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# Robust Adaptive Control of Linear Time-Varying Systems Which are Not Necessarily Slowly Varying

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## Abstract

This paper presents an indirect adaptive control scheme for discrete linear systems whose parameters are not necessarily slowly varying. It is assumed that system parameters are modelled as linear combinations of known bounded functions with unknown constant coefficients. Unknown coefficients are estimated using a recursive least squares algorithm with a dead zone and a forgetting factor. A control law which makes the estimated model exponentially stable is constructed. With this control law and a state observer, all based on the parameter estimates, it is shown that the resulting closed-loop system is globally stable and robust to bounded external disturbances and small unmodelled plant uncertainties.

## 1 Introduction

In many applications, the plant to be controlled is time-varying. However, most research on adaptive control deals with time-invariant systems. Some results do exist on the adaptive control of linear systems with slowly varying parameters (see, for example, [1], [2], [3] and [4]). However, the results for the systems which are not necessarily slowly varying are very few. Adaptive control schemes proposed by Xianya and Evans [5] and later Zheng [6] are for those systems, but these schemes can not be applied to the systems which are not stably invertible. Tsakalis and Ioannou [7] proposed an indirect adaptive pole-placement scheme for continuous linear time-varying systems which may be fast time-varying.

Recently, Kamen, Bullock and Song [8] developed an indirect adaptive control scheme for multi-input multi-output linear discrete-time systems which are not necessarily slowly varying nor stably invertible. In [8], a projection scheme was used in the parameter estimation to ensure the reachability of the estimated model. In this paper, projection scheme will be replaced by search techniques in such a way that they do not restrict the estimation process. Furthermore, the adaptive control scheme will be modified to achieve more robustness by including a dead zone in the estimation algorithm. As in [8], we assume that the plant is specified by an n-th order input/output difference equation and time-variations of coefficients are modelled as linear combinations of known bounded functions with unknown coefficients. The unknown coefficients are then estimated by a recursive least squares scheme with a dead zone and a forgetting factor. As a control law, a generalized Kleinman's method [9] is taken. Its advantage over other methods such as the pole-placement [10] and transfer function approach [11] is ease of implementation, especially when the reachability index $N \geq n$ where $n$ is the system order. With this control law and a state observer, all based on the estimated parameter values, it is shown that the resulting closed-loop system is globally stable and roust to bounded external disturbances and small unmodelled plant uncertainties.

## 2 Preliminaries

We consider a single-input single-output (SISO) time varying discrete-time system described by n-th order input/output difference equation

$$y(k) = \sum_{i=1}^{n} a_i(k)y(k-i) + \sum_{i=1}^{n} b_i(k)u(k-i) + d(k) \qquad (1)$$

where $u(k)$ is the control, $y(k)$ is the output, $d(k)$ is the bounded external disturbance and $n$ is the system order which is assumed known. In practice, some a priori knowledge about time-variation of system parameters $a_i(k)$ and $b_i(k)$ are often available, so we assume that $a_i(k)$ and $b_i(k)$ are linear combinations of known bounded functions with unknown constant coefficients. However, there may be some errors in the modelling of parameter time-variations. We, therefore, express $a_i(k)$ and $b_i(k)$ in the form

$$a_i(k) = \sum_{j=1}^{r}(a_{ij} + \Delta a_{ij}(k))f_j(k) \quad i = 1, \ldots, n \qquad (2)$$

$$b_i(k)=\sum_{j=1}^{r}(b_{ij}+\Delta b_{ij}(k))f_j(k)\ \ i=1,\ldots,n \quad (3)$$

where $a_{ij}$ and $b_{ij}$ are unknown constant scalars, $r$ is a known integer, $f_j(k)$ are known real-valued bounded functions of $k$, and $\Delta a_{ij}(k)$ and $\Delta b_{ij}(k)$ are mismatch terms which are also referred to as *Unmodelled Plant Uncertainties* or simply *Plant Uncertainties*. Note that the rate of time-variation of $a_i(k)$ and $b_i(k)$ depends on the rate of time-variation of $f_j(k)$ which can be arbitrarily fast.

We define the parameter vector $\theta$, the regression vector $\phi(k)$ and the plant uncertainty vector $\delta(k)$ by

$$\theta'=[a_{11}\cdots a_{1r}\cdots a_{n1}\cdots a_{nr}\,b_{11}\cdots b_{nr}] \quad (4)$$

$$\begin{aligned}\phi'(k-1)=[&f_1(k)y'(k-1)\cdots f_r(k)y'(k-1)\\ &\cdots f_1(k)y'(k-n)\cdots f_r(k)y'(k-n)\\ &f_1(k)u(k-1)\cdots f_r(k)u(k-n)]\end{aligned} \quad (5)$$

$$\begin{aligned}\delta'(k)=[&\Delta a_{11}(k)\cdots\Delta a_{nr}(k)\\ &\Delta b_{11}(k)\cdots\Delta b_{nr}(k)]\end{aligned} \quad (6)$$

where " $'$ " denotes the transpose of a matrix. Then the input/output difference equation (1) can be written in a compact form

$$y(k)=(\theta+\delta(k))'\phi(k-1)+d(k) \quad (7)$$

For the construction of a state feedback control law, we consider the canonical observable realization of (1) given by

$$\begin{aligned}x(k+1)&=F(k)x(k)+G(k)u(k)\\ &\quad+H'd(k+1) &(8)\\ y(k)&=Hx(k) &(9)\end{aligned}$$

where

$$\begin{aligned}F(k)&=\begin{bmatrix}a_1(k+1) & 1 & 0\cdots0\\ \vdots & & \\ a_{n-1}(k+n-1) & 0 & 0\cdots1\\ a_n(k+n) & 0 & 0\cdots0\end{bmatrix}\\ G(k)&=\begin{bmatrix}b_1(k+1)\\ \vdots\\ b_{n-1}(k+n-1)\\ b_n(k+n)\end{bmatrix}\\ H&=[\,1\,0\cdots0\,].\end{aligned}$$

If a bounded row vector $L(k)$ exists such that $x(k+1)=(F(k)-G(k)L(k))x(k)$ is asymptotically stable,then $L(k)$ will be called a stabilizing feedback gain. In particular, $L(k)$ can be computed from the $N$-step reachability grammian defined by

$$\begin{aligned}Y_N(k)=&\sum_{i=k}^{k+N-1}\Phi(k+N,i+1)G(i)\cdot\\ &G'(i)\Phi'(k+N,i+1).\end{aligned} \quad (10)$$

where $\Phi(k,i)$ denote the transition matrix for the system $x(k+1)=F(k)x(k)$. If a pair $(F(k),G(k))$ is uniformly reachable in $N$ steps, i.e.,

$$Y_N(k)\geq\varepsilon I \text{ for some } \varepsilon>0\ \ \forall k\in Z^+ \quad (11)$$

where $Z^+$ denotes a set of nonnegative integers, then a stabilizing feedback gain $L(k)$ which will be referred as to *Kleinman feedback gain* is

$$\begin{aligned}L(k)=G'(k)&\Phi'(k+N+1,k+1)Y_{N+1}^{-1}(k)\cdot\\ &\Phi(k+N+1,k+1)F(k).\end{aligned} \quad (12)$$

The proof that $L(k)$ ia a stabilizing feedback gain follows from the work of Moore and Anderson [9]. This type of control law will be used for the adaptive control scheme. Then we need the following assumption:

*A.1*: The observable realization of (1) described by (8)-(9) is uniformly reachable in $N$ steps with $N\geq n$.

The above assumption ensures that there is a stabilizing feedback for the system to be controlled. However, the system parameters are not exactly identified in general, so this assumption does not guarantee the reachability of the estimated model.

When the parameter estimate $\theta(k)$ violates the reachability condition, we can use search techniques to find a substitute for $\theta(k)$ (call it $\bar\theta(k)$) which satifies the reachability condition. Furthermore, $\bar\theta(k)$ is required to retain some properties of the estimation process which are crucial in proving the closed-loop stability. In case of linear time-invariant systems, there exist some works in this line of study (see, for example, [12], [13]). The idea proposed by Lozano-Leal and Goodwin [12] will be adopted in our setup where we treat linear time-varying systems.

Now, we want to find a proper adaptive control algorithm which makes the closed-loop system globally stable, that is, all signals in the system converge to zero for any initial conditions. We would also like to ensure that the adaptive control scheme is robust to unmodelled dynamics and external disturbances. In this paper, robustness to errors in the modelling of parameter time-variations (see equations (2) and (3)) and bounded external disturbances will be considered. If the mismatch term $\delta(k)$ is slowly varying, the estimation algorithm can track, to some extent, the variation of $\delta(k)$ by using a forgetting factor. Thus, the use of a forgetting factor will improve robustness to these modelling errors. However, it is still possible that the parameter estimates are updated along the wrong direction and even makes the closed-loop system unstable. We therefore use a dead zone as well as a forgetting factor to achieve further robustness to the plant uncertainties. We now state the robustness problem of our concern as follows : there exists $D^*>0$ such that for any bounded external disturbances $d(k)$

and for all possible plant uncertainties $\delta^*(k)$ satisfying $\sup\{\|\delta^*(k)\|\} \leq D^*$, the closed-loop system is stable [14].

## 3 Parameter Estimation

Based on the model (7), we shall construct an estimator of the parameter vector $\theta$ by using a recursive least squares scheme with a forgetting factor and a dead zone. We first take a normalized version of (7) which is necessary in the following development.

Define normalized signals as

$$\begin{aligned} y_n(k) &= y(k)/n(k-1) \\ \phi_n(k-1) &= \phi(k-1)/n(k-1) \end{aligned} \tag{13}$$

where

$$n(k-1) = \max\{1, \|\phi(k-1)\|\}$$

and $\|\phi(k)\|$ is the Euclidean norm of $\phi(k)$. Then, (7) is equivalent to

$$y_n(k) = (\theta + \delta(k))'\phi_n(k-1) + d(k)/n(k-1). \tag{14}$$

With the normalized prediction error

$$e_n(k) = y_n(k) - \theta'(k-1)\phi_n(k-1) \tag{15}$$

and the normalized signals defined in (13), the estimation algorithm is given by

$$\theta(k) = \theta(k-1) + \alpha(k)\lambda(k)P(k)\phi_n(k-1)e_n(k) \tag{16}$$

$$P(k) = \frac{1}{\lambda(k)}\{P(k-1) - \frac{\alpha(k)P(k-1)\phi_n(k-1)\phi_n'(k-1)P(k-1)}{1+\phi_n'(k-1)P(k-1)\phi_n(k-1)}\}$$
$$P(0) = P'(0) > 0 \tag{17}$$

$$\lambda(k) = 1 - \frac{1}{\mathrm{tr}P(0)}\,\frac{\phi_n'(k-1)P(k-1)^2\phi_n(k-1)}{1+\phi_n'(k-1)P(k-1)\phi_n(k-1)} \tag{18}$$

where $\lambda(k)$ is a forgetting factor with $0 < \lambda(k) \leq 1$ and $\alpha(k)$ is the dead zone indicator which has a value 0 or 1.

The forgetting factor $\lambda(k)$ defined by (18) keeps the trace of $P(k)$ constant [12]. As a result, parameter updating will not stop in the case when there are model errors in the modelling of (2) and (3).

Now, denote the size of dead zone by $\Delta(k)$. Then $\alpha(k)$ is given by

$$\alpha(k) = \begin{cases} 1 & \text{if } \Delta(k) \leq \|e_n(k)\| \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

If we choose the size of dead zone $\Delta(k)$ as

$$\Delta(k) = \sqrt{\mu(k)}(\|\phi_n(k-1)\|D + W/n(k-1)) \tag{20}$$

$$\mu(k) = 1 + \phi_n'(k-1)P(k-1)\phi_n(k-1)$$

with some constants $D$ and $W$ satisfying

$$D \geq \sup\{\|\delta(k)\|\}$$

$$W \geq \sup\{\|d(k)\|\}$$

then the estimation scheme has the following properties which are crucial in proving the closed-loop stability.

**Lemma 3.1** : *For the parameter estimation algorithm (16) - (18) with the dead zone defined by (19) - (20), applied to the plant model (1) - (3), suppose that* $\sup\{\|d(k)\|\} \leq W$ *and* $\sup\{\|\delta(k)\|\} \leq D$. *Then the following properties hold:*

- *i)* $P(k)$ *is uniformly bounded.*
- *ii)* $\limsup\{\|e_n(k)\| - K(\|\phi_n(k-1)\|D + W/n(k-1))\} \leq 0$ *for some* $K \geq 1$
- *iii)* $\theta(k)$ *is uniformly bounded.*
- *iv)* $\|\theta(k) - \theta(k-1)\| \to 0, \quad as \quad k \to \infty.$

This theorem is a simple extension to the results in [12] (unmodelled plant uncertainties and external disturbances were not considered in [12]). We therefore briefly outline the proof.

*Proof (Outline)*: i) See [12].

ii) Define the functional

$$v(k) = (\theta(k) - \theta)P^{-1}(k)(\theta(k) - \theta) \tag{21}$$

Using (16) - (17) and the property that

$$0 < \lambda(k) \leq 1$$

we have

$$v(k) = \lambda(k)v(k-1) - \alpha(k)\lambda(k)\left(\frac{\|e_n(k)\|^2}{1+\phi_n'(k-1)P(k-1)\phi_n(k-1)} - \|\eta(k-1)\|^2\right) \tag{22}$$

where

$$\eta(k) = \theta'(k)\phi_n(k-1) + d(k)/n(k-1). \tag{23}$$

Then (19) - (20) gives

$$v(k) \leq v(k-1).$$

Since $v(k) \geq 0 \forall k$, $v(k)$ converges. Then ii) follows from (22).

iii) Using the fact that $v(k) \leq \lambda(k)v(k-1)$, we get

$$\|\theta(k) - \theta\|^2 \leq \mathrm{tr}\ (P(0))\sigma(0,k)v(0) \tag{24}$$

where

$$\sigma(i,k) = \lambda(i+1)\lambda(i+2)\cdots\lambda(k) \text{ for } i < k; \quad \sigma(k,k) = 1. \tag{25}$$

Therefore, iii) holds.

iv) Let

$$\lim_{k\to\infty} \sigma(0,k) = \sigma(0,\infty).$$

If $\sigma(0,\infty) = 0$, then $\theta(k) \to \theta$ from (24) and thus iv) holds. Now assume that $\sigma(0,\infty) \neq 0$. Then, $\lambda(k) \to 1$ as $k \to \infty$. and thus from (18)

$$P(k-1)\phi_n(k-1) \to 0. \tag{26}$$

Then iv) follows from (16) and (26). ■

# 4 Adaptive Control Scheme

We shall construct a feedback control law which makes the estimated model exponentially stable and show that this control law makes the closed-loop system robustly globally stable. Consider the observable realization (8) - (9) of the plant to be controlled. To denote that the plant model depends on the parameter vector $\theta$, we shall denote $F(k)$ and $G(k)$ by $F(\theta,k)$ and $G(\theta,k)$, respectively. Now given the estimate $\theta(k)$ of $\theta$ produced by the parameter estimator, we define the estimated model to be

$$x(k+1) = F(\theta(k),k)x(k) + G(\theta(k),k)u(k) \tag{27}$$

where $F(\theta(k),k)$ and $G(\theta(k),k)$ are equal to $F(\theta,k)$ and $G(\theta,k)$ with $\theta$ replaced by the estimate $\theta(k)$.

Suppose that $(F(\theta,k), G(\theta,k))$ with $\theta$ replaced by the estimate $\theta(k)$ violates the reachability condition (11). Then, we shall search a substitute $\bar{\theta}(k)$ for $\theta(k)$ where $\bar{\theta}(k)$ is in the form [12], [13]

$$\bar{\theta}(k) = \theta(k) + P(k)\gamma(k). \tag{28}$$

$\gamma(k)$ in (28) is a vector which is to be chosen if necessary but normally zero.

Define an augmented error $\bar{e}_n(k)$ by

$$\bar{e}_n(k) = y_n(k) - \bar{\theta}'(k-1)\phi_n(k-1). \tag{29}$$

Then

$$\bar{e}_n(k) = e_n(k) - \phi_n'(k-1)P(k-1)\gamma(k-1) \tag{30}$$

and thus $\bar{e}_n(k) \to e_n(k)$ as $k \to \infty$ provided $\gamma(k)$ is bounded. Furthermore,

$$\text{if } \gamma(k) - \gamma(k-1) \to 0 \text{ as } k \to \infty,$$

$$\bar{\theta}(k) - \bar{\theta}(k-1) \to 0.$$

In order that $\bar{\theta}(k)$ retains some properties of the estimation process as in Lemma 3.1, We want to impose the following properties on $\gamma(k)$ :

(a) $\gamma(k)$ is bounded.

(b) $\gamma(k) - \gamma(k-1) \to 0$, as $k \to \infty$.

(c) $\det\,(Y_{N+1}(\bar{\theta}(k),k) \geq \varepsilon$ for some $\varepsilon > 0$, $\forall k \in Z^+$.

Noting that $\det(Y_{N+1}(\theta,k))$ is a polynomial in entries of $\theta$, we can show that $\Delta\theta$ can always be found in a finite number of steps such that

$$\det\,(Y_{N+1}(\theta + \Delta\theta, k) \geq \varepsilon,$$

for some $\varepsilon > 0$ where $\varepsilon$ is independent on $k$ and for fixed $k \in Z^+$. Numerical procedures to find $\gamma(k)$ in a finite number of steps such that it satisfies (a) - (c) will not be treated here in details. Conceptually, a procedure is described as follows: We take in advance a subset $\Omega$ of the parameter space whose element satisfies the reachability condition. $\Omega$ is not necessarily covex nor connected. We choose the direction of $\gamma(k)$ such that the distance from $\theta(k)$ to $\Omega$ is minimized and the magnitude of $\gamma(k)$ is chosen as small as possible while satisfying the reachability condition.

We now assume that the system

$$x(k+1) = F(\theta,k)x(k) + G(\theta,k)u(k) \tag{31}$$

is reachabile in N steps for any $\theta$ replaced by $\bar{\theta}(k)$, $k \in Z^+$. Kleinman feedback gain for the system (31) obtained from (12) will be denoted as $L(\theta,k)$. On the other hand, in order to calculate Kleinman feedback gain for the estimated model by using (12), we need the future values of the parameter estimates. Nevertheless, as shown in the next lemma, $L(\theta(k),k)$ with $\theta$ replaced by $\theta(k)$ leads to a stabilizing feedback gain for the estimated model.

**Lemma 4.1** : *Let $L(\theta,k)$ be Kleinman feedback gain for the system (31) with $\theta$ replaced by $\theta(k)$. Then $L(\theta(k),k)$ is uniformly bounded and*

$$(F(\theta(k),k) - G(\theta(k),k)L(\theta(k),k))$$

*is exponentially stable.*

*Proof*: Since $\{f_j(k)\}$, $j = 1,\cdots,r$ are bounded functions and $\theta(k)$ is uniformly bounded, $L(\theta(k),k)$ is also uniformly bounded.

Now, denote the $N$-step reachability grammians of the system (31) and the estimated model by $Y_N(\theta,k)$ and $Y_N(\theta(k),\cdots,\theta(k+N),k)$, respectively. By property iv) of Lemma 3.1,

$$\|\theta(k) - \theta(k-1)\| \to 0 \text{ as } k \to \infty.$$

Therefore, as $k \to \infty$

$$\det(Y_{N+1}(\theta(k),k)) - \det(Y_{N+1}(\theta(k),\cdots,\theta(k+N),k)) \to 0$$

by continuity of $Y_{N+1}(\theta,k)$ as a function of $\theta$. The remainder of the proof is straightforward. ■

If $L(\theta(k),k)$ is a stabilizing feedback gain, then the adaptive control scheme is realized by

$$u(k) = -L(\theta(k),k)\hat{x}(k) \tag{32}$$

where $\hat{x}(k)$ is the estimated state vector. The estimate $\hat{x}(k)$ is obtained from the state observer

$$\hat{x}(k+1) = F(\theta(k), k)\hat{x}(k) + G(\theta(k), k)u(k) \\ +M(\theta(k), k)(y(k) - H\hat{x}(k)) \quad (33)$$

where

$$M'(\theta(k), k) = [a_1(\theta(k), k+1) \cdots a_n(\theta(k), k+n)]$$

and $a_i(\theta, k+i)$ is $a_i(k+i)$ with the coefficients replaced by elements of $\theta$. All eigenvalues of the observer (33) are located in the origin. Thus this observer has a finite settling time. Combining the control scheme (32), the state observer (33) and the parameter estimation scheme, we have the following theorem.

**Theorem 4.1** *: There exists $D^* > 0$ such that for any design parameters $D$ and $W$ with $0 \leq D < D^*$, and for any bounded external disturbances $d(k)$ and plant uncertainties $\delta^*(k)$ satisfying*

$$\sup\{\|\delta^*(k)\|\} \leq D$$

$$\sup\{\|d(k)\|\} \leq W$$

*the adaptive control law $u(k) = -L(\theta(k), k)\hat{x}(k)$ makes the closed-loop system globally stable.*

*Proof (Sketch)*: We briefly sketch the proof (see [15] for details). If there are no unmodelled plant uncertainties and no external disturbances in the plant model, the recursive least squares algorithm has the following property:

$$\|e(k)\| \leq c_1(k)\|\phi(k-1)\| + c_2(k)$$

where

$$c_1(k) \to 0\,,\; c_2(k) \to 0, \text{ as } k \to \infty.$$

Then, the proof of closed-loop stability directly follows from the proof techniques given in [17], [18]. However, we now have from Lemma 3.1 ii),

$$\limsup\{\|e_*(k)\| - K(\|\phi_*(k-1)\|D \\ +W/n(k-1))\} \leq 0 \text{ with } k \geq 1.$$

Thus, we use a discrete-time version of the Gronwall's Lemma to finally get

$$\|z(k+t)\| \leq K_1 c_1(k))^t\|z(k)\| + c_2(k) \\ \forall k \in Z^+, \text{ for some } K_1 \quad (34)$$

where both $c_1(k)$ and $c_2(k)$ are functions of D and W, and

$$z(k+1) = [\,\psi(k) \;\; \hat{x}(k+1)\,] \quad (35)$$

with

$$\psi'(k) = [y(k), \cdots, y(k-n+1)\, u(k), \cdots, u(k-n+1)\,].$$

Furthermore, for sufficiently large $k$, as $D$ goes to zero,

$$c_1 \rightarrow \gamma_o < 1 \text{ for some } \gamma_o$$
$$c_2 \rightarrow K_2 W \text{ for some } K_2.$$

We, therefore, can choose $D^*$ such that for any plant uncertainties $\delta(k)$ with $\sup\{\|\delta(k)\|\} < D$, the closed-loop is globally stable. ■

# 5 Discussion

We applied the proposed adaptive control scheme to the design of autopilot for air-to-air missiles using bank-to-turn steering. Simulations have shown that the control scheme works very well, even though large errors have been observed in the estimation of some of the time-varying parameters [15], [16].

To achieve the robust stability, we had to assume that the design parameter $D \geq \sup\{\|\delta(k)\|\}$ where $\delta(k)$ is the plant uncertainties. However, simulations without dead zone gave good performance. This fact implies that our adaptive control scheme without dead zone is, to a good extent, robust to the plant uncertainties.

To guarantee the reachability of the estimated model, we combined search technique with the standard recursive least squares algorithm. In this paper, numerical procedures to compute $\gamma(k)$ were not discussed in details, but, in simulations, a very simple scheme (if $\det(Y_{N+1}(\theta(k), k))$ is too small in magnitude, then use the previous feedback gain) gave good performance, which implies that complicate search processes may be unnecessary in practice. Current work is centering on the numerical procedures to choose $\gamma(k)$ which are practically useful.

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