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A VSS Observer-based Sliding Mode Control for Uncertain Systems

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Abstract

A VSS observer-based sliding mode control is described for continuous-time systems with uncertain nonlinear elements, in which the Euclidean norm of unknown element is bounded by a known value. For a case of complete state information, we first derive a sliding mode controller consisting of three parts: a linear state feedback control, an equivalent input and a min-max control. It is then shown that the present attractiveness condition is simpler than that for a case without using the concept of equivalent input. We next design a VSS observer as a completely dual form to the sliding mode controller. Finally, we discuss a case of incomplete state information by applying the VSS observer.

1. Introduction

The sliding mode control or VSS (Variable Structure System) control has an increased interesting, because it can realize a robust control for a trajectory control of robot arm with unknown element [1-3]. However, almost existing sliding mode controls deal with a case when the state-variable can be completely measurable. As in the well-known LQ (Linear Quadratic) control, we need use of an observer to realize a practical sliding mode control for a case of incomplete state information.

There exist some observer-based sliding mode controls. For example, Bondarev *et al.* [4] discussed a sliding mode control by using a Luenberger-type observer. But, they did not at all take into account of a duality between the controller and observer. Recently, Żak *et al.* [5] investigated an observer-based sliding mode control by using a VSS observer, which is dual to the min-max controller derived in Gutman and Palmor [6].

In this paper we state a VSS observer-based sliding mode control for an uncertain dynamical system, where the Euclidean norm of unknown element is bounded by a known value. For a case of complete state information, we first derive a sliding mode controller consisting of three parts: a linear state feedback control, an equivalent input and a min-max control, by applying the strictly positive realness. It is then pointed out that the present attractiveness condition is much simpler than that for a case without using the concept of equivalent input [6]. We next design a VSS observer as a completely dual form to the sliding mode controller. Finally, we discuss a case of incomplete state information by applying the VSS observer.

2. Systems Description

Consider the following continuous-time system described by the state-space model:

$$\dot{x}(t) = Ax(t) + Bu(t) + B\zeta(t, x(t)) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where, $x(t) \in \mathcal{R}^n, u(t) \in \mathcal{R}^r, y(t) \in \mathcal{R}^m, m \geq r$, and $\zeta(\cdot)$ is an uncertain element. It is then assumed that the norm of the uncertain element is bounded by a known scalar ρ that is,

$$\|\zeta(t, x)\| \leq \rho, \quad \rho \geq 0$$

where $\|\cdot\|$ denotes the Euclidean norm, i.e.,

$$\|x\| = \left[\sum_{j=1}^{n} |x_j|^2\right]^{1/2}$$

for any vector $x \in \mathcal{R}^n$

$$\|A\| = \left[\lambda_{\max}(A^T A)\right]^{1/2}$$

for any matrix A, in which $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue and similarly $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of a matrix.

3. VSS Controller for the Case of Complete State Information

In this section, we shall design a VSS controller for the case when the state variables are all available, that is, the case of complete state information. It is assumed that the VSS control input $u(t)$ consists of three inputs:

$$u(t) \triangleq u_{ls}(t) + u_{eq}(t) + u_{mm}(t) \tag{3}$$

where $u_{ls}(t)$ is a linear state feedback input, $u_{eq}(t)$ is an equivalent input when applying the linear state feedback control to the linear system, and $u_{mm}(t)$ is a min-max control input.

3.1 Linear state feedback input

The following assumption will be utilized in the subsequent discussion.

Assumption 1: The pair (A, B) is completely controllable. This implies that we can find a matrix $K_c \in \mathcal{R}^{r\times n}$ such that all eigenvalues of the matrix $A_c \triangleq A - BK_c$ are in the desired location in the open left half-plane.

Then, we have the linear state feedback control such that

$$u_{ls}(t) = -K_c x \tag{4}$$

3.2 Equivalent input

It is assumed that the system (1) with $\zeta(\cdot) = 0$ is controlled by $u_{ls}(t) + u_{eq}(t)$.

Assumption 2: There exist real symmetric positive definite matrices Q_c and P_c, where P_c is the unique solution to the algebraic Lyapunov equation:

$$A_c^T P_c + P_c A_c = -Q_c \tag{5}$$

Then, we define the following switching surface for the control

$$S_c = \{x(t)|\sigma_c(t) \triangleq K_s x(t) = 0\} \tag{6}$$

where $K_s \triangleq B^T P_c$. Since $\dot{\sigma}_c(t) = 0$ in the sliding mode, we have

$$\dot{\sigma}_c(t) = K_s A_c x(t) + K_s B u_{eq}(t) = 0 \tag{7}$$

and therefore

$$u_{eq}(t) = -(K_s B)^{-1} K_s A_c x(t) \tag{8}$$

where $(K_c B)$ is assumed to be nonsingular. Subsequently, when using $u_{ls}(t) + u_{eq}(t)$, the equivalent system reduces to

$$\dot{x}(t) = [I - B(K_s B)^{-1} K_s] A_c x(t) \tag{9}$$

3.3 Min-max input

It is here assumed that $u_{mm}(t)$ is applied to the system (1) with $\zeta(\cdot) \neq 0$, where $u_{mm}(t)$ is defined as a min-max controller [6]:

$$u_{mm}(t) = \begin{cases} -\dfrac{\sigma_c}{\|\sigma_c\|}\bar{\rho}_c & \text{for all } x(t) \notin S_c \\ u_{mm} \in \{B\eta_c \in \mathcal{R}^n | \|\eta_c\| \leq \bar{\rho}_c\} & \text{for all } x(t) \in S_c \end{cases} \tag{10}$$

Here, $\bar{\rho}_c$ is to be determined. When defining the generalized Lyapunov function as $W_c = \frac{1}{2}\sigma_c^T \sigma_c$, we have

$$\begin{aligned}\dot{W}_c &= \sigma_c^T \dot{\sigma}_c = \sigma_c^T (K_s \dot{x}) \\ &= \sigma_c^T (K_s A_c x + K_s B u_{eq} - K_s B \frac{\sigma_c}{\|\sigma_c\|}\bar{\rho}_c + K_s B\zeta)\end{aligned}$$

because a sufficient condition for the attractiveness of $x(t)$ to S_c is $\dot{W}_c = \sigma_c^T \dot{\sigma}_c < 0$ for $\sigma_c \neq 0$ [7]. Using (4) and (8) in above, it follows that

$$\dot{W}_c = -\sigma_c^T (B^T P_c B)\frac{\sigma_c}{\|\sigma_c\|}\bar{\rho}_c + \sigma_c^T (B^T P_c B)\zeta \tag{11}$$

Noting that

$$\begin{aligned}\lambda_{\min}(B^T P_c B)\|\sigma_c\|^2 &\leq \sigma_c^T (B^T P_c B)\sigma_c \\ &\leq \lambda_{\max}(B^T P_c B)\|\sigma_c\|^2\end{aligned}$$

and using the property of a vector norm:

$$\begin{aligned}\sigma_c^T (B^T P_c B)\zeta &\leq \|B^T P_c B\zeta\| \, \|\sigma_c\| \\ &\leq \|B^T P_c B\| \, \|\zeta\| \, \|\sigma_c\| \\ &\leq \lambda_{\max}(B^T P_c B)\| \, \|\sigma_c\|\rho\end{aligned}$$

it is seen that (11) can be written as

$$\dot{W}_c \leq -\lambda_{\min}(B^T P_c B)\|\sigma_c\|^2 \frac{\bar{\rho}_c}{\|\sigma_c\|} + \lambda_{\max}(B^T P_c B)\|\sigma_c\|\rho \tag{12}$$

Henceforth, if

$$\bar{\rho}_c > \frac{\lambda_{\max}(B^T P_c B)}{\lambda_{\min}(B^T P_c B)}\rho \tag{13}$$

then S_c is globally asymptotically attractive at everywhere.

Thus, it is found that the present attractiveness condition is simpler than that for the case without using the equivalent input [6]. The block diagram for this case is depicted in Fig. 1.

4. Example 1

To illustrate the preceding results, consider the following second-order system:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u + \begin{bmatrix}0 \\ 1\end{bmatrix} 7.35 \sin x_1$$

where $7.35 \sin x_1$ is assumed to be unknown, but $\rho = |7.35 \sin x_1| = 7.35$ is known. Letting $K_c = [k_{c1} \quad k_{c2}]$, it follows that

$$A_c = \begin{bmatrix}0 & 1 \\ -k_{c1} & -(1 + k_{c2})\end{bmatrix}$$

A necessary and sufficient condition that assures the stability of the matrix A_c is that, in the characteristic equation $|A_c - \lambda I| = \lambda^2 + (1 + k_{c2})\lambda + k_{c1}$, coefficients $1 + k_{c2}$ and k_{c1} have the same sign and $1 + k_{c2} > 0$. Since

$$\begin{aligned}&A_c^T P_c + P_c A_c \\ &= \begin{bmatrix}-2k_{c1}p_{c2} & p_{c1} - p_{c2}(1 + k_{c2}) - k_{c1}p_{c3} \\ p_{c1} - p_{c2}(1 + k_{c2}) - k_{c1}p_{c3} & 2p_{c2} - 2p_{c3}(1 + k_{c2})\end{bmatrix}\end{aligned}$$

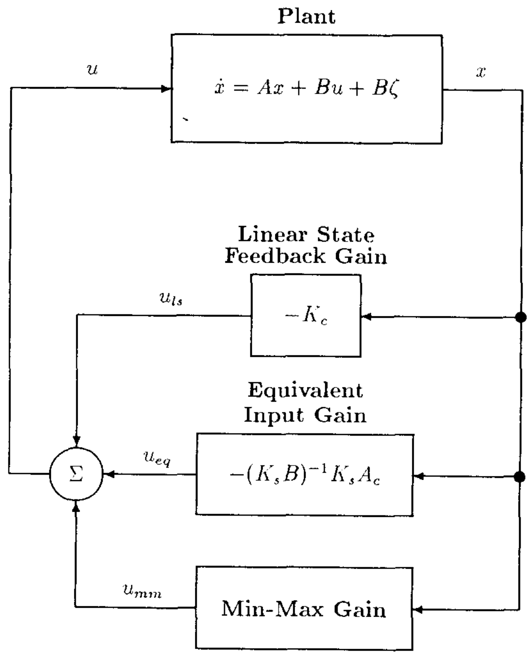

Fig. 1 A VSS control system with complete state information.

setting for example $K_s = [1 \quad 1]$, we have

$$p_{c2} = p_{c3} = 1$$

because $K_s = B^T P_c$. If we choose $K_c = [6 \quad 4]$, then

$$Q_c = \begin{bmatrix} 12 & 11 - p_{c1} \\ 11 - p_{c1} & 8 \end{bmatrix}$$

Since Q_c must be positive definite, it is enough to choose that $p_{c1} = 4$, so that

$$Q_c = \begin{bmatrix} 12 & 7 \\ 7 & 8 \end{bmatrix} > 0, \quad P_c = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix} > 0$$

The linear state feedback input is then given by

$$u_{ls} = -6x_1 - 4x_2$$

and the equivalent input is given by

$$\begin{aligned} u_{eq} &= -[K_sB]^{-1}K_sA_cx \triangleq K_{eq}x \\ &= -[1 \quad 1]\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x = 6x_1 + 4x_2 \end{aligned}$$

Therefore, the equivalent linear system matrix reduces to

$$A_c + BK_{eq} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$

In this case, it should be noted that the equivalent linear system is identical to the original linear part. Furthermore, if we choose $\bar{\rho}_c = 8$, then the switching input becomes

$$u_{mm} = -8 \operatorname{sgn}(x_1 + x_2)$$

For the case without using the equivalent input, we must further check the positive definiteness of $\hat{Q}_c$, which satisfies the equation [6]:

$$\begin{aligned} &(P_cBB^TP_c)A_c + A_c^T(P_cBB^TP_c) \\ &\quad = \begin{bmatrix} -12 & -10 \\ -10 & -8 \end{bmatrix} \triangleq -\hat{Q}_c \end{aligned}$$

Clearly, it is found that $\hat{Q}_c$ is not positive definite or negative definite. Therefore, we must further select $\bar{\rho}_c$ such that

$$\bar{\rho}_c > \rho + |6x_1 + 4x_2|$$

5. VSS Observer

In this section, we shall design a VSS observer as a completely dual form to the VSS controller described in section 3. It is assumed that the observer is of full-order given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + M_l(e) + M_{eq}(e) + M_{mm}(e) \quad (14)$$

where $\hat{x}(t)$ denotes the state of the observer and $e(t) \triangleq x(t) - \hat{x}(t)$ is estimation error. Here, $M_l(e)$ is the linear compensative term, $M_{eq}(e)$ is the equivalent compensative term and $M_{mm}(e)$ is the min-max compensative term.

5.1 Linear observer

Assumption 3: The pair (C, A) is completely observable. This implies that we can find a matrix $K_e \in \mathcal{R}^{n\times m}$ such that all eigenvalues of the matrix $A_e \triangleq A - K_eC$ are in the desired location in the open left half-plane.

Then we have, from the well-known Luenberger observer, the linear compensative term:

$$\begin{aligned} M_l(e) &= K_e[y(t) - C\hat{x}(t)] \\ &= K_eCe(t) \end{aligned} \quad (15)$$

5.2 Equivalent compensative term

It is assumed that the system (1) and (2) with $\zeta(\cdot) = 0$ is estimated via an observer using $M_l(e) + M_{eq}(e)$.

Assumption 4: There exist real symmetric positive definite matrices Q_e and P_e, where P_e is the unique solution to the algebraic Lyapunov equation:

$$A_e^TP_e + P_eA_e = -Q_e \quad (16)$$

Then, we define the following switching surface for the estimation

$$S_e = \{e(t)|\sigma_e(t) \triangleq G_se(t) = 0\} \quad (17)$$

where $G_s \triangleq B^T P_e$. Furthermore, it is assumed that there exist $F_1, F_2 \in \mathcal{R}^{r\times m}$ such that

$$F_1 C = B^T P_e \tag{18}$$

$$F_2 C = B^T P_e A_e \tag{19}$$

Defining $M_{eq}(e) \triangleq BL_{eq}(e)$, since in the sliding mode $\dot{\sigma}_e(t) = 0$

$$\dot{\sigma}_e(t) = G_s A_e e(t) - G_s B L_{eq}(e) = 0 \tag{20}$$

Using (19) gives

$$L_{eq}(e) = (G_s B)^{-1} F_2 C e(t) \tag{21}$$

where $(G_s B)$ is assumed to be nonsingular. Subsequently, when using $M_l(e) + M_{eq}(e)$, the equivalent error system becomes

$$\begin{aligned}\dot{e}(t) &= A_e e(t) - BL_{eq}(e)\\ &= [I - B(G_s B)^{-1} G_s] A_e e(t)\end{aligned} \tag{22}$$

5.3 Min-max observer

It is here assumed that the system (1) and (2) with $\zeta(\cdot) \neq 0$ is estimated from an observer with $M_{mm}(e)$. The min-max compensative term $M_{mm}(e)$ is given by [5]:

$$M_{mm}(e) = \begin{cases} \dfrac{BF_1Ce}{\|F_1Ce\|}\bar{\rho}_e & \text{for all } e(t) \notin S_e \\ M_{mm}(e) \in \{B\eta_e \in \mathcal{R}^n \mid \|\eta_e\| \le \bar{\rho}_e\} & \text{for all } e(t) \in S_e \end{cases} \tag{23}$$

where $\bar{\rho}_e$ is to be determined. Using the observer consisting of (15),(21)and (23), the estimation error equation becomes

$$\dot{e}(t) = A_e e(t) - BL_{eq}(e) - \frac{BF_1Ce}{\|F_1Ce\|}\bar{\rho}_e + B\zeta \tag{24}$$

for $e(t) \notin S_e$. When defining the generalized Lyapunov function as $W_e = \frac{1}{2}\sigma_e^T\sigma_e$, the sufficient condition for assuring the attractiveness of $e(t)$ to the switching surface S_e is

$$\begin{aligned}\dot{W}_e &= \sigma_e^T\dot{\sigma}_e = \sigma_e^T(G_s\dot{e})\\ &= \sigma_e^T(G_sA_ee - G_sBL_{eq}(e)\\ &\quad - G_sB\frac{F_1Ce}{\|F_1Ce\|}\bar{\rho}_e + G_sB\zeta)\end{aligned}$$

Using (19),(21) and $G_s = B^TP_e$ in above equation gives

$$\dot{W}_e = -\sigma_e^T(B^TP_eB)\frac{\sigma_e}{\|\sigma_e\|}\bar{\rho}_e + \sigma_e^T(B^TP_eB)\zeta \tag{25}$$

Taking into account the fact of section 3.3, we have

$$\dot{W}_e \le -\lambda_{\min}(B^TP_eB)\|\sigma_e\|^2\frac{\bar{\rho}_e}{\|\sigma_e\|} + \lambda_{\max}(B^TP_eB)\|\sigma_e\|\rho \tag{26}$$

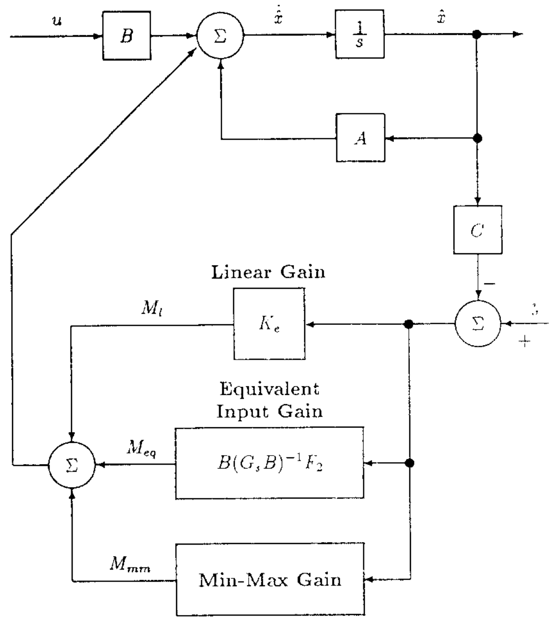

Fig. 2 A VSS observer.

Hence, if

$$\bar{\rho}_e > \frac{\lambda_{\max}(B^TP_eB)}{\lambda_{\min}(B^TP_eB)}\rho \tag{27}$$

then S_e is globally asymptotically attractive at everywhere.

Fig.2 shows the block diagram of the present VSS observer.

6. Example 2

To illustrate the design of VSS observer using an equivalent compensative term, let us return to Example 1 but now with $C = [1 \quad 1]$.

In this case, the linear part of the system is completely observable. Letting $K_e = [k_{e1} \quad k_{e2}]^T$, we have

$$A_e = \begin{bmatrix} -k_{e1} & 1 - k_{e1} \\ -k_{e2} & -(1 + k_{e2}) \end{bmatrix}$$

A necessary and sufficient condition that assures the stability of the matrix A_e is that, in $|A_e - \lambda I| = \lambda^2 + (1 + k_{e1} + k_{e2})\lambda + k_{e1} + k_{e2}$, coefficients $1 + k_{e1} + k_{e2}$ and $k_{e1} + k_{e2}$ have the same sign and $1 + k_{e1} + k_{e2} > 0$. Using $B^TP_e = F_1C$ while maintaining the symmetry of P_e yields

$$p_{e2} = p_{e3} = F_1$$

Applying this result to the relation of $B^TP_eA_e = F_2C$ gives

$$-F_1[k_{e1}+k_{e2}, \quad k_{e1}+k_{e2}] = F_2[1, \quad 1]$$

so that

$$F_2 = -F_1(k_{e1}+k_{e2})$$

If $K_e = [1, \quad 0.5]^T$ and $F_1 = 1$, then $F_2 = -1.5$. Substituting these results into $A_e^T P_e + P_e A_e = -Q_e$ results in

$$Q_e = \begin{bmatrix} 2p_{e1}+1 & 3 \\ 3 & 3 \end{bmatrix}$$

Since Q_e must be positive definite, it is enough to choose that $p_{e1} = 1.5$. Subsequently, we have

$$P_e = \begin{bmatrix} 1.5 & 1 \\ 1 & 1 \end{bmatrix} > 0, \quad Q_e = \begin{bmatrix} 4 & 3 \\ 3 & 3 \end{bmatrix} > 0$$

The linear compensative term is then given by

$$M_l = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} (y - \hat{x}_1 - \hat{x}_2)$$

and the equivalent compensative term is also given by

$$M_{eq} = BL_{eq} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-1.5(y - \hat{x}_1 - \hat{x}_2)]$$

The equivalent linear error system matrix reduces to

$$[I - B(G_s B)^{-1} G_s] A_e = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

The switching compensative term is given by

$$M_{mm} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} 8 \operatorname{sgn}(y - \hat{x}_1 - \hat{x}_2)$$

For the case without using the equivalent compensative term, we must further check the positive definiteness of $\hat{Q}_e$, which satisfies the equation [5]:

$$(P_e BB^T P_e) A_e + A_e^T (P_e BB^T P_e) = \begin{bmatrix} -1.5 & -1.5 \\ -1.5 & -1.5 \end{bmatrix} \triangleq -\hat{Q}_e$$

Clearly, it is held that $\hat{Q}_e \geq 0$.

7. Example 3

Next consider the same problem as in Example 2, but with $C = [1 \quad 0]$. This case also assures that the linear part of the system is completely observable.

Computing $B^T P_e = F_1 C$, it follows that $p_{e3} = 0$. This means that the condition of $P_e > 0$ does not hold. Hence, the VSS observer based on the condition of strictly positive realness can not be designed for this case.

This is a particular case when the sliding motion is generated through the value of a single component of the error states, rather than a linear combination of both components, as studied in Slotine *et al.* [8].

8. VSS Controller for the Case of Incomplete State Information

In this section, assume that $x(t)$ is not available to construct the VSS controller. Instead, we may use the VSS observer described above. For such a case, the VSS control input given by (3) is replaced by

$$u(t,\hat{x}) \triangleq u_{ls}(t,\hat{x}) + u_{eq}(t,\hat{x}) + u_{mm}(t,\hat{x}) \qquad (28)$$

The linear state feedback input becomes

$$u_{ls}(t,\hat{x}) = -K_c \hat{x}(t) \qquad (29)$$

The switching surface (6) is also exchanged by

$$\bar{S}_c \triangleq \{\hat{x}(t) | \bar{\sigma}_c(t) \triangleq K_s \hat{x}(t) = 0\} \qquad (30)$$

Since $\dot{\bar{\sigma}}_c(t) = 0$ in the sliding mode, using (14) with $u(t,\hat{x}) = u_{ls}(t,\hat{x}) + u_{eq}(t,\hat{x})$ gives

$$\begin{aligned} \dot{\bar{\sigma}}_c(t) &= K_s A_c \hat{x}(t) + K_s B u_{eq}(t,\hat{x}) + K_s K_e C e(t) \\ &\quad + K_s B (G_s B)^{-1} F_2 C e(t) + K_s M_{mm}(e) = 0 \end{aligned} \qquad (31)$$

Hence,

$$\begin{aligned} u_{eq}(t,\hat{x}) &= -(K_s B)^{-1} K_s [A_c \hat{x}(t) + K_e C e(t) \\ &\quad + B(G_s B)^{-1} F_2 C e(t) + M_{mm}(e)] \end{aligned} \qquad (32)$$

Furthermore, the min-max controller becomes

$$u_{mm}(t,\hat{x}) = \begin{cases} -\dfrac{\bar{\sigma}_c}{\|\bar{\sigma}_c\|} \bar{\rho}_c & \text{for all } \hat{x}(t) \notin \bar{S}_c \\ u_{mm} \in \{B\eta_c \in \mathcal{R}^n \,|\, \|\eta_c\| \leq \bar{\rho}_c\} & \text{for all } \hat{x}(t) \in \bar{S}_c \end{cases} \qquad (33)$$

9. Conclusions

We have described a VSS observer-based sliding mode control for continuous-time systems with uncertain nonlinear elements, where the Euclidean norm of unknown element is bounded by a known value. For a case of complete state information, by applying a strictly positive realness, we first derived a sliding mode controller consisting of three parts: a liner state feedback control, an equivalent input and a min-max control. It was then shown that the present attractiveness condition was simpler than that for a case without using the concept of equivalent input. We next designed a VSS observer as a completely dual form to the sliding mode controller. Finally, we dealt with a case of incomplete state information by applying the VSS observer.

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