.

The notations adopted are;

Raw Material Procurement

C_{ih} ; Stock holding cost per unit per year for the raw material h type i.

S_{ih} ; cost of placing a purchase order of raw material h type i.

T_{ih} ; purchase cycle of orders of raw material h type i.

Manufacturing process

j : manufacturing echelon. $j=1, \dots, J,$

b : number of sub-batches of manufacturing lot ,

T_m : manufacturing cycle (Time between successive manufacturing) ,

T : scheduling cycle ,

d : requirement per work day,

p : manufacturing output per work day,

S : a manufacturing set-up cost ,

s : a set-up cost for sub-batch ,

t_j : processing time for jth manufacturing operation,

w : work day per year ,

y : number of minutes available for manufacturing per work day ,

Distribution process

- N : total number of retailers,
- r_ℓ/d_ℓ : replenishment rate/ demand rate at retailer ℓ ,
- n_ℓ : number of set-up at retailer ℓ during a manufacturing cycle
- S_ℓ : a set-up cost at retailer ℓ ,
- h_ℓ : stock holding per unit per year at retailer ℓ .

(i) Cost of carrying finished good Inventory

The manufacturing rate for finished good depends on the processing time at the last manufacturing echelon, $\frac{1}{t_J}$ unit per minute. Thus the average carrying cost is given by [See Figure 3.]

$$\frac{1}{2}C_JQ(1-dt_J/y) \dots \dots \dots \quad (1)$$

(ii) Carrying cost of Work-in-process Inventory

The work-in-process inventory incurs from the start of the first manufacturing operation to $(J-1)^{st}$ operation.

Szendrovits[7] gives a general form of manufacturing cycle (T_m) as

$$T_m = \frac{Q}{b} \left[t_1 + t_2 + (b-1) \sum_{j=1}^2 (t_j - t_{j-1}) I_j \right] \dots \dots \quad (2)$$

where t_0 is defined to be zero,

$$\text{and } I_j = \begin{cases} 0, & \text{if } t_j \leq t_{j-1} \\ 1, & \text{otherwise} \end{cases}$$

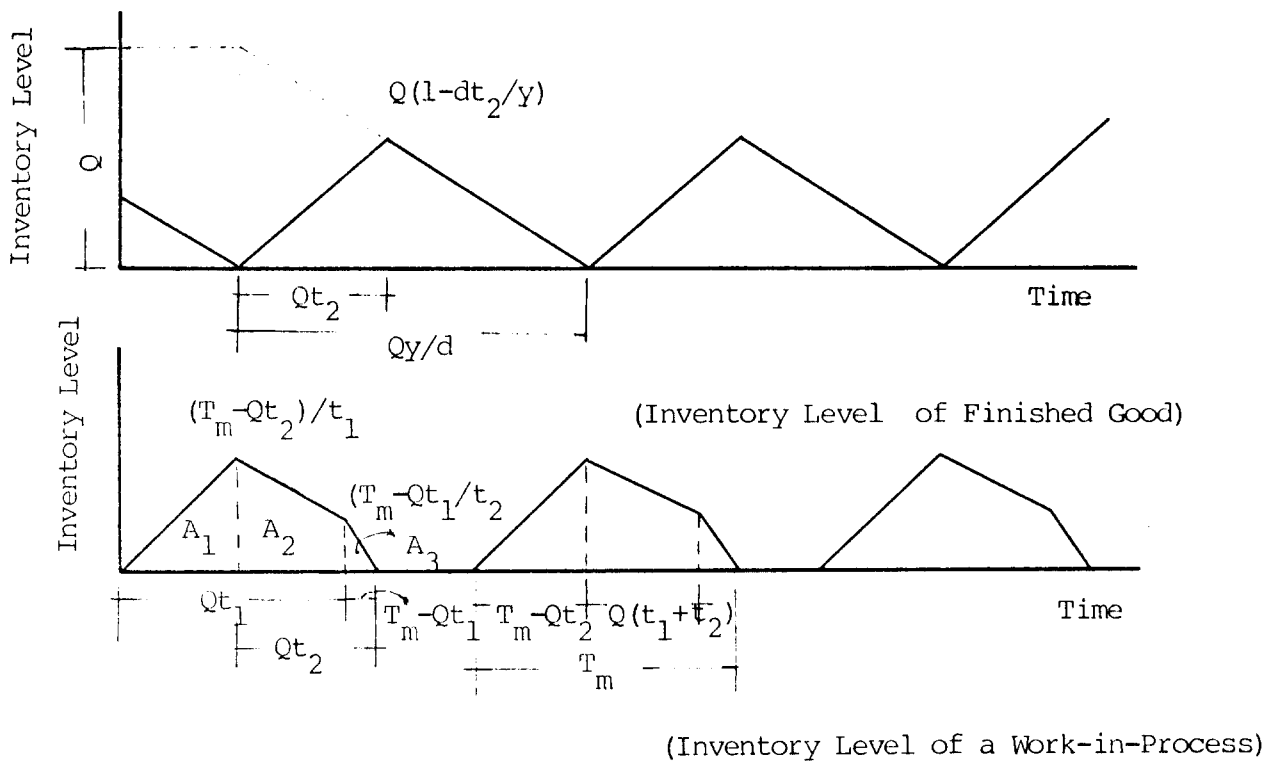


Figure 3. Inventory Level of Finished Good and Work-in-Process

It can be easily extended to the case of j th echelon case as

$$T_j = \frac{Q}{b} \left[t_j + t_{j+1} + (b-1) \left(t_j + (t_{j+1} - t_j) I_j \right) \right] \quad (3)$$

$$\text{where } I_j = \begin{cases} 0, & \text{it } t_j \geq t_{j+1} \\ 1, & \text{otherwise,} \end{cases}$$

$$j = 1, \dots, J-1$$

The amount of work-in-process inventory for j th manufacturing echelon is given by

$$\begin{aligned}
& \frac{1}{2} \left((T_j - Qt_{j+1})/t_j \right) \cdot (T_j - Qt_{j+1} + Qt_j - T_j) \\
& + \frac{1}{2} \left((T_j - Qt_1)/t_2 \right) \cdot (T_j - Qt_j + Qt_j + Qt_{j+1} - T_j) \\
& = QT_j - Q^2(t_j + t_{j+1})/2 \tag{3-1}
\end{aligned}$$

$$\left(\begin{array}{l} \text{the average} \\ \text{work-in-process} \\ \text{inventory in } j\text{th echelon} \end{array} \right) = d \cdot \left\{ T_j - Q(t_j + t_{j+1})/2 \right\} / Y$$

Thus, the annual work-in-process inventory cost for all echelons of the manufacturing is given by deviding eq. (3-1) by Qy/d as

$$\sum_{j=1}^{J-1} C_j d(T_j - Q(t_j + t_{j+1})/2)/Y \tag{4}$$

(iii) Cost of carrying and ordering of raw material inventories

For simplicity , we assumed all J types of raw materials to be analysed, i.e. the i^{th} raw materials are used only for the j th manufacturing echelon. Thus there are m_i kinds of raw materials used in manufacturing echelon i .

$$\text{Thus, } \left[\begin{array}{l} \text{the requirement amount of} \\ \text{raw material } h, \text{ type } i \end{array} \right] = K_{ih} \cdot Q,$$

$$\text{cover period} = K_{ih} Q \cdot \frac{Y}{d} \text{ minutes (see Figure 4).}$$

As in Figure 4, there are K_{ih} triangle sub-areas and $K_{ih} \cdot (K_{ih}-1)/2$ quadrangle sub-areas under the raw material h , type i curve.

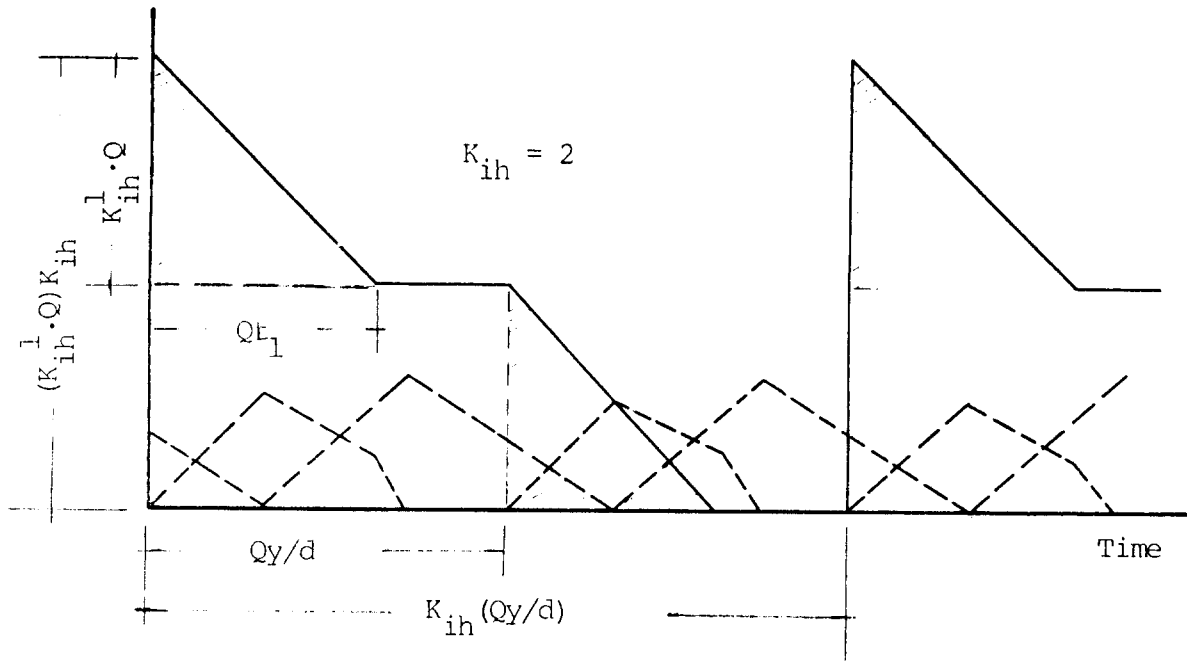


Figure 4. Raw Material Inventory Level

The total area under the curve is given by

$$K_{ih} \frac{1}{2} Q^2 \cdot t_i + \frac{1}{2} K_{ih} (K_{ih} - 1) \frac{Q^2 Y}{d} ,$$

and (Average amount of raw material inventory)

$$\begin{aligned} &= \left[K_{ih} \cdot \frac{1}{2} \cdot Q^2 t_i + \frac{1}{2} K_{ih} (K_{ih} - 1) \frac{Q^2 Y}{d} \right] / (K_{ih} \frac{QY}{d}) \\ &= \frac{1}{2} Q \left(\frac{dt_i}{Y} + K_{ih} - 1 \right) \end{aligned}$$

Thus the raw material inventory cost for all h, and i is given as

$$\sum_{i=1}^J \sum_{h=1}^{M_i} \frac{1}{2} C_{ih} Q \left(\frac{dt_i}{Y} + K_{ih} - 1 \right) \quad (5)$$

The annual ordering cost is given by

$$\frac{S_{ih} Wd}{K_{ih} Q} \quad (5-1)$$

(iv) Annual Set-up Cost

The number of manufacturing lots in a year is $\frac{w d}{Q}$.

Thus, the total cost of set-up for manufacturing lots and the sub-batches per year is given by

$$\frac{wd S}{Q} + \frac{wd s}{Q} \quad (6)$$

(v) Annual Costs for Distribution Echelons

When single cycle policy (n_1, \dots, n_N) [5] is used, the annual

inventory and transportation costs for retailer ℓ are given as:

$$\text{annual set-up cost} = \frac{d \cdot w \cdot n_{\ell}}{Q} \cdot S_{\ell}$$

$$\text{annual carrying cost} = \frac{d_{\ell} \cdot r_{\ell} \cdot t_{\ell}}{2} (1 - d_{\ell}/r_{\ell}) \cdot h_{\ell}$$

$$\begin{aligned} \text{transportation cost} &= t_{\ell} d_{\ell} C_{t_{\ell}} \\ &= \frac{Q}{d \cdot w \cdot n_{\ell}} \cdot d_{\ell} C_{t_{\ell}} \end{aligned}$$

Thus, in the distribution process, the total annual cost

$$= \sum_{\ell=1}^N \left(\frac{d \cdot w \cdot n_{\ell} S_{\ell}}{Q} + \frac{1}{2} \frac{d_{\ell} Q}{d \cdot w \cdot n_{\ell}} (1 - d_{\ell}/r_{\ell}) h_{\ell} + \frac{Q}{d \cdot w \cdot n_{\ell}} \cdot d_{\ell} C_{t_{\ell}} \right) \cdot \cdot \cdot (7)$$

By equations (1) - (7), the total cost function is given

$$\begin{aligned} &F(Q, b, k_{ih}, n_{\ell} : i=1, \dots, J ; h=1, \dots, m_i ; \ell=1, \dots, N) \\ &= (wds+bs)/Q + \frac{1}{2} C_j Q (1 - dt_j/y) + \sum_{j=1}^{J-1} C_j \cdot \frac{d}{y} \cdot \left\{ T_j - Q(t_j + t_{j+1})/2 \right\} \\ &+ \sum_{i=1}^J \sum_{h=1}^{m_i} \left\{ S_{ih} w \cdot d / (K_{ih} Q) + \frac{1}{2} C_{ih} \cdot Q \left(\frac{dt_i}{y} + K_{ih} - 1 \right) \right\} \\ &+ \sum_{\ell=1}^N \frac{d \cdot w \cdot n_{\ell} S_{\ell}}{Q} + \frac{1}{2} \frac{d_{\ell} Q}{d \cdot w \cdot n_{\ell}} (1 - d_{\ell}/r_{\ell}) h_{\ell} + \frac{Q}{d \cdot w \cdot n_{\ell}} d_{\ell} \cdot C_{t_{\ell}} \end{aligned} \quad (8)$$

where T_j is defined in equation (3)

$$I_j = \begin{cases} 0, & \text{if } t_j \geq t_{j+1} \\ 1, & \text{otherwise} \end{cases}$$

By substituting T_j in equation (8) and rearranging, the total cost function yields

$$\begin{aligned}
& F(Q, b, K_{ih}, n_\ell; i=1, \dots, J, h=1, \dots, m_i, \ell=1, \dots, N) \\
&= (w \cdot d \cdot S + w \cdot d \cdot b_s) / Q + Q \left\{ \frac{1}{2} C_j \left(1 - \frac{dt_i}{Y}\right) + \sum_{j=1}^{J-1} C_j \frac{d}{jY} \left(\frac{t_j - t_{j+1}}{2} + (t_{j+1} - t_j) I_j\right) \right. \\
&\quad \left. + \sum_{i=1}^J \sum_{h=1}^{m_i} \frac{1}{2} C_{ih} \left(\frac{dt_i}{Y} - 1\right) + \frac{1}{b} \sum_{j=1}^{J-1} C_j \frac{d}{jY} (t_{j+1} - (t_{j+1} - t_j) I_j) \right\} \\
&\quad + \sum_{i=1}^J \sum_{h=1}^{m_i} \left\{ S_{ih} w \cdot d / (Q \cdot K_{ih}) + \frac{1}{2} C_{ih} K_{ih} \cdot Q \right\} \\
&\quad + \sum_{\ell=1}^N \frac{dwS}{Q} n_\ell + Q \sum_{\ell=1}^N \left\{ \frac{d_\ell}{n_\ell d \cdot w} \left(h_\ell - \frac{h_\ell d_\ell}{r_\ell} + c_{t_\ell}\right) \right\}
\end{aligned}$$

Let, $G = w \cdot d \cdot s,$

$D = w \cdot d \cdot s,$

$$\begin{aligned}
R &= \frac{1}{2} C_j \left(1 - \frac{dt_j}{Y}\right) + \sum_{j=1}^{J-1} C_j \frac{d}{jY} \left(\frac{t_j - t_{j+1}}{2} + (t_{j+1} - t_j) I_j\right) \\
&\quad + \sum_{i=1}^J \sum_{h=1}^{m_i} \frac{1}{2} C_{ih} \left(\frac{dt_i}{Y} - 1\right),
\end{aligned}$$

$$Z = \sum_{j=1}^{J-1} C_j \frac{d}{jY} (t_{j+1} - (t_{j+1} - t_j) I_j),$$

$G_{ih} = w \cdot d \cdot s_{ih}, \quad \forall i \text{ and } h,$

$B_{ih} = \frac{1}{2} C_{ih},$

$$A_l = d \cdot w \cdot s_l,$$

$$B_l = \frac{d_l}{d \cdot w} \left(h_l - \frac{h_l d_l}{r_l} + c_{t_l} \right),$$

then, the system cost function is given as

$$F(Q, b, K_{ih}, n_l) = (G + Db)/Q + Q(R + \frac{1}{b}Z) + \sum_{i=1}^J \sum_{h=1}^{m_i} \left(\frac{G_{ih}}{Q_{ih}} + B_{ih} \cdot Q_{ih} \right) + \sum_{l=1}^N \left(A_l \frac{n_l}{Q} + B_l \frac{Q}{n_l} \right)$$

Since the manufacturing Cycle time, T_m , must not exceed the scheduling time, T ,

$$T_m + \alpha \cdot Q \leq T \tag{9}$$

where α is additional delay factor,

$$T = Qy/d, \text{ and } T_m = \sum_{i=1}^{J-1} (T_i - Qt_{i+1})$$

By substituting T and T_m into equation (9), b is given as

$$b \geq \frac{\sum_{j=1}^{J-1} \left\{ t_{j+1} - (t_{j+1} - t_j) I_j \right\}}{\left\{ \frac{y}{d} - \left(\alpha + t_1 + \sum_{j=1}^{J-1} (t_{j+1} - t_j) I_j \right) \right\}}$$

Since b is an integer greater than or equal to one,

$$b \geq [B]$$

Thus, our problem is

$$\begin{aligned} \text{Min } F(Q, B, K_{ih}, n_\ell; i=1, \dots, J, h=1, \dots, m_i, \ell=1, \dots, N) \\ = (G + Db)/Q + Q(R + 1/b \cdot Z) + \sum_{i=1}^J \sum_{h=1}^{m_i} \left(\frac{G_{ih}}{Q_{ih}} + B_{ih} \cdot Q_{ih} \right) \\ + \sum_{\ell=1}^N \left(A_\ell \frac{n_\ell}{Q} + B_\ell \frac{Q}{n_\ell} \right) \end{aligned}$$

$$\text{ST. } Q_{ih} = K_{ih} \cdot Q, \quad \forall i, h$$

$$K_{ih} \geq 1,$$

$$n_\ell \geq 1, \quad \forall \ell$$

$$b \geq [B]$$

Solution Algorithm

The objective function, $F(Q, b, K_{ih}, n_\ell)$ is a non-linear polynomial with second degree of cross product terms, where Q is continuous, and the other decision variables are integers. Thus the problem is a non-linear integer program. The function $F(\cdot)$ is convex over Q, b, Q_{ih} and n_ℓ . Unfortunately the optimal decision variable is represented as the function of the other decision variables, thus it cannot be obtained without prior knowledge of the others.

Thus, as in Hadley [3] and Goyal [1], we adopt a heuristic method to solve the problem as follow;

Step 1. Set initial values as;

$$b = 1$$

$$K_{ih} = 1, \forall i, h ; i=1,2,; h=1,\dots, m_i$$

$$n_\ell = 1, \forall \ell=1,\dots,N,$$

and set $n=1$

Step 2. Obtain n^{th} iteration values and let $Q = Q^{(n)}$ such as

$$Q^{(n)} = \sqrt{\frac{2wd \left\{ S + bs + \sum_{i=1}^J \sum_{h=1}^{m_i} (S_{ih}/K_{ih}) + \sum_{\ell=1}^N n_\ell \cdot S_\ell \right\}}{C_j (1-dt_j/y) + M_1 + M_2 + M_3}}$$

$$\text{where, } M_1 = 2 \cdot \frac{d}{y} \sum_{j=1}^{J-1} C_j \left\{ \frac{1}{b} (t_{j+1} - (t_{j+1} - t_j) I_j) + \frac{t_j - t_{j-1}}{2} + (t_{j+1} - t_j) I_j \right\},$$

$$M_2 = \sum_{i=1}^J \sum_{h=1}^{m_i} \left\{ C_{ih} (dt_i/y + K_{ih} - 1) \right\},$$

$$M_3 = \sum_{\ell=1}^N \left\{ \frac{d_\ell}{dw \cdot n_\ell} (1 - d_\ell/r_\ell) h_\ell + \frac{1}{dw \cdot n_\ell} d_\ell C_{t_\ell} \right\},$$

$$\text{and } I_j = \begin{cases} 0, & \text{if } t_j \geq t_{j+1} \\ 1, & \text{otherwise} \end{cases}$$

Step 3. Obtain the value of

$$b = b(Q^{(n)})$$

$$K_{ih} = K_{ih}(Q^{(n)})$$

$$n_\ell = n_\ell(Q^{(n)})$$

where $b(Q^{(n)})$, $k_{ih}(Q^{(n)})$ and $n_\ell(Q^{(n)})$ are determined as following;

$$(i) \frac{\partial F(b)}{\partial b} = 0, \quad b_0 = Q \sqrt{\frac{\sum_{j=1}^{J-1} C_j \{t_{j+1} - (t_{j+1} - t_j) I_j\}}{w \cdot y \cdot s}}$$

$$b = \left[\begin{array}{c} \text{Min} \\ F(b) ([b_0], [b_0] + 1), [B] \end{array} \right]^+$$

where, $F(b) = F(\text{only with } b \text{ terms})$

$$= \frac{wdsb}{Q} + \frac{Q}{b} \sum_{j=1}^{J-1} \left\{ C_j \frac{d}{y} (t_{j+1} - (t_{j+1} - t_j) I_j) \right\}$$

$$(ii) \frac{2F(K_{ih})}{2K_{ih}} = 0, \quad K_{iho} = \frac{1}{Q} \sqrt{\frac{2wds_{ih}}{C_{ih}}}$$

$$K_{ih} = K_{in}(Q^{(n)}) = \left[\begin{array}{c} \text{Min} \\ F(K_{ih}) ([K_{iho}], [K_{iho}] + 1), [1] \end{array} \right]^+$$

$$\text{where, } F(K_{ih}) = \sum_{i=1}^J \sum_{h=1}^{m_i} \left\{ wd S_{ih} / (K_{ih} \cdot Q) + C_{ih} Q K_{ih} / 2 \right\},$$

$$(iii) \frac{\partial F(n)}{\partial n_\ell} = 0, \quad n_{\ell 0} = Q \sqrt{\frac{1}{awS_\ell} \left\{ \frac{1}{l} \frac{d_\ell}{dw} (1 - \frac{d_\ell}{r_\ell}) h_\ell + \frac{d_\ell}{dw} C_{t_\ell} \right\}}$$

$$n_\ell = n_\ell(Q^{(n)}) = \left[\begin{array}{c} \text{Min} \\ F(n) ([n_{\ell 0}], [n_{\ell 0}] + 1), [1] \end{array} \right]^+$$

$$\text{where, } F(n_\ell) = \frac{dwn_\ell}{Q} S_\ell + \frac{1}{2} \frac{d_\ell Q}{dwn_\ell} (1 - \frac{d_\ell}{r_\ell}) h_\ell + \frac{Qd_\ell}{dwn_\ell} C_{t_\ell}$$

Step 4. Stopping Rule

The search procedure terminates in n^{th} iteration

$$\text{if, } b(Q^{(n)}) = b(Q^{(n-1)}),$$

$$K_{ih}(Q^{(n)}) = K_{ih}(Q^{(n-1)}), \quad \forall i, h$$

$$n_\ell(Q^{(n)}) = n_\ell(Q^{(n-1)}), \quad \forall \ell$$

otherwise, set $n = n + 1$ and go to Step 2.

Numerical Example

Following example problem with 5 manufacturing echelons and 5 retailers is examined by a computer program coded after above heuristic algorithm.

Example problem:

production echelon	1	,	2	,	3	,	4	,	5
unit processing time	0.7	,	1.10	,	0.6	,	1.2	,	0.6
carrying cost \$/unit-year	0.10	,	0.15	,	0.30	,	0.29	,	0.4
No. of raw material required at echelon	2	,	2	,	2	,	1	,	3

$$w = 250 \text{ work days/year}$$

$$y = 480 \text{ minutes/work day}$$

$$d = 200 \text{ units/work day}$$

$$\alpha = 0.4 \text{ minutes/unit}$$

$$S = \$300 \text{ per set-up}$$

$$s = \$20 \text{ per sub-batch}$$

Unit cost of carrying raw materials, \$/unit/year ,

$$C_{11} = 0.05 \quad , \quad C_{12} = 0.07 \quad , \quad C_{21} = 0.02 \quad , \quad C_{22} = 0.01$$

$$C_{31} = 0.09 \quad , \quad C_{32} = 0.03 \quad , \quad C_{41} = 0.01 \quad , \quad C_{51} = 0.02$$

$$C_{52} = 0.15 \quad , \quad C_{53} = 0.09$$

Cost of ordering of raw material, S/order

$$\begin{aligned}
 S_{11} &= 32.0 & , & & S_{12} &= 47.0 & , & & S_{21} &= 6.0 & , & & S_{22} &= 25 \\
 S_{31} &= 30.0 & , & & S_{32} &= 39.0 & , & & S_{41} &= 47.0 & , & & S_{51} &= 40.0 \\
 S_{52} &= 6.0 & , & & S_{53} &= 6.0 & , & & & & & & & &
 \end{aligned}$$

Various unit costs of retailers

Retailer No.	demand rate	Replenishment rate	Carrying cost S/unit/year	Ordering cost \$/order	Transportation cost \$/unit
1	7,000.00	11,000.00	10.00	100.00	15.00
2	5,000,00	9,000,00	20.00	50.00	20.00
3	15,000.00	20,000.00	50.00	20.00	6.00
4	3,000.00	5,000.00	15.00	10.00	30.00
5	1,000.00	19,000.00	90.00	120.00	20.00

The solution of above example problem is represented in table 1.

MANUFACTURING LOT SIZE	TOTAL COST IN \$	CONSUMPTION CYCLE IN DAYS	MANUFACTURING CYCLE IN DAYS	NUMBER OF SUB-BATCHES
19656.00	40445.83	98.28	80.99	9

COST OF PHASE1= 8726.78
 COST OF PHASE2= 31719.06

.....

MATERIAL ORDERING QUANTITY IN UNITS

RAW MATERIAL NUMBER

ECHOLON	1	2	3
1	19656.00	19656.00	
2	19656.00	19656.00	
3	19656.00	19656.00	
4	19656.00		
5	19656.00	19656.00	19656.00

NUMBER OF CONSUMPTION CYCLES BETWEEN SUCCESSIVE RAW MATERIAL ORDERS

ECHOLON	1	2	3
1	1	1	
2	1	1	
3	1	1	
4	1		
5	1	1	1

NUMBER OF ORDER-CYCLE OF RETAILER J BETWEEN SUCCESSIVE PRODUCTION SET-UP

RETAILER	NUMBER OF ORDERS	CYCLE TIME OF RETAILER J	NUMBER OF ORDER QUANTITY
RET. 1	13	7.560	211.680
RET. 2	19	5.173	103.453
RET. 3	38	2.586	155.179
RET. 4	39	2.520	30.240
RET. 5	23	4.273	170.922

Table 1. Results of Example Problem.

To compare the above result with that of other methods, a number of results of the same example problem are represented in Table 2.

Method	Economic Lot Size	Remarks
Classical EOQ Model	10,000.00	Only one -echelon manufacturing process is considered without both raw material and in-process inventory
Goyal's Method [7]	10,000.12	Raw material and finished good inventory are considered but not in-process inventory
Szendrovits' Model [7]	7,216.29	Multi-echelon manufacturing with both in-rpocess and finished good inventory is considered
Proposed Integrated Model	19,656.00	The raw material procurement, Multi-echelon manufacturing, and two-echelon distribution process of finished good are unified into one model

Table 2. Comparison of Lot Size by Various Methods

As in Table 2, the first three models result in similar lot sizes, where only manufacturing process (one-echelon or multi-echelon) is considered, but our proposed integrated model results in significant lot size compared

with others as expected. This result is mainly due to the fact that multi-echelon manufacturing and distribution process are incorporated into a model.

Conclusional Remark

The model suggested in this paper is well fitted to an integrated manufacturing and distribution system where the operation is multi-echelon structure. There is well known conflict of interacting interests among procurement of raw materials, multi-echelon manufacturing and distribution policy of finished good.

This paper has illustrated the optimal manufacturing lot size, affected by the inventory costs of raw materials, in-process and finished good in both manufacturing and distribution process.

A mathematical model for a single product has been formulated, and a heuristic solution algorithm of this model is developed.

This model may be extended to incorporate a probabilistic demand for finished good, although it would be much more complicated.

References

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