# ON-LINE FAULT DETECTION METHOD ACCOUNTING FOR MODELLING FRRORS

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#### Abstract

This paper proposes a robust on-line fault detection method for uncertain systems. It is based on the fault detection method [10] accounting for modelling errors, which is shown to have superior performance over traditional methods but has some computational problems so that it is hard to be applied to on-line problems. The proposed method in this paper is an on-line version of the fault detection method suggested in [10]. Thus the method has the same detection performance robust to model uncertainties as that of [10]. Moreover, its computational burden is shown to be considerably lessened so that it is applicable to on-line fault detection problems.

# T INTRODUCTION

As automatic control systems become more and more complex and expensive, the fault detection method is essential for improving the supervision and monitoring as part of the overall process control especially in advanced processes with the highest demands on reliability and safety, e.g. aircrafts and nuclear power plants. In this context the fault is understood as any kind of abnormal change in the system characteristics that leads to an undesired performance in the system under consideration. If a fault occured it has to be detected as early as possible, which is called as a fault detection. This may be followed by isolating the source of it, namely a fault diagnosis. The next step would be a fault evaluation that means characterizing the extent and significance of the fault. And according to this assessment, some proper actions could be taken.

Previously supervision of technical processes was restricted to checking directly measurable variables for upward or downward transgression of fixed limits or trends. With the aid of process models, estimation and decision methods, it is now possible to monitor nonmeasurable variables like process states, process parameters and other characteristic quantities related

to the process. We concentrate on the fault detection problem based on the estimated parametric models. This can be done by checking if the estimated parameters are within a certain tolerance of the normal value.

The fault detection would be straightforward if exact system models were available and systems were noise-free. Yet noise affects all real systems and modelling errors that are by no means avoidable in practice cause differences between the estimated parameters even in the absence of faults, which lead to false alarms. Fault detection methods must be sensitive to the appearance of faults, but insensitive (robust) to other changes like noise, modelling errors, operating points, normal signal variations, etc. Even though robustness issues have been recognized in several areas including control, estimation and system identification, robust methods in fault detection have only been developed lately [1-4].

In recent papers, Weiss[5], Horak[6] and Carlsson et, al.[7] have presented some fault detection methods which account for modelling errors. These methods, however, seem to work fine only on low order noise-free Although Emami-Naeini et. al.[8] have systems. suggested a fault detection method for linear systems having modelling errors and noise, their method is restricted to the sensor failure detection in linear systems and not suitable for the detection of faults in plant dynamics and nonlinear systems. Kwon et. al. [9,10] have proposed a robust fault detection method for uncertain systems having modelling errors, noise and nonlinearities. But their method is based on off-line scheme and so less suitable for on-line detection of abrupt changes and other applications where fast on-line decisions are important.

In addition to taking the robustness issue into consideration, the issue of computational complexity should be considered in the fault detection problem since it is closely related with the rapid response to the occurance of a fault. A distinction is made between off-line and on-line detection problems; In off-line problems the detection is based on observations over the complete time interval of interest and in on-line problems a detection decision must be made at each time moment based on past observations. From an off-line point of view, multiple

changes may be found by global search; from an on-line viewpoint, the changes are assumed to be detected one after another. The needs in real-time applications have stimulated the development of an on-line fault detection method.

In this paper, we are primarily concerned with the idea of deriving a practically appealing scheme that extends the work in [9,10]. The fault detection method suggested here accounts for the effect of noise and model mismatch. Modelling errors are depicted by the additive form and the nominal model denominator is fixed via prior experiments in order to quantify the uncertainty bound on the parameter estimation. fixed-interval-sliding-window recursive parameter estimation algorithm with no matrix inversion is presented to provide methodology for handling the real-time fault detection. And the on-line counterpart of the robust fault detection method proposed by Kwon et. al. [9,10] is suggested.

The layout of this paper is as follows: According to the basic introductory background in Section I, an efficient parameter estimation algorithm is adopted in Section II. Then the on-line fault detection method is proposed in Section III. In Section IV, the comparison of computational requirements between the new algorithm presented here and the existing method [9,10] is done, which shows the advantages of the new algorithm. Finally conclusions are in Section V.

# II. SYSTEM DESCRIPTION AND IDENTIFICATION SCHEME

# 2.1 System Description

All mathematical models are only approximate descriptions for real systems. One can describe the model uncertainty with additive form of unmodelled dynamics and measurement noise as shown in Fig. 2.1 [10].

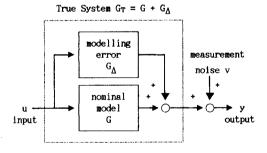


Fig. 2.1 System description including unmodelled dynamics and measurement noise [10].

It is assumed that the true system  $G_T$  and the nominal model G are stable and causal. We also assume that the measurement noise v is a zero mean white noise with variance  $\sigma_V{}^2$ .

The nominal model is taken to be of the form:

$$G(z^{-1}, \theta(k)) = \frac{B(z^{-1}, \theta(k), n_b)}{F(z^{-1}, n_f)}$$
(2.1)

where

$$\begin{split} B(z^{-1},\theta(k),n_b) \; &\equiv \; b_1(k)z^{-1} \; + \; b_2(k)z^{-2} \; + \\ & \cdots \cdots \; + \; b_{n_b}(k)z^{-n_b} \\ \\ F(z^{-1},n_f) \; &\equiv \; 1 \; + \; f_1z^{-1} \; + \; f_2z^{-2} \; + \; \cdots \; + \; f_{n_f}z^{-n_f} \\ \theta(k) \; &\equiv \; [b_1(k) \; b_2(k) \; \dots \; b_{n_b}(k)]^T. \end{split}$$

The denominator  $F(z^{-1},n_f)$  is to be fixed which can be determined from a priori information about the system or by some prior experiments on the parameter estimation for the system [11]. The output of the system satisfies

$$y(k) = G_{T}(q^{-1})u(k) + v(k)$$

$$= G(q^{-1})u(k) + G_{\Delta}(q^{-1})u(k) + v(k)$$

$$= B(q^{-1}, \theta(k), \eta_{b})u_{F}(k) + \eta(k)$$
(2.2)

where q-1 denotes a delay operator and

$$u_{\mathbf{F}}(\mathbf{k}) \equiv \frac{1}{\mathbf{F}(\mathbf{q}^{-1}, \mathbf{n}_{\mathbf{f}})} u(\mathbf{k}) \tag{2.3}$$

$$\eta(k) \equiv G_{A}(q^{-1})u(k) + v(k).$$
(2.4)

# 2.2 Recursive Parameter Estimation

For the parameter estimation, Eq. (2.2) can be represented in standard linear regression form as

$$\mathbf{y}(\mathbf{k}) = \boldsymbol{\phi}^{\mathrm{T}}(\mathbf{k})\,\boldsymbol{\theta}(\mathbf{k}) + \boldsymbol{\eta}(\mathbf{k}) \tag{2.5}$$

where

$$\phi(k) = [u_F(k-1) \ u_F(k-2) \ \dots \ u_F(k-n_b)]^T$$

The least squares method then gives the estimated parameters as  $\ensuremath{\mathsf{a}}$ 

$$\hat{\theta}(k) = [\Phi^{T}(k)\Phi(k)]^{-1}\Phi^{T}(k)Y(k) \qquad (2.6)$$

where

$$\Phi(k) = [\phi(k-N+1) \phi(k-N+2) \dots \phi(k)]^T$$
 (2.7)

$$Y(k) = [y(k-N+1) \ y(k-N+2) \ .... \ y(k)]^T$$
 (2.8)

and N is the number of data in the interval of interest.

Note that the estimator (2.6) is of nonrecursive form and so unsuitable for on-line applications. A recursive procedure is required to speed up the above least squares estimation. This corresponds to finding the recursive least squares estimate of the current parameters of the process, based on 'N' most recent observations.

A general form of the recursive least squares estimate is given for the case of the growing-window problem as follows [12]:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P^{-1}(k) \phi(k) [y(k) - \phi^{T}(k) \hat{\theta}(k-1)]$$
(2.9)

$$P(k) = \lambda(k) P(k-1) + \phi(k) \phi^{T}(k)$$
, (2.10)

where  $\lambda$  is the forgetting factor.

The present problem, however, is the fixed-interval-sliding-window problem and we are constrained to use only 'N' most recent observations for k-N+1 $\leq$ t $\leq$ k. Since the oldest data at time t = k-N should have no influence on the estimate, we need the deletion procedure to have  $\hat{\theta}_d(k-1)$ ,  $P_d(k-1)$  which suffered the deletion of (k-N)th data effect from  $\hat{\theta}(k-1)$  and P(k-1) in Eqs. (2.9) and (2.10). The basic strategy is depicted in Fig. 2.2.

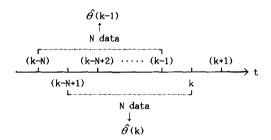


Fig. 2.2 The basic scheme of the recursive FISW (fixed-interval-sliding-window) parameter estimation.

The desired recursive form of fixed-intervalsliding-window least squares method (FISW-LSM) can be obtained as follows [13]:

$$\hat{\theta}(k) = \hat{\theta_d}(k-1) + \mathbb{P}^{-1}(k) \, \phi(k) [y(k) - \phi^T(k) \, \hat{\theta_d}(k-1)]$$
 where

$$P(k) = \lambda(k) P_{d}(k-1) + \phi(k) \phi^{T}(k)$$

$$\hat{\theta_{d}}(k-1) = [I - P^{-1}(k-1) \phi(k-N) \phi^{T}(k-N)]^{-1}$$

$$\cdot [\hat{\theta}(k-1) - P^{-1}(k-1) \phi(k-N) y(k-N)]$$
(2.12)

$$P_d(k-1) = \lambda^{-1}(k-1) [P(k-1) - \phi(k-N)\phi^T(k-N)].$$
(2.13)

To avoid inverting P(k) at each step, we introduce

$$P(k) \equiv P^{-1}(k)$$

$$P_d(k) \equiv P_{d-1}(k)$$
.

Finally the recursive FISW parameter estimation is achieved without matrix inversion, which has an estimation cycle as follows:

1. Delete the effect of (k-N)th data from  $\hat{\partial} \, (k-1)$  and P(k-1).

$$\begin{cases} \hat{\theta}_{\mathbf{d}}(k-1) \\ = \{I - P(k-1)\phi(k-N)[\phi^{T}(k-N)P(k-1)\phi(k-N)-1\}^{-1} \\ \cdot \phi^{T}(k-N)\} [\hat{\theta}(k-1) - P(k-1)\phi(k-N)y(k-N)] \\ (2.14a) \end{cases}$$

$$\begin{vmatrix} P_{d}(k-1) \\ = \lambda (k-1) \left[ P(k-1) - \frac{P(k-1) \phi (k-N) \phi^{T}(k-N) P(k-1)}{\phi^{T}(k-N) P(k-1) \phi (k-N) - 1} \right]$$
(2.14b)

2. Conduct the updating process with new data at t = k.

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}_{d}(k-1) + K(k)[y(k) - \phi^{T}(k)\hat{\theta}_{d}(k-1)] \\ \text{where} \\ K(k) = \frac{P_{d}(k-1)\phi(k)}{\lambda(k) + \phi^{T}(k)P_{d}(k-1)\phi(k)} \\ P(k) = [I - K(k)\phi^{T}(k)] P_{d}(k-1) / \lambda(k) \end{cases}$$

Assuming the matrix  $\Phi(k)$  has full rank for k > N, the recursively derived estimate  $\hat{\theta}(k)$  with  $\lambda = 1$  is identical to its nonrecursive counterpart in Eq. (2.6).

Notice that the matrix P(k) is defined only when the matrix  $\Phi^{T}(k)\Phi(k)$  is nonsingular. Since

$$\Phi^{T}(k)\Phi(k) = \sum_{t=k-N+1}^{k} \phi(t)\phi^{T}(t)$$

it follows that  $\Phi^T(k)\Phi(k)$  is always singular if k is smaller than N. In order to obtain an initial condition for P, it is thus necessary to choose k=N such that  $\Phi^T(k)\Phi(k)$  is nonsingular and determine

$$P(N) = [\Phi^{T}(N)\Phi(N)]^{-1}$$

$$\hat{\theta}(N) = P(N) \Phi^{T}(N) Y(N).$$

The recursive equations can then be used from  $k \ge N$  [14].

With respect to the computation time, the proposed algorithm is more economical than the nonrecursive LSM with moving window. It is useful in situations where the system undergoes some abrupt changes and, for data analysis purposes, one desires to know exactly on which observations each of the succeeding estimates is based.

### 2.3 Estimation Error

From Eqs. (2.5) and (2.6) we can derive the following expression for the estimation error:

$$\widetilde{\theta}(\mathbf{k}) \equiv \widehat{\theta}(\mathbf{k}) - \theta(\mathbf{k}) = [\Phi^{\mathsf{T}}(\mathbf{k})\Phi(\mathbf{k})]^{-1}\Phi^{\mathsf{T}}(\mathbf{k})S(\mathbf{k})$$
(2.15)

where

$$S(k) = [\eta(k-N+1) \eta(k-N+2) \dots \eta(k)]^{T},$$
 (2.16)

Using Eq. (2.4) and denoting the impulse response of  $G_{\Delta}$  as  $\{h(\cdot)\}$ ,  $\eta(k)$  can be expressed as

$$\eta(k) = \sum_{t=k-N+1}^{k} u(t)h(k-t) + v(k) , k \ge N$$
 (2.17)

assuming that u(t)=0 for  $t \le k-N$  and h(t)=0 for t < 0. We obtain the following relationship from (2.16)

$$S(k) = \Psi(k) H(k) + V(k)$$
,  $k \ge N$  (2.18)

where

$$\Psi(k) = \begin{bmatrix} u(k-N+1) & 0 & 0 \\ u(k-N+2) & u(k-N+1) & 0 \\ \vdots & \vdots & \ddots & \vdots \\ u(k) & u(k-1) & \cdots & u(k-N+1) \end{bmatrix}$$
(2.19)

 $H(k) = [h(0) h(1) ... h(N-1)]^T$ 

$$V(k) = [v(k-N+1) \ v(k-N+2) \ ... \ v(k)]^{T}$$

Before proceeding we need to say something about the unmodelled impulse response  $\{h(\cdot)\}\$  in Eq. (2.17). It seems to make no sense to assume this was known since it would then hardly qualify as the unmodelled dynamics. So {h(·)} is taken as a stochastic process here and we assume that a priori knowledge is available for  $\{h(\cdot)\}$ . This procedure is discussed in detail in [15] where the term 'stochastic embedding' is used to describe the procedure giving an a priori distribution In this paper we will simply assume to  $\{h(\cdot)\}.$ knowledge of the mean and covarinace function for this distribution. Given information about the second order statistics of h and v we can then evaluate the expected value of the estimation error.  $E\{\tilde{\theta}(k)\tilde{\theta}^{T}(k)\}.$  This is the basis of the fault detection method to be investigated below.

# III. FAULT DETECTION METHOD

# 3.1 Nonrecursive Scheme

A robust fault detection to unmodelled dynamics with the aid of parameter estimation can be achieved by the following method which is mainly devoted to in [10].

Suppose that there are two sets of data  $I_n$  and  $I_f$ , which are nonfaulty data and suspected faulty data respectively. The estimated parameter  $\hat{\theta}$  may take different values in each experiment:

$$\hat{\theta} = \begin{cases} \hat{\theta}_n , & \text{for data set } I_n \\ \hat{\theta}_f , & \text{for data set } I_f . \end{cases}$$
 (3.1)

We assume that the noises are uncorrelated between the two experiments.

The fault detection procedure now amounts to comparing  $\hat{\theta}_n$  and  $\hat{\theta}_f$  and to deciding if the observed changes can be explained satisfactorily in terms of the effects of noise and/or undermodelling. If not, then we may conclude that a system fault has occured. The covariance function of  $(\hat{\theta}_n - \hat{\theta}_f)$  under nonfaulty condition is used as the measure of the uncertainty due to noise and undermodelling, which is obtained in the following result:

**Theorem 3.1** [10]: Given two sets of estimated parameters as in Eq. (3.1) for the underlying system in Fig. 2.1, the covariance of  $(\hat{\theta}_n - \hat{\theta}_f)$  is as follows under nonfaulty condition:

$$C = Cov(\hat{\theta}_n - \hat{\theta}_f) = \mathbb{E}\{[\hat{\theta}_n - \hat{\theta}_f][\hat{\theta}_n - \hat{\theta}_f]^T\}$$
$$= [Q_n - Q_f] R [Q_n - Q_f]^T + [P_n + P_f] \sigma_{\mathbf{v}^2}$$
(3.2)

where

$$Q_i = P_i \Phi_i^T \Psi_i$$

$$P_i = [\Phi_i^T \Phi_i]^{-1} , i = n, f$$

$$R = E \{HH^T\}$$

E denotes the expectation with respect to the underlying probability space, and  $\Phi$  and  $\Psi$  are as in (2.7) and (2.19) respectively.

The first term in the right of Eq. (3.2) accounts for the effects of undermodelling and the difference in input signals for the two experiments. Note that if there is no undermodelling or if the inputs are identical, this term vanishes. The second term on the right side of Eq. (3.2) corresponds to measurement noise. The higher SNR (signal-to-noise ratio) is, the smaller the norm of this term. Also note that Eq. (3.2) is of nonrecursive form and so unsuitable for the application to om-line detection problems.

# 3.2 Recursive Scheme

For the recursive computation, Eq. (3.2) can be rewritten from the on-line point of view:

$$C(k) = [Q_n - Q_f(k)]R[Q_n - Q_f(k)]^T + [P_n + P_f(k)]\sigma_v^2,$$

$$k \ge N$$
(3.3)

where

$$Q_f(k) = P_f(k) \Phi_f(k) \Psi_f(k)$$
 (3.4)

$$P_f(k) = [\Phi_f^T(k)\Phi_f(k)]^{-1}$$
.

In Eq. (3.3)  $Q_n$  and  $P_n$  are obtained from the nonfaulty data for the fixed interval at a certain operating condition by prior experiments.  $P_f(k)$  can utilize the value which has been already saved on the way to estimating parameter  $\hat{\mathcal{G}}_f(k)$ . In order to obtain  $Q_f(k)$  rapidly we present a recursive algorithm to compute  $\Phi^T(k)\Psi(k)$  in Eq. (3.4).

**Theorem 3.2:** The term  $\Phi^T(\cdot)\Psi(\cdot)$  in Eq. (3.4) can be computed by the following recursion:

$$\Phi^{T}(k)\Psi(k) = \Phi^{T}(k-1)\Psi(k-1) + \phi(k)\psi_{N}^{T}(k) - u(k-N)\Phi^{T}(k-1), \quad k \ge N$$
 (3.5)

where

$$\psi_{i}(k) \equiv [u(k) u(k-1) ... u(k-i+1) O_{N-i}]^{T}$$
 (3.6)

and  $\phi$ (k) is given by Eq. (2.5). In (3.6)  $\mathbb{Q}_j$  denotes the null vector of j columns.

**Proof** : By the definition (3.6) for  $\psi_i(\cdot)$ , we can express  $\Psi(\mathbf{k})$  of Eq. (2.19) as follows :

$$\Psi(k) = [ \psi_1(k-N+1) \ \psi_2(k-N+2) \ \dots \ \psi_N(k) ]^T.$$

We have then

$$\Phi^{\mathsf{T}}(\mathsf{k})\Psi(\mathsf{k})$$

$$= \left[ \phi(\mathsf{k}-\mathsf{N}+1) \ \phi(\mathsf{k}-\mathsf{N}+2) \ \dots \ \phi(\mathsf{k}) \ \right] \left[ \begin{array}{c} \psi_1^{\mathsf{T}}(\mathsf{k}-\mathsf{N}+1) \\ \psi_2^{\mathsf{T}}(\mathsf{k}-\mathsf{N}+2) \\ \vdots \\ \psi_N^{\mathsf{T}}(\mathsf{k}) \end{array} \right]$$

$$= \sum_{i=1}^{N} \phi(k-N+i) \psi_{i}^{T}(k-N+i)$$
 (3.7)

$$= \sum_{i=1}^{N-1} \phi(k-N+i) \psi_i^T(k-N+i) + \phi(k) \psi_N^T(k), \quad (3.8)$$

Eq. (3.7) gives

$$\Phi^{T}(k-1)\Psi(k-1) = \sum_{i=1}^{N} \phi(k-1-N+i) \psi_{i}^{T}(k-1-N+i)$$

$$= \sum_{i=0}^{N-1} \phi(k-N+i) \psi_{i+1}^{T}(k-N+i). \quad (3.9)$$

Since

$$\psi_{i+1}(k-N+i)$$
=  $[u(k-N+i) \dots u(k-N+1) u(k-N) O_{N-i-1}]^T$   
=  $\psi_i(k-N+i) + [O_i u(k-N) O_{N-i-1}]^T$  (3.10)

substituting (3.10) into (3.9) yields

$$\begin{split} & \Phi^{\mathsf{T}}(\mathsf{k}\!-\!1) \Psi(\mathsf{k}\!-\!1) \\ &= \sum_{i=0}^{\mathsf{N}-1} \phi(\mathsf{k}\!-\!\mathsf{N}\!+\!i) \{ \psi_i^{\mathsf{T}}(\mathsf{k}\!-\!\mathsf{N}\!+\!i) + [\ \mathsf{O}_i \ \mathsf{u}(\mathsf{k}\!-\!\mathsf{N}) \ \mathsf{O}_{\mathsf{N}-i-1} \ ] \} \\ & \stackrel{\mathsf{N}-1}{=} \sum_{i=1}^{\mathsf{N}-1} \phi(\mathsf{k}\!-\!\mathsf{N}\!+\!i) \psi_i^{\mathsf{T}}(\mathsf{k}\!-\!\mathsf{N}\!+\!i) + \Phi^{\mathsf{T}}(\mathsf{k}\!-\!1) \mathsf{u}(\mathsf{k}\!-\!\mathsf{N}). \end{split}$$

Eqs. (3.8) and (3.11) give Eq. (3.5), which completes the proof.

 $\nabla\nabla\nabla$ 

Note that Theorem 3.2 makes it possible to compute C(k) of Eq. (3.3) rapidly since Eq. (3.5) is computed recursively. We can derive a recursive equation for C(k), if necessary, using Eqs. (2.14) and (3.5).

Of course, the derivation of the covariance C(k) depends upon prior knowledge of  $\sigma v^2$  and R. This data can be obtained from prior experimentation with nonfault systems based on some simplifying assumptions. For instance, in recent literature on robust adaptive control [16,17] it has been assumed that the unmodelled dynamics are bounded by an exponential function. The corresponding stochastic assumption would be

$$E\{h(k)h(j)\} = r(k)\delta_{k,i}$$

where 
$$r(k) = \sigma_0^2 e^{-\beta k}$$
;  $k, j = 0, 1, \cdots$  (3.12)

In Eq. (3.12),  $2/\beta$  can be considered as the 'average' time constant for the class of unmodelled dynamics. Given the simple description (3.12), we can estimate  $\sigma \circ^2$  and  $\beta$  from a sequence of prior experiments on nonfault systems.

The matrix C(k) of (3.3) can now be used to formulate appropriate test variables for robust fault detection. For example, we may use

$$T_1(k) = [\hat{\theta}_n - \hat{\theta}_f(k)]^T C^{-1}(k) [\hat{\theta}_n - \hat{\theta}_f(k)]$$
 (3.13)

$$T_2(k) = [\hat{\theta}_n - \hat{\theta}_f(k)]^T [diag(C(k))]^{-1} [\hat{\theta}_n - \hat{\theta}_f(k)]$$

which are based on the idea of comparing the observed value of  $[\hat{\theta}_n - \hat{\theta}_f(k)][\hat{\theta}_n - \hat{\theta}_f(k)]^T$  with its expected value, i.e. the covariance C(k). If the test variable is larger than a proper threshold, we take this as an evidence that the system parameters have changed, i.e. a fault has occured.

Note that the detection algorithm proposed in this section is applicable to on-line fault detection problems since it is of recursive form and has a low computational burden, which will be shown in the next section.

# IV. COMPARISON OF COMPUTATIONAL BURDEN

In passing from nonrecursive scheme to recursive one, considerable improvements in the computational aspect are expected. As shown in Table 4.1, where the required numbers of multiplications are compared, it can be seen that the improvements of the new algorithm for fault detection are obvious.

Table 4.1 Comparison of computational requirements (number of multiplications) between nonrecursive and recursive schemes.

Object	Nonrecursive scheme	Recursive scheme
$\hat{ heta}(k)$	$[(^{3}/_{2})n_{b}^{2} + n_{b}]N$ + $(n_{b}^{6})/2$	$(3/2)n_b^3 + 6n_b^2 + 6n_b$
C(k)	$(1/2) n_b N^2$ + $(n_b^2 + n_b) N$ + $(1/2) n_b^2$	$(2n_b^2 + 3n_b)N$ + $(3/2)n_b^2$
T <sub>1</sub> (k)	пь <sup>6</sup> + пь <sup>2</sup> + пь	n <sub>b</sub> <sup>6</sup> + n <sub>b</sub> <sup>2</sup> + n <sub>b</sub>
Total computational burden	O (N²)	O (N)

where n<sub>b</sub>: number of parameters to be estimated N: number of data in the fixed interval.

Note that the total computational burden of nonrecursive and recursive schemes can be summarized as order of  $N^2$  and order of N, respectively since N is usually much larger than  $n_b$  in Table 4.1. As a consequence, the algorithms presented in this paper is shown to be more convenient for practical applications to on-line fault detection than the nonrecursive algorithms in [9,10].

#### V. CONCLUSIONS

The aim of this paper is to design an on-line fault detection method robust to modelling error and measurement noise. An effort has been made in this paper to overcome the computational complexity problem in existing methods. The recursive fixed-intervalsiding-window filter (2.14) and the recursive algorithm for C based on Theorem 3.2 have given a contribution to this purpose. Since the proposed algorithm is a recursive version of the robust fault detection method in [9,10], it is supposed to exhibit the same detection performance as that of [9,10]. Moreover, it is applicable to on-line fault detection problems since its computational burden is considerably lessened as shown in Table 4.1.

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