DESIGN OF A NEW COMMAND TO LINE-OF-SIGHT GUIDANCE LAW VIA FEEDBACK LINEARIZATION TECHNIQUE

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ABSTRACT

This paper describes the application of the recently developed feedback linearization technique to the design of a new command to line-of-sight (CLOS) guidance law for skid-to-turn (STT) missiles. The key idea lies in converting the three dimensional CLOS guidance problem to the tracking problem of a time-varying nonlinear system. Then, using a feeedback linearizing approach to tracking in nonlinear systems, we design a three dimensional CLOS guidance law that can ensure zero miss distance for a randomly maneuvering target. Our result may shed new light on the role of the feedforward acceleration terms used in the earlier CLOS guidance laws. Furthermore, we show that the new CLOS guidance law can be computationally simplified without performance degradation. This is made possible by dropping out the terms in the new CLOS guidance law, which obey the well-known matching condition.

NOMENCLATURE

ψ_m	Yaw angle of the missile
θ_m	Pitch angle of the missile
ϕ_{mc}	Roll angle command
σ_t	Azimuth angle of the LOS to target
γ_t	Elevation angle of the LOS to target
σ_m	Azimuth angle of the LOS to missile
γ_m	Elevation angle of the LOS to missile
$\Delta\sigma$	σ_m-σ_t
$\Delta\gamma$	γ_m-γ_t
T	Thrust force
D	Drag force
M	Mass of the missile
g	Gravity acceleration
a_x	Axial acceleration of the missile
a_{yc}	Yaw acceleration command
a_{zc}	Pitch acceleration command
v_m	Missile velocity
R_m	Missile range from the ground tracker
R_t	Target range from the ground tracker
$s\theta$	$\sin\theta$
$c\theta$	$\cos\theta$
$\|A\|$	Induced norm of a matrix A
$\lvert x \rvert$	Euclidean norm of a vector x
(X_I,Y_I,Z_I)	Inertial frame
(X_M,Y_M,Z_M)	Missile body frame
(X_L,Y_L,Z_L)	LOS frame
(x_m,y_m,z_m)	Missile position in the inertial frame
(R_p,c_1,e_2)	Missile position in the LOS frame
i_M, i_L	Unit vectors corresponding, respectively, to the X_M, X_L axes
i_I, j_I, k_I	Unit vectors corresponding, respectively, to the X_I, Y_I, Z_I axes

I. INTRODUCTION

The principle of command to line-of-sight (CLOS) guidance [1-7] is to force the missile to fly as nearly as possible along the instantaneous line joining the ground tracker and the target, which is called the line-of-sight (LOS). The CLOS guidance has been regarded as a low-cost guidance concept because it emphasizes placement of avionics on the launch platform,' as opposed to on board the expendable weapon.

The CLOS guidance laws for skid-to-turn (STT) missiles are composed of two lateral acceleration commands, pitch and yaw. Each of them is shaped by the sum of an error compensation acceleration term to null deviation of the missile from the LOS to target and a feedforward acceleration term to make the missile chase the LOS rotation. The feedforward acceleration term plays an important role when large LOS rates occur, as is the case with CLOS guidance for short-range air defence intercept scenarios. In the earlier results [2-4], however, this feedforward acceleration term has been derived approximately under some restrictive assumptions. In [2,3], it is assumed that the missile is on the LOS to target and its velocity vector lies on the so called flyplane. In [4], the lateral acceleration of the missile projection point onto the LOS to target is adopted as the feedforward acceleration term.

In this paper, we focus our efforts on elucidating the precise expression and role of feedforward acceleration term for the case of general pursuit situation. We first show that the general three dimensional CLOS guidance problem can be converted to a nonlinear tracking problem. This is our key result. Thereby, the recently developed feedback linearization technique [5-8,10,11] can be easily applied to the CLOS guidance problem. We propose a new CLOS guidance law. It involves a term which is, in the form, similar to the feedforward acceleration term used in the earlier results [2-4]. This term in our new CLOS guidance law is required to transform the nonlinear tracking problem into a linear one, while the feedforward acceleration term in the earlier results is used to make the missile chase the LOS rotation. Thus, our result may shed new light on the role of the feedforward acceleration terms used in the earlier CLOS guidance laws. It is shown that our CLOS guidance law can drive miss distance to zero against a randomly maneuvering target in the three dimensional space. It is, however, computationally complex. In this context, we attempt to simplify the new CLOS guidance law so that the computational burden is reduced without performance degradation. To the authors' knowledge, however, our paper presents the first result to apply the feedback linearization technique to missile guidance.

Ⅱ. PROBLEM FORMULATION

In this section, we show that three dimensinal CLOS guidance problem can be formulated as a tracking problem of a time-varying nonlinear system.

In modelling the pursuit dynamics of missile and target, we assume that

A1: Compared with the overal guidance loop, the autopilot and ground tracker dynamics are fast enough to be neglected.

A2: The total angle-of-attack is small enough to be neglected.

These assumptions have been generally accepted in the design and analysis of missile guidance laws. However, as will be discussed soon, the assumption A2 is not the precondition for the desired performance of our new CLOS guidance law but is introduced only for simplicity of our developments.

The three dimensional pursuit situation is depicted in Fig. 1. Under the prescribed assumptions, motion of the missile in the inertial frame can be represented by

$$\begin{aligned} \ddot{x}_m &= a_x c\theta_m c\psi_m - a_{yc}(s\phi_{mc} s\theta_m c\psi_m + c\phi_{mc} s\psi_m) \\ &\quad - a_{zc}(c\phi_{mc} s\theta_m c\psi_m - s\phi_{mc} s\psi_m) , \\ \ddot{y}_m &= a_x c\theta_m s\psi_m - a_{yc}(s\phi_{mc} s\theta_m s\psi_m - c\phi_{mc} c\psi_m) \\ &\quad - a_{zc}(c\phi_{mc} s\theta_m s\psi_m + s\phi_{mc} c\psi_m) , \\ \ddot{z}_m &= a_x s\theta_m + a_{yc} s\phi_{mc} c\theta_m + a_{zc} c\phi_{mc} c\theta_m - g , \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{\psi}_m &= a_{yc} c\phi_{mc}/(v_m c\theta_m) - a_{zc} s\phi_{mc}/(v_m c\theta_m) , \\ \dot{\theta}_m &= a_{yc} s\phi_{mc}/v_m + a_{zc} c\phi_{mc}/v_m - g c\theta_m/v_m . \end{aligned} \tag{2}$$

Here, v_m is the velocity of the missile given by

$$v_m = (\dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2)^{1/2} , \tag{3}$$

and a_x is the axial acceleration of the missile given by

$$a_x = (T-D)/M . \tag{4}$$

Note that the assumption A2 is used only in deriving a simplified model (2) of missile attitude dynamics. As will be seen in Section Ⅳ and Ⅴ, the derivation of our CLOS guidance laws (38), (41) does not rely on the simplified model (2) of missile attitude dynamics. Hence, omission of A2 does not result in any degradation of guidance performance.

Next, we define tracking error in order to convert the CLOS guidance problem into a tracking problem. As mentioned in Section Ⅰ, the concept of CLOS guidance is to guide the missile onto the LOS to target. Therefore, a reasonable choice of tracking error may be

$$e \triangleq \begin{bmatrix} \sigma_t \\ \gamma_t \end{bmatrix} - \begin{bmatrix} \sigma_m \\ \gamma_m \end{bmatrix} \tag{5}$$

Even in the case of small tracking error, however, this selection can cause large miss distance as the missile flies farther from the launch point.

To overcome this shortcoming, we consider a different choice of tracking error. We first define the LOS frame as is shown in Fig. 2. The coordinates (R_p, e_1, e_2) indicated in Fig. 2 represent the missile position in the LOS frame. We define the tracking error as

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \triangleq \begin{bmatrix} -s\sigma_t x_m + c\sigma_t y_m \\ -s\gamma_t c\sigma_t x_m - s\gamma_t s\sigma_t y_m + c\gamma_t z_m \end{bmatrix} . \tag{6}$$

Note that $|e|$ just represents the smallest distance from the missile to the LOS to target. Therefore, the missile eventually will hit the target if the tracking error is driven to zero before the target crosses the missile.

So far, we have shown that the three dimensional CLOS guidance problem can be formulated as a tracking problem. From notational convenience, the column vector $[x_1 \ldots x_n]^T$ will occasional be denoted by $(x_1, \ldots, x_n)$. $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (x_m, y_m, z_m, \dot{x}_m, \dot{y}_m, \dot{z}_m, \psi_m, \theta_m)$, $u = (u_1, u_2) = (a_{yc}, a_{zc})$ and $y_d = (y_{d1}, y_{d2}) = (\sigma_t, \gamma_t)$. Using these notations and regarding the axial acceleration of the missile as a time function, we can write the equations (1), (2) and (6) in the following state space form:

$$\dot{x} = f_0(x,t) + \sum_{i=1}^{2} f_i(x) u_i , \quad e = E(x, y_d) \tag{7}$$

where

$$\begin{aligned} f_0(x,t) = (&x_4, x_5, x_6, a_x(t) c x_7 c x_8, a_x(t) s x_7 c x_8, \\ &a_x(t) s x_8 - g, 0, -g c x_8/(x_4^2 + x_5^2 + x_6^2)^{1/2}), \end{aligned} \tag{8}$$

$$\begin{aligned} f_1(x) = (&0,0,0, -s\phi_{mc} s x_8 c x_7 - c\phi_{mc} s x_7, -s\phi_{mc} s x_8 s x_7 + c\phi_{mc} c x_7, \\ &s\phi_{mc} c x_8, c\phi_{mc}/\{(x_4^2 + x_5^2 + x_6^2)^{1/2} c x_8\}, s\phi_{mc}/(x_4^2 + x_5^2 + x_6^2)^{1/2}), \end{aligned} \tag{9}$$

$$\begin{aligned} f_2(x) = (&0,0,0, -c\phi_{mc} s x_8 c x_7 + s\phi_{mc} s x_7, -c\phi_{mc} s x_8 s x_7 - s\phi_{mc} c x_7, \\ &c\phi_{mc} c x_8, -s\phi_{mc}/\{(x_4^2 + x_5^2 + x_6^2)^{1/2} c x_8\}, c\phi_{mc}/(x_4^2 + x_5^2 + x_6^2)^{1/2}), \end{aligned} \tag{10}$$

$$E(x, y_d) = (-s\sigma_t x_1 + c\sigma_t x_2, -s\gamma_t c\sigma_t x_1 - s\gamma_t s\sigma_t x_2 + c\gamma_t x_3) . \tag{11}$$

Now, the tracking problem we want to solve is to find a control law u to drive the tracking error e to zero.

Ⅲ. FEEDBACK LINEARIZING APPROACH TO TRACKING IN NONLINEAR SYSTEMS

In this section, we describe our approach to the tracking problem formulated in the preceding section. Our approach is basically motivated by the recently developed input-output linearization technique [5-8]. The general idea of the input-output linearization technique is to linearize the input-output dynamic characteristics of a nonlinear system via an appropriate nonlinear feedback control law. This input-output linearization technique facilitates the controller design of nonlinear systems since the resulting systems have linear input-output dynamic charateristics. This technique has been extended to the tracking problem of nonlinear systems [9,12].

The practical use of the input-output linearization technique has been severely limited by the difficulties in verifying the conditions for input-output linearization and solving a set of partial differential equations to obtain the desired control law and state transformation. However, there is a special class of nonlinear systems, for which it is not necessary to solve a set of partial differential equations [5,7,10]. In the similar way, we can characterize the special class of tracking problems, to which the approach proposed in [12] can be easily applied. For readable development of our results, we describe it here although its characterization is easily deducible from the results in [5,7,10].

Let R^+ be the set of nonnegative real numbers. Consider the following nonlinear tracking problem.

$$\dot{x} = f_0(x,t) + \sum_{i=1}^{m} f_i(x,t) u_i , \quad e = E(x, y_d) \tag{12}$$

where $f_i: R^n \times R^+ \to R^n$, $u_i(t) \epsilon R$, $i = 1, \ldots, m$; $f_0: R^n \times R^+ \to R^n$, $E: R^n \times R^s \to R^m$, $y_d: R^+ \to R^s$; $x(t) \epsilon R^n$, $e(t) \epsilon R^m$, $t \epsilon R^+$. Here, the time function y_d represents the desired trajectory of the system output. For the system (12), define the vector fields X_0, X_i, $i = 1, \ldots, m$, by

$$X_0 = \partial/\partial t + \sum_{j=1}^{n} f_{0,j}(x,t) \partial/\partial x_j ,$$

$$X_i = \sum_{j=1}^{n} f_{i,j}(x,t)\partial/\partial x_j, \quad i=1,\ldots,m \quad , \tag{13}$$

where $f_{i,j}$, x_j are the jth components of f_i, x respectively.

Suppose that the nonlinear system (12) satisfies:

B1: There exist nonnegative integers d_i, $i=1,\ldots,m$ such that

$$[X_1X_0^kE_i(x,y_d(t)) \ \ldots \ X_mX_0^kE_i(x,y_d(t))]=0, \quad x\in R^n, \ t\in R^+, \ k=0,\ldots,d_i-1, \tag{14}$$

$$[X_1X_0^{d_i}E_i(x,y_d(t)) \ \ldots \ X_mX_0^{d_i}E_i(x,y_d(t))]\neq 0, \quad x\in R^n, \ t\in R^+. \tag{15}$$

and

B2: $y_d \in C^{d_0}$ where $d_0=\max\{d_i+1, \ i=1,\ldots,m\}$.

Note that when

$$[X_1E_i(x,y_d(t)) \ \ldots \ X_mE_i(x,y_d(t))]\neq 0, \ x\in R^n, \ t\in R^+, \tag{16}$$

the assumption B1 is still satisfied by $d_i=0$.

Define the functions D^*, A^* by

$$D^*(x,t,Y_d(t)) = \begin{bmatrix} X_1X_0^{d_1}E_1(x,y_d(t)) & \ldots & X_mX_0^{d_1}E_1(x,y_d(t)) \\ \vdots & & \vdots \\ X_1X_0^{d_m}E_m(x,y_d(t)) & \ldots & X_mX_0^{d_m}E_m(x,y_d(t)) \end{bmatrix} \tag{17}$$

$$A^*(x,t,Y_d(t))=(X_0^{d_1+1}E_1(x,y_d(t)),\ldots,X_0^{d_m+1}E_m(x,y_d(t))) \tag{18}$$

where $Y_d(t)=(y_d(t),y_d^{(1)}(t),\ldots,y_d^{(d_0)}(t)]\in R^{s(d_0+1)}$ and $y_d^{(j)}$ is the jth derivative of y_d.

Now, we show that the control law and state transformation for input-output linearization of the system (12) can be easily found if the following condition holds.

B3: $D^*(x,t,Y_d(t))$ is nonsingular, $x\in R^n$, $t\in R^+$.

Define the mappings α, β and T by

$$\alpha(x,t,Y_d(t)) = -[D^*(x,t,Y_d(t))]^{-1}A^*(x,t,Y_d(t)) \ , \tag{19}$$

$$\beta(x,t,Y_d(t)) = [D^*(x,t,Y_d(t))]^{-1} \ , \tag{20}$$

$$T(x,t,Y_d(t))=(T_1(x,t,Y_d(t)),\ldots,T_m(x,t,Y_d(t))) \tag{21}$$

where

$$T_i(x,t,Y_d(t))= (E_i(x,y_d(t)),X_0E_i(x,y_d(t)),\ldots,X_0^{d_i}E_i(x,y_d(t))) \ . \tag{22}$$

Then, it is not difficult to see that the system (12) with the control law:

$$u = \alpha(x,t,Y_d(t)) + \beta(x,t,Y_d(t))\bar{u} \ , \tag{23}$$

is transformed through the state transformation:

$$\bar{x} = T(x,t,Y_d(t)) \ , \tag{24}$$

into the decoupled linear system:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} \ , \quad e = \bar{C}\bar{x} \tag{25}$$

where $x_i(t)\in R^{d_i+1}$, $\bar{A}=\mathrm{diag}\bar{A}_i$, $\bar{B}=\mathrm{diag}\bar{b}_i$, $\bar{C}=\mathrm{diag}\bar{c}_i$, and

$$\bar{A}_i = \begin{bmatrix} 0 & I_{d_i} \\ \vdots & \\ 0 \cdots & 0 \end{bmatrix} \in R^{(d_i+1)\times(d_i+1)} \ ,$$

$$\bar{b}_i=(0,0,\ldots,1)\in R^{d_i+1},$$

$$\bar{c}_i=[1 \ 0 \ \ldots \ 0]\in R^{1\times(d_i+1)}, \ i=1,\ldots,m. \tag{26}$$

Hence, we can easily find the desired control law and state transformation for input-output linearization under the assumptions B1, B2 and B3.

Now, take the new input $\bar{u}$ in (23) by

$$\bar{u}_i = K_i\bar{x}_i = K_iT_i(x,t,Y_d(t)), \quad i=1,\ldots,m \tag{27}$$

so that

$$\bar{A}_i+K_i\bar{b}_i, \ i=1,\ldots,m, \text{ are stable matrices.} \tag{28}$$

Then, the tracking error will tend to zero since the closed-loop system given by (12), (23) and (27) has the same input-output dynamic characteristics as the stably decoupled linear system (25) with (27).

IV. DESIGN OF NEW CLOS GUIDANCE LAW

In this section, we design a new CLOS guidance law, by applying the feedback linearizing approach described in Section III to the tracking problem formulated in Section II. To do so, we make the following assumption in addition to the assumptions A1, A2.

A3: $\mathbf{i}_M \cdot \mathbf{i}_L > 0, \quad t\geq 0$.

In other words, we assume that the missile flies with its X_M-axis upward in the Y_L-Z_L plane until target interception. This assumption is valid in the usual pursuit situations of missile and target. Note that the unit vectors $\mathbf{i}_M$, $\mathbf{i}_L$ can be represented in the inertial frame as

$$\mathbf{i}_M = c\theta_m c\psi_m \mathbf{i}_I + c\theta_m s\psi_m \mathbf{j}_I + s\theta_m \mathbf{k}_I \ ,$$

$$\mathbf{i}_L = c\gamma_t c\sigma_t \mathbf{i}_I + c\gamma_t s\sigma_t \mathbf{j}_I + s\gamma_t \mathbf{k}_I \ , \tag{29}$$

respectively. Therefore, the assumption A3 can be stated alternatively as follows

A3': $c\gamma_t c\theta_m c(\psi_m-\sigma_t) + s\gamma_t s\theta_m > 0, \quad t\geq 0$.

Direct computation yields

$$\begin{aligned} &X_1E_1=X_2E_1=X_1E_2=X_2E_2=0 \ , \\ &X_1X_0E_1=-s\phi_{mc}sx_8s(x_7-\sigma_t)+c\phi_{mc}c(x_7-\sigma_t) \ , \\ &X_2X_0E_1=-c\phi_{mc}sx_8s(x_7-\sigma_t)-s\phi_{mc}c(x_7-\sigma_t) \ , \\ &X_1X_0E_2=s\phi_{mc}\{c\gamma_t cx_8+s\gamma_t sx_8c(x_7-\sigma_t)\}+c\phi_{mc}s\gamma_t s(x_7-\sigma_t) \ , \\ &X_2X_0E_2=c\phi_{mc}\{c\gamma_t cx_8+s\gamma_t sx_8c(x_7-\sigma_t)\}-s\phi_{mc}s\gamma_t s(x_7-\sigma_t) \ . \end{aligned} \tag{30}$$

By the assumption A3 or A3',

$$\det \begin{bmatrix} X_1X_0E_1 & X_2X_0E_1 \\ X_1X_0E_2 & X_2X_0E_2 \end{bmatrix} = c\gamma_t cx_8c(x_7-\sigma_t)+s\gamma_t sx_8 > 0, \tag{31}$$

Thus, the assumption A3 implies B1 and B2. In addition, if the flight path of target in the three dimensional space is smooth enough so that

A4: $\sigma_t, \ \gamma_t \in C^2$,

we see that all conditions B1-B3 in Section III are satisfied with $n=8$, $m=2$, $s=2$ and $d_1=d_2=1$. Through some tedious calculation, we have

$$D^*(x,t,Y_d(t)) =$$

$$\begin{bmatrix} -s\phi_{mc}sx_8s(x_7-\sigma_t)+c\phi_{mc}c(x_7-\sigma_t) & -c\phi_{mc}sx_8s(x_7-\sigma_t)-s\phi_{mc}c(x_7-\sigma_t) \\ s\phi_{mc}\{c\gamma_tcx_8+s\gamma_tsx_8c(x_7-\sigma_t)\}+ & c\phi_{mc}\{c\gamma_tcx_8+s\gamma_tsx_8c(x_7-\sigma_t)\}- \\ c\phi_{mc}s\gamma_ts(x_7-\sigma_t) & s\phi_{mc}s\gamma_ts(x_7-\sigma_t) \end{bmatrix} \tag{32}$$

$$A^*(x,t,Y_d(t)) =$$

$$\begin{bmatrix} (-\ddot{\sigma}_tc\sigma_t+\dot{\sigma}_t^2s\sigma_t)x_1-(\ddot{\sigma}_ts\sigma_t+\dot{\sigma}_t^2c\sigma_t)x_2- \\ 2\dot{\sigma}_tx_4c\sigma_t-2\dot{\sigma}_tx_5s\sigma_t+a_x(t)cx_8s(x_7-\sigma_t) \\ \\ (\ddot{\sigma}_ts\sigma_ts\gamma_t+\dot{\sigma}_t^2c\sigma_ts\gamma_t+2\dot{\sigma}_t\dot{\gamma}_ts\sigma_tc\gamma_t-\ddot{\gamma}_tc\gamma_tc\sigma_t+\dot{\gamma}_t^2s\gamma_tc\sigma_t)x_1- \\ (\ddot{\sigma}_tc\sigma_ts\gamma_t-\ddot{\sigma}_t^2s\sigma_ts\gamma_t+2\dot{\sigma}_t\dot{\gamma}_tc\sigma_tc\gamma_t+\ddot{\gamma}_tc\gamma_ts\sigma_t-\dot{\gamma}_t^2s\gamma_ts\sigma_t)x_2- \\ (\ddot{\gamma}_ts\gamma_t+\dot{\gamma}_t^2c\gamma_t)x_3+2(\dot{\sigma}_ts\sigma_ts\gamma_t-\dot{\gamma}_tc\gamma_tc\sigma_t)x_4- \\ 2(\dot{\gamma}_tc\gamma_ts\sigma_t+\dot{\sigma}_tc\sigma_ts\gamma_t)x_5-2\dot{\gamma}_tx_6s\gamma_t+ \\ \{sx_8c\gamma_t-cx_8s\gamma_tc(x_7-\sigma_t)\}a_x(t)-c\gamma_tg \end{bmatrix} \tag{33}$$

$$T(x,t,Y_d(t)) =$$

$$\begin{bmatrix} -x_1s\sigma_t+x_2c\sigma_t \\ -\dot{\sigma}_tx_1c\sigma_t-\dot{\sigma}_tx_2s\sigma_t-x_4s\sigma_t+x_5c\sigma_t \\ -x_1s\gamma_tc\sigma_t-x_2s\gamma_ts\sigma_t+x_3c\gamma_t \\ (\dot{\sigma}_ts\sigma_ts\gamma_t-\dot{\gamma}_tc\gamma_tc\sigma_t)x_1-(\dot{\gamma}_tc\gamma_ts\sigma_t+\dot{\sigma}_tc\sigma_ts\gamma_t)x_2- \\ \gamma_tx_3s\gamma_t-x_4c\sigma_ts\gamma_t-x_5s\sigma_ts\gamma_t+x_6c\gamma_t \end{bmatrix} \tag{34}$$

Using the following identities with the definition of e_1, e_2 in (6) and the identities on $\dot{e}_1$, $\dot{e}_2$ in (34),

$$\begin{aligned} R_p &= x_1c\gamma_tc\sigma_t+x_2c\gamma_ts\sigma_t+x_3s\gamma_t \ , \\ \dot{R}_p &= -(\dot{\sigma}_tc\sigma_ts\gamma_t+\dot{\gamma}_ts\gamma_tc\sigma_t)x_1-(\dot{\gamma}_ts\gamma_ts\sigma_t-\dot{\sigma}_tc\sigma_tc\gamma_t)x_2+ \\ &\quad \dot{\gamma}_tx_3c\gamma_t+x_4c\sigma_tc\gamma_t+x_5s\sigma_tc\gamma_t+x_6s\gamma_t \ , \end{aligned} \tag{35}$$

we can write A^* in a more useful form:

$$A^*(x,t,Y_d(t)) =$$

$$\begin{bmatrix} (2\dot{\sigma}_t\dot{\gamma}_ts\gamma_t-\ddot{\sigma}_tc\gamma_t)R_p+\dot{\sigma}_t^2e_1+(2\dot{\sigma}_t\dot{\gamma}_tc\gamma_t+\ddot{\sigma}_ts\gamma_t)e_2- \\ 2\dot{\sigma}_t\dot{R}_pc\gamma_t+2\dot{\sigma}_t\dot{e}_2s\gamma_t+a_x(t)cx_8s(x_7-\sigma_t) \\ \\ -(\ddot{\gamma}_t+\dot{\sigma}_t^2s\gamma_tc\gamma_t)R_p-\ddot{\sigma}_te_1s\gamma_t+(\dot{\sigma}_t^2s^2\gamma_t+\dot{\gamma}_t^2)e_2- \\ 2\dot{\gamma}_t\dot{R}_p-2\dot{\sigma}_t\dot{e}_1s\gamma_t+\{sx_8c\gamma_t-cx_8s\gamma_tc(x_7-\sigma_t)\}a_x(t)-c\gamma_tg \end{bmatrix} \tag{36}$$

Using (32)-(34), we can construct the control law of the form (23) which transforms the system (7) into the decoupled system (25) with $m=2$ and $d_1=d_2=1$. Now, if we choose the new input u in (27) so that

$$K_1=K_2=[-(\lambda^2+\omega^2) \quad -2\lambda]>0, \quad \lambda>0, \quad \omega>0, \tag{37}$$

the condition (28) is satisfied and hence the tracking error will converge to zero.

The final form of our new CLOS guidance law is given by

$$\begin{aligned} u &= [D^*(x,t,Y_d(t))]^{-1}\{KT(x,t,Y_d(t))-A^*(x,t,Y_d(t))\} \\ &= - \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \begin{bmatrix} p_1+q_1 \\ p_2+q_2 \end{bmatrix} /a \end{aligned} \tag{38}$$

where

$$\begin{aligned} & a=s\gamma_ts\theta_m+c\gamma_tc\theta_mc(\psi_m-\sigma_t), \\ & b_1=s\gamma_ts\theta_mc(\psi_m-\sigma_t)+c\sigma_tc\theta_m, \quad b_2=s\theta_ms(\psi_m-\sigma_t), \\ & b_3=-s\gamma_ts(\psi_m-\sigma_t), \quad b_4=c(\psi_m-\sigma_t), \\ & p_1=-(\lambda^2+\omega^2)e_1-2\lambda\dot{e}_1+(2\dot{\sigma}_t\dot{\gamma}_ts\gamma_t-\ddot{\sigma}_tc\gamma_t)R_P-2\dot{\sigma}_t\dot{R}_Pc\gamma_t+ \\ & \qquad c\theta_ms(\psi_m-\sigma_t)(T-D)/M \ , \\ & p_2=-(\lambda^2+\omega^2)e_2-2\lambda\dot{e}_2-(\ddot{\gamma}_t+\dot{\sigma}_t^2s\gamma_tc\gamma_t)R_P-2\dot{\gamma}_t\dot{R}_P+ \\ & \qquad \{s\theta_mc\gamma_t-c\theta_ms\gamma_tc(\psi_m-\sigma_t)\}(T-D)/M \ , \\ & q_1=+\dot{\sigma}_t^2e_1+(2\dot{\sigma}_t\dot{\gamma}_tc\gamma_t+\ddot{\sigma}_ts\gamma_t)e_2+2\dot{\sigma}_t\dot{e}_2s\gamma_t \ , \\ & q_2=-\ddot{\sigma}_te_1s\gamma_t+(\dot{\sigma}_t^2s^2\gamma_t+\dot{\gamma}_t^2)e_2-2\dot{\sigma}_t\dot{e}_1s\gamma_t \ . \end{aligned} \tag{39}$$

Now, it should be clear that the missile guided by this guidance law always will hit any randomly maneuvering target if the gains in (37) are chosen sufficiently fast.

Most of the guidance informations required by our guidance law (38) can be aquired directly from a ground tracker and an on-board inertial navigation unit (INU). Specifically, such informations are R_m, σ_t, $\dot{\gamma}_t$, σ_t, $\dot{\gamma}_t$, $\Delta\sigma$, $\Delta\gamma$, ψ_m and θ_m. Since R_P, e_1 and e_2 cannot be measured directly, these quantities ought to be computed indirectly from the polar informations available from the ground tracker in the following way.

$$\begin{aligned} R_P &= R_mc(\Delta\gamma+\gamma_t)c\gamma_tc\Delta\sigma+R_ms(\Delta\gamma+\gamma_t)s\gamma_t \ , \\ e_1 &= R_mc(\Delta\gamma+\gamma_t)s\Delta\sigma \ , \\ e_2 &= R_ms(\Delta\gamma+\gamma_t)c\gamma_t-R_mc(\Delta\gamma+\gamma_t)s\gamma_tc\Delta\sigma \ . \end{aligned} \tag{40}$$

The derivative informations $\dot{R}_P$, $\dot{e}_1$, $\dot{e}_2$, $\ddot{\sigma}_t$ and $\ddot{\gamma}_t$ can be estimated with band-limited differentiators or Kalman filters. Also the time history of (T-D)/M can be estimated from experimental data.

Now, we give some comments on the earlier results closely related to ours. Our guidance law involves some terms that are, in the form, similar to the feedforward acceleration and error compensation acceleration terms used in the earlier results [2-4]. The feedforward acceleration term was to make the missile chase the LOS rotation while the error compensation acceleration term was to null deviation of the missile from the LOS to target. In the earlier results, these terms have been calculated under some restrictive assumptions. In [2,3], it is assumed that the missile is on the LOS to target and its velocity vector lies on the flyplane which is the imaginary plane spanned by the target velocity and the LOS to target. In [4], the lateral acceleration of the missile projection point onto the LOS to target is adopted as the feedforward acceleration term. Therefore, their guidance laws may undergo some performance degradation in the case that such restrictive assumptions are not valid. From this context, our result, in some sense, completes the earlier works in [2-4]. The recently developed nonlinear control technique has proven to be a useful tool in doing so.

In the practical viewpoint, the new CLOS guidance law (38) is computationally complex. Moreover, the promised performance of our guidance law cannot be fully achieved due to modelling errors such as the ground tracker and autopilot dynamics. Therefore, it may be more practical to find a guidance law which requires less computation but does not degrade guidance performance significantly. In the next section, we explore this problem.

V. SIMPLIFICATION OF NEW CLOS GUIDANCE LAW

In this section, we attempt to simplify the new CLOS guidance law in (38) to reduce computational burden without performance degradation. For this aim, we drop out some terms in A^* to obtain the following simplified CLOS guidance law:

$$u = [D^*(x,t,Y_d(t))]^{-1}\{KT(x,t,Y_d(t))-\hat{A}^*(x,t,Y_d(t))\}$$
$$= -\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}/a \quad (41)$$

Comparing two guidance laws in (38) and (41), we can see that such a simplification reduces the computational burden almost by half. In what follows, we show that the simplified guidance law in (41) can provide almost the same guidance performance as the original one in (38) provided that K is chosen sufficiently large.

Let $\Delta A^* \triangleq A^*-\hat{A}^*$. Then, we see that ΔA^* has the form:

$$\Delta A^* = \Delta KT \quad (42)$$

where

$$\Delta K = \begin{bmatrix} \dot{\sigma}_t^2 & 0 & 2\dot{\sigma}_t\dot{\gamma}_t c\gamma_t+\ddot{\sigma}_t s\gamma_t & 2\dot{\sigma}_t s\gamma_t \\ -\ddot{\sigma}_t s\gamma_t & -2\dot{\sigma}_t s\gamma_t & \dot{\sigma}_t^2 s^2\gamma_t+\dot{\gamma}_t^2 & 0 \end{bmatrix} \quad (43)$$

Note that the target acceleration is bounded and hence that $\dot{\sigma}_t$, $\dot{\gamma}_t$, $\ddot{\sigma}_t$ and $\ddot{\gamma}_t$ are bounded during the guidance. Therefore, we can assume without loss of generality that

A5: There exists a constant $\delta > 0$ such that

$$\|\Delta K\| < \delta, \quad t \geq 0 . \quad (44)$$

Now, choose λ, ω in (37) so that

$$\lambda\{(\lambda^2+\omega^2+1)-\sqrt{(\lambda^2+\omega^2+1)^2-4\omega^2}\}/2\omega > \delta \quad (45)$$

Then, let $\bar{A}_K=\bar{A}+\bar{B}K$. Since $\bar{A}_K$ has simple structure, it can be transformed to a real canonical form. To see this, take

$$P_K = \begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix} \qquad P_0 = \begin{bmatrix} 1 & 0 \\ -\lambda & -\omega \end{bmatrix} \quad (46)$$

Then,

$$\hat{A}_K=P_K^{-1}\bar{A}_K P_K = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \quad A = \begin{bmatrix} -\lambda & -\omega \\ \omega & -\lambda \end{bmatrix} \quad (47)$$

Furthermore,

$$\|P_K\| = [(\lambda^2+\omega^2+1)+\sqrt{(\lambda^2+\omega^2+1)^2-4\omega^2}]/2,$$
$$\|P_K^{-1}\| = \|P_K\|/\omega. \quad (48)$$

Choose a Lyapunov-like function V by

$$V(t) = 1/2\,|v(t)|^2, \quad v(t) = P_K^{-1}T(x,t,Y_d(t)) . \quad (49)$$

Since T, x, Y_d are all C^1, we can take the total time derivative of $\bar{x}=T(x,t,Y_d(t))$ along the solution trajectory of the closed-loop system (7) and (41). Then, we have

$$\dot{\bar{x}} = (\bar{A}_K+\bar{B}\Delta K)\bar{x} . \quad (50)$$

By (47)-(50),

$$\dot{V}=\bar{x}^T(P_K^{-T}P_K^{-1}\bar{A}_K+\bar{A}_K^T P_K^{-T}P_K^{-1})\bar{x}/2+\bar{x}^T P_K^{-T}P_K^{-1}\bar{B}\Delta K\bar{x}$$
$$= v^T(\hat{A}_K+\hat{A}_K^T)v+v^T P_K^{-1}\bar{B}\Delta K P_K v$$
$$\leq -2(\lambda-\|P_K^{-1}\|\,\|P_K\|\,\|\bar{B}\|\,\delta)\,|v|^2$$
$$\leq -2\mu V \quad (51)$$

where

$$\mu=\lambda-\delta[(\lambda^2+\omega^2+1)+\sqrt{(\lambda^2+\omega^2+1)^2-4\omega^2}]/2\omega . \quad (52)$$

This implies that

$$|v(t)| \leq |v(0)|\,e^{-\mu t} . \quad (53)$$

This with the fact that

$$e = \bar{C}P_K v , \quad \bar{C}P_K = \bar{C} \quad (54)$$

yields the desired result:

$$|e(t)| = \rho e^{-\mu t}, \quad t\geq 0 \quad (55)$$

where

$$\rho= |T(x(0),0,Y_d(0))|\,[(\lambda^2+\omega^2+1)+\sqrt{(\lambda^2+\omega^2+1)^2-4\omega^2}]/2\omega. \quad (56)$$

From (45) and (55), we see that, if λ and ω are chosen sufficiently large, the simplified CLOS guidance law in (41) will guarantee zero miss distance against any target with bounded acceleration. To prove this, we have closely followed the arguments in [15] since the neglected term $[D^*]^{-1}\Delta A^*$ satisfies the well-known matching condition on modelling errors. In the earlier results (See the references in [15]), the matching condition was required for robust control of uncertain systems. Here, we have used it for simplification of controllers.

VI. CONCLUSION

We have presented a novel approach to the three dimensional CLOS guidance problem. We convert cleverly the CLOS guidance problem to a nonlinear tracking problem so that the recently developed approach to robust tracking in nonlinear systems can be applied effectively. Our work differs from the earlier one mainly in that nonlinearity of guidance mechanism is fully taken into account in the process of CLOS guidance law design. Our result can be easily extended to bank-to-turn (BTT) missiles.

By simulations, we verified guidance performance of our new and simplified CLOS guidance laws and investigated the effect of the autopilot and ground tracker dynamics on guidance performance. The results will be presented in the conference.

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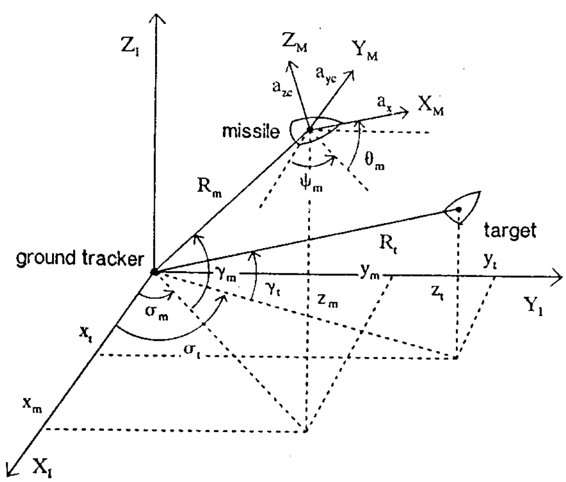

Fig. 1. Three dimensional pursuit situation

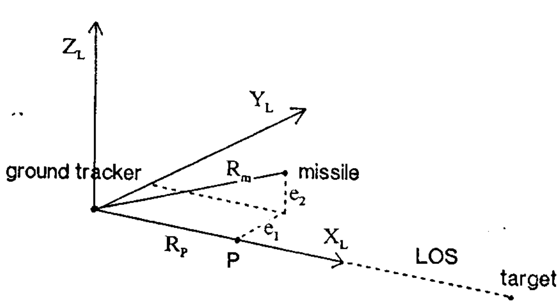

Fig. 2. Definition of the tracking error