A DISCRETE ITERATIVE LEARNING CONTROL METHOD WITH APPLICATION TO ELECTRIC SERVO MOTOR CONTROL

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Abstract — In this paper, an iterative learning control algorithm for unknown linear discrete systems is proposed by employing a parameter estimator together with an inverse system model. Regardless of initial error and inherent parameter uncertainty, a good tracking control performance is obtained using the proposed learning control algorithm characterized by recursive operations. A sufficient condition for convergency is provided to show the effectiveness of the proposed algorithm. To investigate the performance of the algorithm a series of simulations and experiments were performed for the tracking control of a servo motor.

I. Introduction

One of the superior capabilities of human workers over their mechanical counterparts is that in some circumstance, they can make use of their experience, either successful or unsuccessful, to improve the performance. Such an ability is called "learning capability", an inborn nature of living animals. On the other hand, a mechanical system will repeat same mistakes until it is readjusted by the operator because it does not have this ability.

Recently, there has been a growing interest in intelligent control system, particularly, in learning control systems. The iterative learning control as a type of the learning control systems accomplishes its learning through repetition or sequence of operations. In the iterative learning control, the control input sequence for next operation is determined by utilizing the experiences obtained from the past operations such that the state trajectory of the plant tracks the given desired trajectory as closely as possible within a finite number of iterations. Several research works are reported on designing such an iterative learning controllers. Arimoto et al. [1–3] proposed an iterative learning control scheme called "Betterment Process". In the algorithm, the time rate of error is utilized to generate a new control input which, in turn, improves the performance of the next operation. The algorithm is extended to discrete system by Togai et al. [4]. But their formulations require a priori knowledge of the plant in order to determine stable gain matrix. To alleviate the difficulty involved in determining stable gain matrix of previous work, Oh et al.[5] combined a parameter estimation technique with learning control method for a class of linear periodic systems. However, since the sufficient condition for convergency in [5] is expressed in terms of linear operators, it is not easy to examine a priori whether the system satisfies the condition. Moreover, the method in [5] which is obtained for continuous time systems is not easily extended to discrete systems due to the noncausality of the discrete time version of the linear operators. On the other hand, since the zero initial error condition should be assumed for every trial in the previous work [1–5], not only stability but the convergency is not guaranteed when the initial error between the plant output and the desired trajectory is not zero. Even when an initial position preset mechanism is installed in the learning controller to identically reset the starting point for every operation the convergency of the overall learning control system remains uncertain because it is highly dependent upon the accuracy of the preset mechanism.

To overcome the limitations of the previous studies, an iterative learning control method for unknown linear discrete systems is proposed. Specifically, the proposed algorithm is composed of two parts : The first one is the parameter estimator part which estimates the unknown parameters of the plant to provide the information on the plant characteristics. In contrast to the indirect method used in the previous study [5], a direct parameter estimation scheme which directly evaluates the controller gains is proposed to reduce the computational burden. The second one is the controller part which generates control input for the next operation based on the knowledge of the past operations and the estimated inverse model of the plant. Regardless of nonzero initial and the parameter estimation error, a sufficient condition for convergency is provided. Also, simulation and experimental studies are performed for the servo motor control to demonstrate the effectiveness of the proposed algorithm.

To help understanding the mathematical derivations in later sections, some notation are defined as followings:
For a matrix A, the transpose of A is denoted as A^T and A^+ represents a generalized inverse of A [9]. I_n and 0_n denote the n × n identity matrix and null matrix, respectively, and diag(·) denotes diagonal matrix. For given a finite dimensional vector x, $\|x\|$ denotes $\max_j |x^j|$ and $\|x(\cdot)\|_\Omega$ is defined as $\sup_{k\in\Omega} \|x(k)\|$. Also, the norm $\|A\|$ denotes the natural matrix norm induced by the vector norm [10], that is,

$$\|A\| \overset{\Delta}{=} \sup \{\|Ax\|/\|x\| : \|x\| \neq 0\}.$$

II. An Iterative Learning Control Algorithm for Discrete Systems.

Consider a linear discrete system described by

$$x(k+1) = \Phi\, x(k) + \Gamma\, u(k) : \; x(0) = \zeta_0 \quad k \in \Omega \quad (1)$$

where x is the $n \times 1$ state vector and u is the $m \times 1$ control input vector. Also, Φ is the $n \times n$ system matrix assumed to be stable, Γ is the $n \times m$ input matrix. The set Ω is defined as $\{ k : 0 \leq k \leq N \}$ where N is the final step number.

Let $\{ x_d(k) : k \in \Omega \}$ be given the desired state trajectory and $\varepsilon^\star$ be given as a tolerance bound. We wish to find a control input sequence $\{ u(k) : k \in \Omega \}$ in the iterative manner such that the corresponding output trajectory $\{ x(k) : k \in \Omega \}$ of the system in (1) satisfies

$$\| e(\cdot) \|_\infty = \| x_d(\cdot) - x(\cdot) \|_\infty \leq \varepsilon^\star .$$

Now, we construct an iterative learning controller for the system in (1) as follows:

$$x_i(k+1) = \Phi\, x_i(k) + \Gamma\, u_i(k) \qquad (2.a)$$
$$e_i(k) = x_d(k) - x_i(k) \qquad (2.b)$$
$$u_{i+1}(k) = u_i(k) + \hat{\Gamma}_i^+ \{ e_i(k+1) - \hat{\Phi}_i\, e_i(k) \} \qquad (2.c)$$

where $\hat{\Gamma}_i$ and $\hat{\Phi}_i$ denote the estimated values of Γ and Φ, respectively and the suffix i denote the iteration ordinal number of trial. In the algorithm ,an initial input sequence for first trial $\{ u_1(k) : k \in \Omega \}$ is arbitrarily chosen. The details of the parameter estimation algorithm for $[\, \hat{\Gamma}_i : \hat{\Phi}_i \,]$ in the (2.c) are described in the chapter III.

Now, the sufficient condition [6–8] for the convergency of the proposed algorithm is shown by the following theorem.

Theorem 1 : Consider a stable linear discrete system in (1). If the initial error $e_i(0) = 0$ for all i and if the eigenvalues of matrix $I_m - \hat{\Gamma}_i^+ \Gamma$ are all within the unit circle, then, the iterative learning controller in (2) yields

$$\| e(\cdot) \|_\infty = 0, \qquad \text{as } i \longrightarrow \infty. \qquad (3)$$

Some preliminary results necessary for the proof of convergence will be shown in advance.

Defining the $n(N+1) \times 1$ vector $\mathcal{X}_i$ and the $mN \times 1$ vector $\mathcal{U}_i$ as

$$\mathcal{X}_i \triangleq [\, x_i(0)^T\ x_i(1)^T \cdots x_i(N)^T]^T$$
$$\mathcal{U}_i \triangleq [\, u_i(0)^T\ u_i(1)^T \cdots u_i(N-1)^T \,]^T$$

and the $n(N+1) \times mN$ matrix P and the $n(N+1) \times n$ matrix Q as

$$P \triangleq \begin{bmatrix} 0 & & & & \\ \Gamma & & & \bigcirc & \\ \Phi\Gamma & \Gamma & & & \\ \vdots & & \ddots & & \\ \Phi^{N-1}\Gamma & \Phi^{N-2}\Gamma & \cdots & \Phi\Gamma & \Gamma \end{bmatrix} \qquad (4)$$

$$Q \triangleq \begin{bmatrix} I_n \\ \Phi \\ \Phi^2 \\ \vdots \\ \Phi^N \end{bmatrix} \qquad (5)$$

the response of the system described in (1) can be written as

$$\mathcal{X}_i = P\, \mathcal{U}_i + Q\, x_i(0). \qquad (6)$$

If we assume that the inverse system of the plant described by (1) takes following form

$$u(k) = \Gamma^+(x(k+1) - \Phi\, x(k)), \quad k \in \Omega, \qquad (7)$$

then an $mN \times n(N+1)$ matrix R is defined

$$R \triangleq \begin{bmatrix} -\Gamma^+\Phi & \Gamma^+ & & & \bigcirc \\ & -\Gamma^+\Phi & \Gamma^+ & & \\ & & \ddots & & \\ \bigcirc & & & -\Gamma^+\Phi & \Gamma^+ \end{bmatrix} \qquad (8)$$

Here, it is important to note to the following properties:

$$R\,P = I \quad \text{and} \qquad (9)$$
$$R\,Q = 0. \qquad (10)$$

By letting $\mathcal{U}_d = [\, u_d(0)^T\ u_d(1)^T \cdots u_d(N-1)^T \,]^T$ be a sequence of the control input which generates the given desired trajectory $\mathcal{X}_d = [\, x_d(0)\ x_d(1)^T \cdots x_d(N)^T \,]^T$ preset for finite control period, we obtain

$$\mathcal{X}_d = P\, \mathcal{U}_d + Q\, x_d(0). \qquad (11)$$

Also, by substituting $\{ \Phi, \Gamma \}$ in (7) with $\{ \hat{\Phi}_i, \hat{\Gamma}_i \}$, an $mN \times m(N+1)$ matrix $\hat{R}_i$ is defined which represents the estimated value of R. Then, by using the matrix $\hat{R}_i$, the proposed control scheme described in (2.c) can be rewritten as

$$\mathcal{U}_{i+1} = \mathcal{U}_i + \hat{R}\, \mathcal{E}_i \qquad (12)$$

where, the $n(N+1) \times 1$ vector $\mathcal{E}_i$ can be defined as

$$\mathcal{E}_i \triangleq [\, e_i(0)^T\ e_i(1)^T \cdots e_i(N)^T \,]^T$$

Now, based upon above preliminary results, the theorem 1 will be proven.

Proof of Theorem 1 : By subtracting (6) from (11), the error equation obtained as

$$\mathcal{E}_i = P\,(\mathcal{U}_d - \mathcal{U}_i) + Q\, e_i(0) \qquad (13)$$

Premultiplying the both sides of (13) with the R matrix and using the properties of (9) and (10), the following equation is obtained

$$R\, \mathcal{E}_i = R\, P\, (\, \mathcal{U}_d - \mathcal{U}_i \,) + R\, Q\, e_i(0)$$
$$= \mathcal{U}_d - \mathcal{U}_i. \qquad (14)$$

If we define $\mathcal{Z}_i$ as $\mathcal{U}_d - \mathcal{U}_i$, it follows from (13), (14) and the condition $e_i(0) = 0$ that

$$\mathcal{E}_i = P\, \mathcal{Z}_i \qquad (15)$$
$$\mathcal{Z}_i = R\, \mathcal{E}_i$$
$$= \mathcal{U}_d - \mathcal{U}_i \qquad (16)$$

Hence, using (15) and (16), we get

$$\mathcal{Z}_{i+1} = \mathcal{Z}_i - (\, \mathcal{U}_{i+1} - \mathcal{U}_i \,)$$

$$
\begin{aligned}
&= \mathscr{Z}_i - \hat{R}_i\, \mathscr{E}_i \\
&= \mathscr{Z}_i - \hat{R}_i\, P\, \mathscr{Z}_i \\
&= H_i\, \mathscr{Z}_i
\end{aligned}
\tag{17}
$$

where the $mN \times mN$ matrix H_i denotes $(I_{mN} - \hat{R}_i P)$ which will be called learning transition matrix in the sequel. Here, the learning transition matrix H_i can be expressed as the following lower triangular block matrix :

$$
H_i = \begin{bmatrix} h_1 & & & & \bigcirc \\ h_2 & h_1 & & & \\ \vdots & & \ddots & & \\ \vdots & & & h_1 & \\ h_N & h_{N-1} & \cdots & h_2 & h_1 \end{bmatrix}
\tag{18}
$$

$$
h_j = \begin{cases} I_m - \hat{\Gamma}_i^{\dagger}\Gamma & \text{for } j = 1 \\ -\hat{\Gamma}_i^{\dagger}\tilde{\Phi}\Phi^{j-1}\Gamma & \text{for } j > 1 \end{cases}
$$

where $\tilde{\Phi}$ represents $(\Phi - \hat{\Phi})$. If we consider (17), it is easy to see that $\mathscr{Z}_i$ converges to zero only under the condition that all eigenvalues of the learning transition matrix H_i are located within the unit circle in complex plane. Since H_i is a lower triangular block matrix, this is equivalent to the condition that the eigenvalues of $(I_m - \hat{\Gamma}_i^{\dagger}\Gamma)$ are all within the unit circle in complex plane. Hence, it is obvious that

$$
\mathscr{Z}_i = \mathscr{U}_d - \mathscr{U}_i \longrightarrow 0 \quad \text{as } i \longrightarrow \infty \tag{19}
$$

and from (15),

$$
\mathscr{E}_i \longrightarrow 0 \quad \text{as } i \longrightarrow \infty. \tag{20}
$$

Therefore, the statement in (3) is obvious.
This completes the proof.

Several remarks are provided to avoid ambiguity regarding the theorem 1. First, it is noted that in agreement with previous work [1], only the accuracy of a priori knowledge or estimation regarding the input matrix Γ determines whether the iterative learning control is convergent or not since the eigenvalues of the learning transition matrix H_i are determined by from input matrix related terms. Secondly, it is noted that uniform convergency can be assured if $\{\hat{\Gamma}_i, \hat{\Phi}_i\}$ is suitably estimated in such a way that the learning transition matrix satisfies a more strong condition, $\| H_i \| < 1$, than the condition regarding the eigenvalues of the matrix H_i in Theorem 1.

In the next chapter, a direct parameter estimation scheme is proposed and its suitability for the proposed control are checked.

III. A direct parameter estimation scheme.

To complete the proposed learning control method, we now propose a direct parameter estimation scheme, in which the estimation is performed with respect to controller gain $[\,\Gamma^+ : \Gamma^+\Phi\,]$ directly, not with respect to system matrix $[\,\Phi : \Gamma\,]$ as can be seen in the preceded study [5]. In comparison with the indirect method, the proposed direct method reduces the computation time as well as memory size. Based upon the inverse model defined in (8), the following equation holds

$$
u(k) = \theta\phi(k), \qquad k \in \Omega \tag{21}
$$

where the $m \times 2n$ parameter matrix θ and $2n \times 1$ data vector ϕ are defined as

$$
\begin{aligned}
\theta &\stackrel{\Delta}{=} [\Gamma^+ : -\Gamma^+\Phi\,] \\
\phi(k) &\stackrel{\Delta}{=} [\, x(k+1)^T : x(k)^T\,]^T.
\end{aligned}
$$

Therefore, we wish to identify $2nm$ parameters of θ. To do this, let the matrix θ be partitioned as:

$$
\theta = \begin{bmatrix} \theta^{1T} \\ \vdots \\ \theta^{mT} \end{bmatrix} \tag{22}
$$

where θ^{jT} is a $2n \times 1$ row vector. Then, (21) can also be described by using the partitioned vector as

$$
u^j(k) = \theta^{jT}\phi(k), \qquad j = 1, \cdots, m \tag{23}
$$

where $u^j(k)$ is the j th element of the $m \times 1$ input vector $u(k)$. Now, we may apply a conventional recursive estimation method [11] with regard to the relation (21). That is, for $j = 1, \cdots, m$, let the estimated parameter vector $\hat{\theta}_i^j$ in the i th operation be given by

$$
\hat{\theta}_i^j(k+1) = \hat{\theta}_i^j(k) + G_i^j(k+1)\phi_i^j(k)^T\epsilon_i^j(k), \tag{24}
$$

where $G_i^j(k)$ is the estimator gain matrix and $\delta_i^j(k)$ is defined as [11]

$$
\epsilon_i^j(k) \stackrel{\Delta}{=} u_i^j(k) - \hat{\theta}_i^j(k)^T\phi_i(k). \tag{25}
$$

Here, for each operation trial, all the initial values are given appropriately as in the conventional recursive estimation method [11].

Now, the standard parameter estimation algorithms for the stable plant provides the following properties [12,13], for all $j = 1, \cdots, m$ and $i = 1, \cdots$

- $\hat{\theta}_i^j(k)$ is bounded for all k, (26)
- $\lim_{k \to \infty} \{\, \hat{\theta}_i^j(k) - \hat{\theta}_i^j(k-h)\} = 0$ for any finite integer h (27)
- $\lim_{k \to \infty} \{\, \epsilon_i^j(k)\,\}^2 = 0,$ (28)

Since the parameter estimation algorithm is continued for every trial in the proposed algorithm, $i \to \infty$ implies $k \to \infty$. And the standard estimation algorithm satisfies above conditions (26–28), we get the following important properties of the proposed parameter estimation method:

1) It follows from (26) and (27) that $\hat{R}_i$ is bounded and asymptotically invariant as the number of iteration increases,
2) From (28), it is obvious that

$$
\begin{aligned}
\mathscr{U}_i - \hat{R}_i\, \mathscr{X}_i &= \mathscr{U}_i - \hat{R}_i\, P\, \mathscr{U}_i \\
&= H_i\, \mathscr{U}_i \\
&= 0 \qquad \text{as } i \longrightarrow \infty\,.
\end{aligned}
\tag{29}
$$

Since the above properties are satisfied for arbitrary control input sequence $\mathscr{U}_i$, the conditions to satisfies (29) can be obtained as

$$H_i = 0 \text{ and } \tag{30}$$

Hence, the convergency of the proposed control scheme is guaranteed since the proposed parameter estimation scheme reduces both H_i down to zero as iteration proceeds.

The structure of the proposed learning control method in (2) with the parameter estimator in (24–25) is shown in Fig.1.

IV. Numerical Example

To show the effectiveness of the proposed learning control method, a series of computer simulations are performed for the mathematical model of an electric servo motor system. Also, through the examples, the effect of the parameter miss–matching is investigated to check the applicability of the proposed algorithm using the biased parameter estimator.

To show the performance of the proposed learning control scheme, a servo motor system shown in Fig.3 is modeled as a 2nd order system and is given by:

$$x(k+1) = \begin{bmatrix} 0.93126 & 0.01188 \\ -5.74869 & 0.29368 \end{bmatrix} x(k) + \begin{bmatrix} 0.06874 \\ -5.74869 \end{bmatrix} u(k),$$
$$x(k) = [\ \alpha(k)\ ,\ \dot{\alpha}(k)\ ]^T, \tag{31}$$

where α, $\dot{\alpha}$ represent the angular position and velocity, respectively. The sampling time T is chosen to be 0.02 sec and learning operation is assumed to be performed during k $\in$ [0,200]. Let the desired state trajectory $x_d(k) = [\ \alpha_d(k)\ ,\ \dot{\alpha}_d(k)\ ]^T$ be given by

$$\alpha_d(k) = \begin{cases} 25\pi(1-\cos(\pi(kT-1))),\ \text{rad.} & 2 \le kT \le 3 \\ 0, & \text{elsewhere} \end{cases}, \tag{32}$$

$$\dot{\alpha}_d(k) = \begin{cases} 25\pi^2 \sin(\pi(kT-1)),\ \text{rad./sec} & 2 \le kT \le 3 \\ 0, & \text{elsewhere} \end{cases}. \tag{33}$$

The results in Theorem 1 are checked by letting controller parameters $[\ \hat{\Gamma}^+ : \hat{\Gamma}^+\hat{\Phi}\ ]$ be fixed for all trials. Since the eigenvalues of the learning transition matrix H_i are all identical to a value $\lambda = (1-\hat{\Gamma}^+\Gamma)$ when m = 1, the effects of the λ to the convergency are investigated. Fig.2 (a) shows the mean absolute error convergence when λ is assigned by 0.3, 0.0, –0.9 and 1.1, respectively while setting $\hat{\Phi} = \Phi$ for all cases. If $\hat{\Phi} = \Phi$ i.e. $H_i = \mathrm{diag}(\lambda)$, the uniform convergence is guaranteed when $\lambda < 1$ since the ratio of the error written as $\|\mathscr{E}_{i+1}\| / \|\mathscr{E}_i\| = \lambda$ is less than 1 for all i. For the case $\lambda = 1.1$, as expected, the error quickly diverges in contrast to other cases. For a certain value of λ, the $1 - \hat{\Gamma}^+\Gamma = \lambda$ has infinite number of solutions of the $1 \times n$ row vector $\hat{\Gamma}^+$. However, the convergence characteristics are invariant for any arbitrary choice of the solution $\hat{\Gamma}^+$ since the convergence of $\|\mathscr{E}_i\|$ does not depend upon the particular choice of $\hat{\Gamma}^+$ but upon λ.

V. Experiments

A series of experiments are performed for the tracking control of electric servo motor to show the applicability of the proposed learning control. The experiment is set up for the servo motor model used in the simulation and the setup is shown in Fig.3. The system is composed of a motor drive, a position control board with A/D converter and an IBM–PC computer. An analog servo for internal velocity feedback and a PWM amplifier is provided in the motor drive. The position control board adopting INTEL 8032 acts as a proportional gain position controller to generate the voltage input to analog servo using both the encoder feedback and the command input from the PC. The controller has a 12bit A/D converter to transfer the angular velocity from the tacho generator to the PC. The personal computer computes the proposed learning control algorithms and enables the communication between the PC and the position controller board through a serial I/O port using standard RS–232C. It is noted that the servo motor including the motor drive and the position controller is assumed to be the plant to be controlled by the proposed learning control scheme.

In this experiment, the recursive least square method is adopted as a parameter estimation method. In Fig.4 (a)(b), the tracking responses for the desired trajectory in (32) and (33) are plotted when $x_i(0) = [10\pi,\ 0]^T$. Since the true values of the real plant parameters are unknown, the eigenvalues of the learning transition matrix can not be evaluated theoretically. Nevertheless, the convergency rate can arbitrarily be assigned by substituting $\hat{\Gamma}_i^\dagger$ with $(1-\lambda)\ \hat{\Gamma}_i^\dagger$ in the algorithm (2.c) since in this case, the conditions $H_i = \mathrm{diag}(\lambda)$ and $\hat{R}_i Q = 0$ are ensured as iteration proceeds. Obviously, both position and velocity trajectory track the desired trajectories from the second operation as shown in Fig.4 (a) since the proposed parameter estimation scheme reduces both H_i and $\hat{R}_i Q$ to zero after the first operation. Fig.4 (b) shows the tracking performance for λ is 0.3.

To investigate the adaptation capability of the proposed control against the long term parameter change, the case when the position feedback gain k_p within the plant varies from 1 to 2 after the 5th trial is also considered in the experiment although such a sudden variation is hardly found in real plant. As shown in Fig.5, it can be seen that the mean absolute error is successfully recovered after the 6th trial.

VI. Concluding Remarks

An iterative learning control algorithm for the unknown linear discrete systems was proposed in which the direct parameter estimation was employed with respect to the controller gain matrix directly. A sufficient condition for convergency of the proposed algorithm was given regardless of the initial error and parameter estimation error. The merits of the proposed control schemes in comparison with others are as follows: First, it can effectively cope with the long term parameter variations since the installed parameter estimator works at every trial iteration. Second, for a class of systems which can be approximately considered as a linear system, the convergency speed is very high in comparison with other algorithms. Third, the proposed algorithm shows that the initial error affects only the control accuracy of the transient period but not the convergency of the whole learning system.

References

[1] S. Arimoto, S. Kawamura and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. **1**, no. 2, pp.706–712, 1984.

[2] S. Kawamura, F. Miyazaki, and S. Arimoto, "Applications of learning method for dynamic control of robot manipulators," *Proc. of 24th IEEE Conference on Decision and Control,* pp.1381–1386, 1985.

[3] S. Kawamura, F. Miyazaki, and S. Arimoto "Realization of robot motion based on a learning method," *IEEE Trans. on Systems, Man, Cybernetics,* vol. 18, no. 1, pp.126–134, 1988.

[4] M. Togai and O. Yamano "Learning control and its optimality: Analysis and its Application to controlling industrial robotics," *Proc. IEEE International Conference on Robotics and Automation,* vol.1, 1985.

[5] S.R. Oh, Z. Bien, and I.H. Suh, "An iterative learning control methodwith application for the robot manipulators," *IEEE Journal of Robotics and Automation,* vol. 4, no. 5, pp.508–514, 1988.

[6] T. Sugie and T. Ono, " On a learning control law," System and Control vol.31, no.2, pp.129–135, 1987.

[7] H.J. Park and H.S. Cho, " A learning control algorithm for the linear discrete system," Proc. KACC vol1, pp.326–331, 1988.

[8] D.H. Hwang, Z. Bien and S.R. Oh, " Iterative adaptive control of partially known system under tight servo constraints," Proc. KACC, vol.2, pp. 682–685, 1989."

[9] S. Barnett, *Matrices in Control Theory.* New York, NY: Van Nostrand Reinhold, 1971.

[10] D. G. Luenberger, *Optimization by Vector Space Methods.* New york, NY: John Wiley & Sons,Inc., 1969.

[11] L. Ljung and T. Söderström, *Theory and Practice of Recursive Identification.* London, England: MIT Press., 1983.

[12] G.C. Goodwin, P.J. Ramadge, and P. E. Caines, "Discrete time multivariable adaptive control," *IEEE Journal Automatic Control,* vol. 25, 1980.

[13] R. Lozano and I.D. Landau, "Redesign of explicit and implicit discrete time model reference adaptive control scheme," *Int. J. Control,* vol.33, no.2, pp.247–268, 1981.

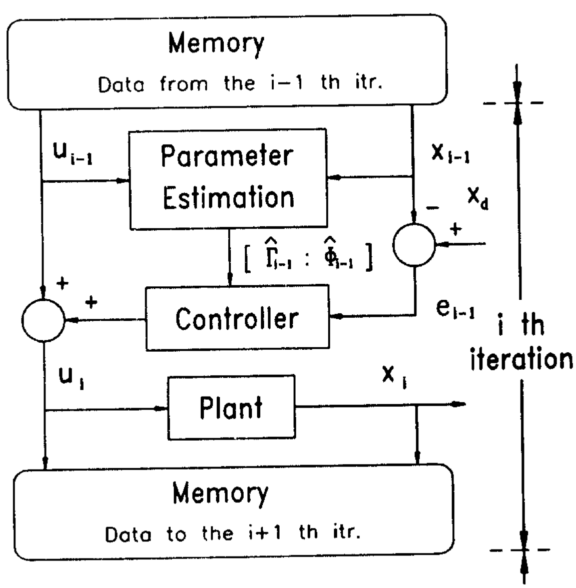


Fig.1 Block diagram of the proposed iterative learning control

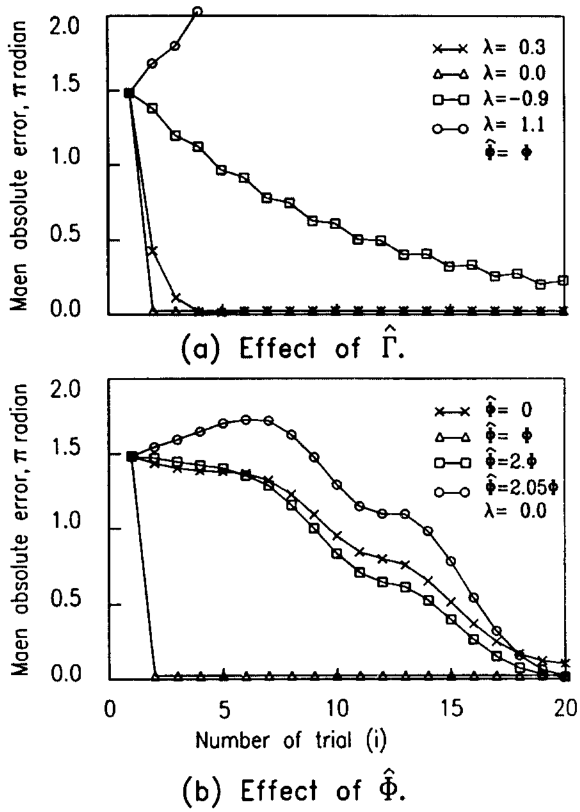


Fig.2 Effect of the parameter miss –matching on the convergency

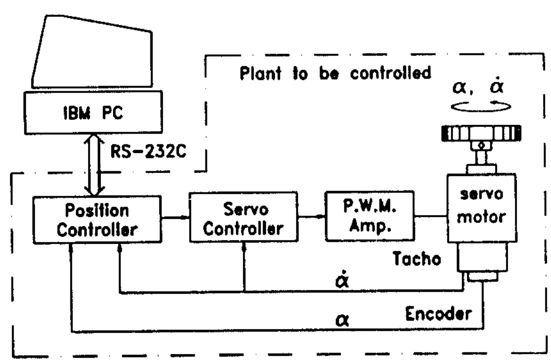
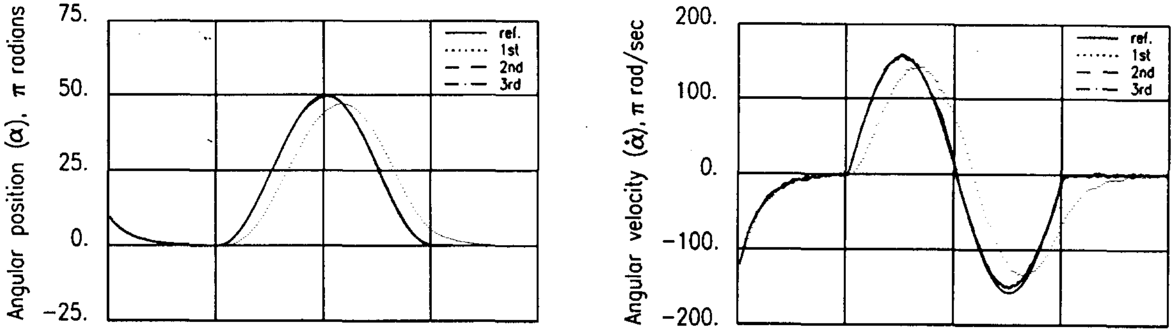
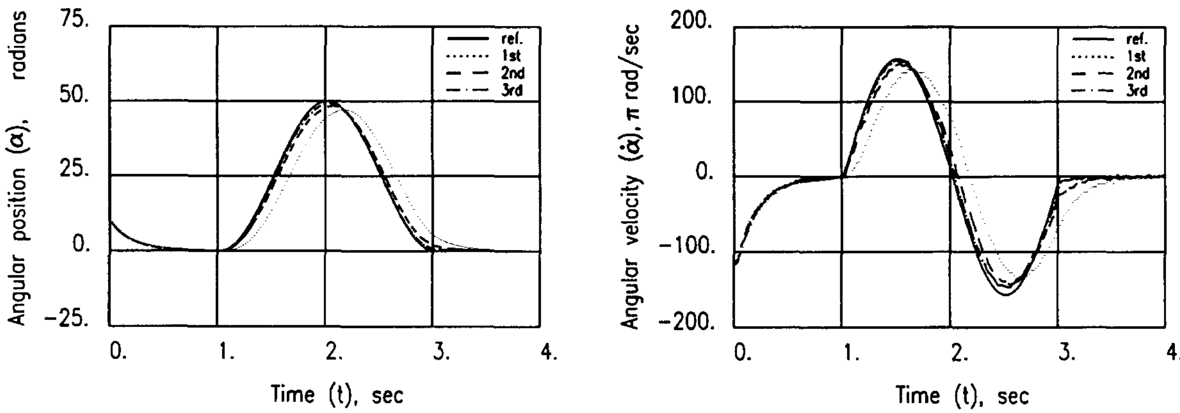


Fig.3 Schematic diagram of the experimental system



(a) $\lambda = 0.$



(b) $\lambda = 0.3$

Fig.4 Experimental results of the proposed iterative learning control

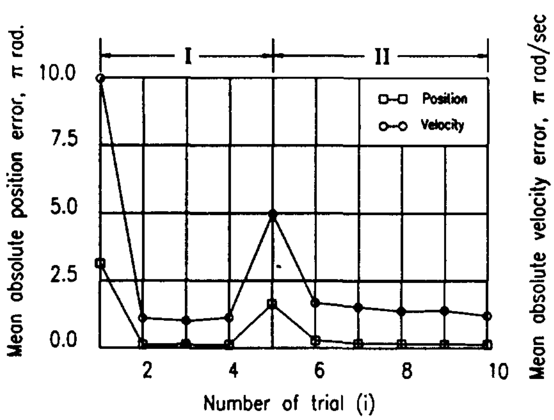


Fig.5 Adaptation performance of the proposed learning control

	Condition	Estimated parameter [$\hat{\Gamma}_i^{*}$: $-\hat{\Gamma}_i^{*}\hat{\Phi}_i$]			
I	Kp = 1.0	0.402	0.027	0.599	0.122
II	Kp = 2.0	−3.885	0.060	4.882	0.127