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A Geometric Approach to Fault Diagnosis Algorithm in Linear Systems

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ABSTRACT

An algorithm for multiple fault diagnosis of linear dynamic systems is proposed. The algorithm is constructed by using of the geometric approach based on observation that, when the number of faulty units of the system is known, the set of faulty units can be differentiated from other sets by checking linear varieties in the measurement data space. It is further shown that the system with t number of faults can be diagnosed within (t+1) sample-time units if the input-output measurements are rich and that the algorithm can be used for diagnosis even when the number of faults is not known in advance.

INTRODUCTION

As systems to be controlled become more and more complex, reliability becomes a vital issue of control system design and, at the same time, an efficient fault diagnosis mechanism is called for as an integral part of the system. The purpose of fault diagnosis is to detect faults in the system at as early a stage as possible before the performance of the system shows signs of marked abnormality, locate the faulty parts and estimate the degree of the fault[1].

Various methods of fault diagnosis have been developed, but most of them are concerned with digital combinational / sequential computing machines [2]. Recently, increasing attention is being paid to the problem of fault diagnosis of dynamic industrial processes [3] [4]. Earlier research includes results on fault detection problems or fault location issues under the assumption that the system has a finite set of known fault modes/states [5][6]. In many practical cases, however, one cannot enumerate all the possible fault modes and characterize the behavior of each component in its faulty state. Another well-known approach of fault diagnosis for dynamic systems is to introduce some redundancies at sensors, actuators and /or the system itself in the form of a functional observer or the like and to analyze residual signals to decide whether or not there exists any fault [7]. The analytical redundancy method, which uses the set of parity equations representing the relations between inputs and outputs, is an effective tool for diagnosis but the method works only if there exist redundant relations in the system.

In case that a fault takes any value in some continuous range and a system has no redundant relation as in SISO systems, one way to determine the location and size of a fault in the system is to construct a model of the system so as to represent a faulty condition in terms of the model parameters. Such an approach may have wide application in the diagnosis of controller malfunctions [8].

To be more specific, consider the plant which is represented by the following linear difference model.

$$y(k) = a_1 y(k-1) + \cdots + a_n y(k-n) + b_0 u(k) + \cdots + b_n u(k-n) \quad , \quad (1)$$

where y(k) and u(k) are the output and the input of the plant. The equation parameters $a_1, a_2, \ldots, b_n$ represent additive and/or multiplicative combination of characteristic values of components, which are often called as physical coefficients [4], such as length, mass, viscosity, resistance, etc. When there is no fault in the plant, that is, when the plant is in normal condition, the equation parameters $a_1, a_2, \ldots, b_n$ have their nominal values.

Suppose that there occurs a change in the characteristic value of a hardware component of the plant. Then some of the equation parameters are deviated from their nominal values. When the changes of parameters are small and slow, several conventional control laws such as feedback control or adaptive control can be applied to accommodate the change of system behavior without serious degradation of system performance. But when changes in some parameters are abrupt and detrimental to system performance, which, we say, is caused by a fault, the faulty parts must be detected and located as early as possible.

For detecting and locating faults, a parameter estimation method can be employed [4], but, again, the scheme may be quite inefficient specially when the number of parameters may be large and/or when the number of allowed measurements is restricted, for example, due to safety reasons. Further it may require additional decision techniques such as a pattern recognition technique [9] to determine which parameters are deviated from their nominal values.

Recently, Ono et al. [1] proposed a geometric approach in which a notion of influence matrix for the diagnostic parameter and a technique of geometric alignment are used to determine the locations and the values of deviated parameters in the dynamic system. The method proves to be effective in handling the degree of influence of each parameter on the estimated variation vector but, since it is based on a recursive parameter estimation, some difficulty may arise in case of on-line real time diagnosis. Further the method is essentially effective only for the case of a single fault.

It is remarked that a typical fault diagnosis assumes that the system under examination is fully known and starts with checking if the system is structurally perturbed: thus, it is natural to suppose that the number of faults that occur simultaneously is small. On the other hand, a parameter estimation scheme, when applied for fault diagnosis, may require more measurements (and thus take time) until the estimated values correctly indicate the number of simultaneous faults. In this sense, any fault diagnosis based on conventional parameter estimation techniques can be inefficient and less informative.

In this paper, a systematic geometric approach for multiple fault diagnosis of linear dynamic systems is proposed. Here the word "geometric approach" is used to indicate the fact that a linear variety generated by measured signals together with a concept of distance is used for the analysis of the system. In the proposed method, it is assumed that a system model with given nominal parameters is available, and by using input/output measurements, each component is examined in regard to whether or not a given criterion is satisfied and thereby, faultness of the component is determined. This proposed approach can be most effective for a class of systems whose time constant is of an order of several seconds or several minutes as in industrial process control systems [8] so that, during one sampling period, the algorithm can perform computations as many times as the number of fault units.

PROBLEM STATEMENT

For concise presentation, we shall also use as an alternative representation of eqn.(1) the equation

$$y(k) = \Phi^T(k)\,\Theta \qquad (2)$$

to be the model of the linear dynamic system under consideration. Here $\Phi(k)=[\ y(k-1)\ y(k-1)\ \dots\ u(k-n)\ ]^T$ is a column vector consisting of old inputs and outputs and present inputs of the system. And $\Theta=[\ a_1\ \ a_2\ \dots\ b_n]^T$ is the parameter vector of the eqn.(1) with dimension N=2n+1.

As a first step for fault diagnosis, we shall view the system as an interconnection of functional "units". Here a unit is a term to represent a physical device (component) or a module and carries with it a nominal characteristic value such as resistance or gain. When one or more characteristic values of units change significantly, we say that the system is in a state of a malfunction or in a faulty condition and that fault diagnosis is to determine the units, and their magnitudes, whose characteristic values have changed more than prespecified values.

To be more definite, let R^N_θ denote the N dimensional Euclidean space of equation parameters . Also let the number of the given functional units of the system be M. We specifically designate Θ_0 to mean the parameter vector in the fault-free condition of the system and Θ_F the parameter vector in some faulty condition, respectively. When the system is in a faulty condition Θ_F , one or more units must have changed their characteristic values from the nominal ones.

Thus, the system (2) falls into faulty condition at some time k if

$$y(k) \neq \Phi^T(k)\Theta_0 \quad , \qquad k=k_0,\ k_0+1,\dots \ . \qquad (3)$$

and in this case there corresponds a vector Θ_F such that

$$y(k) = \Phi^T(k)\Theta_F \quad , \qquad k=k_0,\ k_0+1,\dots \ . \qquad (4)$$

To see how the fault of a unit affects the equation parameter vector (or the behavior) of the system represented by eqn. (2), we say that, with regard to the given functional units of the system, there corresponds a vector set $\{ e_1, e_2, \dots, e_M \}$ in the parameter space R^N_θ such that

$$\Theta_F - \Theta_0 = \alpha_1 e_1 + \dots + \alpha_i e_i + \dots + \alpha_M e_M \qquad (5)$$

where α_i denotes a deviation of the i-th unit from the nominal characteristic value so that α_i becomes nonzero constant if the i-th unit is faulty and $\alpha_i = 0$ if the unit is in normal condition.

Note that if M = N and if $\{ e_1, e_2, \dots, e_M \}$ is the set of orthonomal basis, that is, $e_1 = [1\ 0\ \dots\ 0]$, ... , $e_M = [0\ 0\ \dots\ 1]$, then the components of the parameter vector Θ directly indicate the locations of the units and their characteristic values.

It is remarked that the set $\{ e_1, e_2, \dots, e_M \}$ is not necessarily to be a set of linearly independent vectors. We shall first deal with the case in which the characteristic values of the units and the equation parameters are linearly and algebraically related. The case when the relationship is nonlinear algebraic will be discussed later.

For example, consider a system described by

$$\begin{aligned} y(k) = {} & (p_1 + p_3 + p_4)y(k-1) + (p_1 + p_2 + p_3)y(k-2) \qquad (6) \\ & + (p_2 + p_3)y(k-3) + (p_3 + p_4 + p_5)u(k-3) \ . \end{aligned}$$

where p_1, p_2, p_3, p_4, p_5 denote the characteristic values of functional units. In this case, the system has 4 equation parameters $\theta_1, \theta_2, \theta_3, \theta_4$ and 5 units p_i, i = 1, 2,..., 5, so that N=4 and M=5. Examining eqn. (6), we may set $e_1 = [\ 1\ 1\ 0\ 0]$, $e_2 = [\ 0\ 1\ 1\ 0]$, $e_3 = [\ 1\ 1\ 1\ 1]$, $e_4 = [\ 1\ 0\ 0\ 1]$, $e_5 = [\ 0\ 0\ 0\ 1]$. Consider, for example, the 1st unit that becomes a faulty unit and assume that p_1 deviates from its nominal value by α_1. Then the same amount of deviation is caused in the parameter θ_1 and θ_2 , and therefore we may set $e_1 = [\ 1\ 1\ 0\ 0]$. Also it can be seen that a single fault of any one unit among p_2, p_3, p_4 is not distinguishable from a double fault of the rest, since the set $\{e_2, e_3, e_4\}$ is linearly dependent. Note, however, that under a single fault assumption, any functional unit is distinguishable from another unit because any two vectors among $\{ e_1, e_2, e_3, e_4, e_5 \}$ are linearly independent from each other.

We first consider the fault diagnosis problem when the number of faulty units is known in advance. We shall call the following as the t-fault diagnosis problem (tFDP).

tFDP) Suppose the system (2) whose parameter vector is Θ at its normal operation falls into a faulty condition Θ_F at time k_0, caused by t faulty units. The t-fault diagnosis problem (tFDP) is to determine the location $\{ i_1, i_2, \dots, i_t \}$ of t faulty units among M units { 1, 2,..., M } and their magnitudes of deviations in minimum number of sampling time-steps.

In order to examine the system in a faulty condition O , first observe that the units $\{ i_1, i_2, \ldots, i_t \}$ are faulty if and only if all the measurement vectors $\Phi(k)$ satisfy the equation

$$y(k) = \Phi^T(k)\Theta_F = \Phi^T(k)\Theta_0 + \alpha_{i_1}\Phi^T(k)e_{i_1} + \alpha_{i_2}\Phi^T(k)e_{i_2} + \cdots + \alpha_{i_t}\Phi^T(k)e_{i_t}$$
$$k=k_0,\ k_0+1,\ldots \qquad (7)$$

for some $(\alpha_{i_1}, \alpha_{i_2}, \cdots, \alpha_{i_t})$ with $\alpha_{i_m} \neq 0$ for each m = 1, 2,..., t. Thus, the objective of tFDP is to determine $\{ i_1, i_2, \ldots, i_t \}$ and the corresponding $(\alpha_{i_1}, \alpha_{i_2}, \cdots, \alpha_{i_t})$ from y(k) and o(k) which are measured from the faulty system described by eqn. (5).

FAULT DIAGNOSIS ALGORITHM

* Case when the number of faulty units is known.

Suppose the system (2) becomes faulty at $k= k_0$ and the number of faulty units is known as t. Then, it follows from eqn.(7) that, for $k_0 < k_1 < \ldots < k_m$, a set of m measurements, $s_m = \{\Phi(k_1), \Phi(k_2), \ldots, \Phi(k_m)\}$ satisfy the following equations;

$$\begin{aligned} \alpha_{i_1}\Phi^T(k_1)e_{i_1} + \cdots + \alpha_{i_t}\Phi^T(k_1)e_{i_t} + \Phi^T(k_1)\Theta_0 - y(k_1) &= 0 \\ &\vdots \\ \alpha_{i_1}\Phi^T(k_m)e_{i_1} + \cdots + \alpha_{i_t}\Phi^T(k_m)e_{i_t} + \Phi^T(k_m)\Theta_0 - y(k_m) &= 0 \end{aligned} \qquad (8)$$

For an arbitrary set of t units $J = \{ j_1, j_2, \ldots, j_t \}$ and for a given set $s_m = \{ \Phi(k_1), \Phi(k_2), \ldots, \Phi(k_m) \}$ let a m x t matrix $\Delta_J(s_m)$ and an m x 1 vector $\eta(s_m)$ be defined as follows.

$$\Delta_J(s_m) = \begin{bmatrix} \Phi^T(k_1)e_{j_1} & \Phi^T(k_1)e_{j_2} & \cdots & \Phi^T(k_1)e_{j_t} \\ \Phi^T(k_2)e_{j_1} & \Phi^T(k_2)e_{j_2} & \cdots & \Phi^T(k_2)e_{j_t} \\ \vdots & \vdots & . & \vdots \\ \Phi^T(k_m)e_{j_1} & \Phi^T(k_m)e_{j_2} & \cdots & \Phi^T(k_m)e_{j_t} \end{bmatrix} \qquad (9)$$

$$\eta(s_m) = \begin{bmatrix} y(k_1) - \Phi^T(k_1)\Theta_0 \\ y(k_2) - \Phi^T(k_2)\Theta_0 \\ \vdots \\ y(k_m) - \Phi^T(k_m)\Theta_0 \end{bmatrix} \qquad (10)$$

Also let a linear variety $V_J(s_m)$ be defined in an m-dimensional space as follows.

$$V_J(s_m) = \{ x \mid x = \Delta_J(s_m)\lambda + \eta(s_m),\ \lambda \in R^t,\ \lambda_j \neq 0,\ j=1,2,\ldots,t \} \qquad (11)$$

where λ_j is an element of λ.

Then we can obtain the following result.

Theorem 1. Suppose the set of units $I=\{i_1, i_2, \ldots, i_t\}$ are faulty. Then the null vector 0_m of R^m is in $V_I(s_m)$ for any s_m while, for $J \neq I$, $0_m \notin V_J(s_m)$ if s_m is a measurement set satisfying the relation

$$rank\ (\Delta_J(s_m)) \neq rank\ ([\ \Delta_J(s_m) : \eta(s_m)\ ]) \qquad (12)$$

Here [:] denotes the augumentation of matrices.-

Proof) For the set $I = \{ i_1, i_2, \ldots, i_t \}$, and for any given s_m, eqn.(8) can be written by

$$\Delta_I(s_m)\lambda_I - \eta(s_m) = 0_m \quad \lambda_I = [\ \alpha_{i_1}\ \alpha_{i_2}\ \cdots\ \alpha_{i_t}\ ]^T.$$

Then, from the definition of $V_I(s_m)$ in eqn.(11), x_I becomes 0_m when $\lambda = -\lambda_I$, which shows that $V_I(s_m)$ contains 0_m. Now consider the set of t units J such that $J \neq I$. For a measurement set s_m that satisfies $rank(\ \Delta_J(s_m)\ ) \neq rank(\ [\ \Delta_J(s_m) : \eta(s_m)\ ]\ )$, suppose $0_m \in V_J(s_m)$. Then, from eqn.(11), there exists a solution $\lambda_J \in R^t$ such that $\Delta_J(s_m)\lambda_J + \eta(s_m) = 0_m$, which, however, implies that [10]

$$rank(\ \Delta_J(s_m)\ ) = rank(\ [\ \Delta_J(s_m) : \eta(s_m)\ ]\ ).$$

This condition is contradictory to the condition (12).

Q.E.D.

The above Theorem tells that the linear variety $V_I(s_m)$ of faulty units can be differentiated from $V_J(s_m)$ with $J \neq I$ according to the criterion whether or not the null vector 0_m is contained in $V_J(s_m)$ if the set of I/O measurements s_m satisfying the relation (12).

It is remarked that it can happen that $rank(\ \Delta_J(s_m)\ ) = rank(\ [\ \Delta_J(s_m) : \eta(s_m)\ ]\ )$ for some $J \neq I$ when the output of the system is not "rich" as in the case when the system is in a regulated equilibrium state.

It is also noted that, when the mxt matrix $\Delta_J(s_m)$ is of full rank, m must be greater than t to satisfy the condition (12) since $rank(\ [\ \Delta_J(s_m) : \eta(s_m)\ ]\ )$ must be greater than $rank(\ \Delta_J(s_m)\ )$. Therefore, to satisfy the condition (12) for all $J \neq I$, it is necessary that the number of measurements be greater than the number of faulty units. In fact, if measurements are rich but not contaminated by noise, and if there is no numerical error, then t+1 number of measurements are sufficient to diagnose the faulty system with t faults.

Practically, it may happen that the linear variety for the set of faulty units may not contain the null vector because of measurements noise or numerical error. In this case, it is likely that the linear variety for the set of faulty units lies in the neighborhood of the origin if the effect of noise is small. As a notion to help determine the effect of disturbance in deciding faulty units, we use as a criterion the square of the minimum distance from the origin to the linear variety. More specifically, we utilize the orthogonal projection theorem [11] to find the vector, denoted by $x_J^0(s_m)$, having the minimum distance from the origin to the $V_J(s_m)$ as follows.

$$x_J^0(s_m) = -H_J(s_m)^T\, G_J(s_m)^{-1}\, \Delta_J(s_m) + \eta(s_m) \qquad (13)$$

Here $G_J(s_m)$ is the Gram matrix whose element g_{ij} is $< \delta_i , \delta_j >$ where δ_i is the i-th column vector of $\Delta_J(s_m)$ and < , > denotes an inner product. $H_J(s_m)$ is the vector whose i-th element is $< \eta(s_m) , \delta_i >$. Then we define the distance factor, $d_J(s_m)$ of the linear variety $V_J(s_m)$ to be :

$$d_J(s_m) = \| x_J^0(s_m) \|^2 = \| -H_J(s_m)^T\, G_J(s_m)^{-1}\, \Delta_J(s_m) + \eta(s_m) \|^2 \qquad (14)$$

For practical applications, an appropriate threshold value ϵ may be introduced in consideration of given plant environment and a decision is made in such a way that only if $d_J(s_m)$ is greater than a given threshold ϵ , the set $\{ j_1, j_2, \ldots, j_t \}$ is not the set of faulty units.

When we construct a diagnosis algorithm for a faulty system, all the possible candidates for fault units must be taken into account. For this, we must first begin with forming the collection of subsets consisting of t units as fault candidates. Denote the collection of all subsets as F_t : i.e.,

$$F_t = \{ J_t \mid J_t = \{ j_1, j_2, \ldots, j_t \} \subset \{1, 2, \ldots, M\} \} \quad . \qquad (15)$$

In case that the system continuously generates dependent measurements and there is no sign of new information content in the measurements, we must stop the diagnosis procedure for the safety of the system after concluding that the system cannot be diagnosed. For this, an appropriate maximum number $m_{\max}$ of measurements should be introduced in the diagnosis procedure.

Now, as a solution for the tFDP problem, we can suggest the following algorithmic procedure if the system can be described as eqn.(7) when the system falls into a faulty condition.

Let there be given

Step 1. Let $m = t + 1$;
Step 2. Take m measurements to form s_m ;
Step 3. Let Q_F=0[temporary set of candidate units] ;
Step 4. Choose a J_t out of F_t [refer (18)] ;
Step 5. Compute $d_J(s_m)$ [refer (17)] ;
Step 6. If $d_J(s_m) < \epsilon$, save J_t in Q_F ;
Else, continue ;
Step 7. If F is empty set, go to Step 8 ;
Else, go to Step 4 ;
Step 8. F_t <- Q_F [Put the contents of Q_F into F_t] ;
Step 9. If F_t has more than one element,go to Step 10;
Else, go to Step 11 ;
Step 10. If $m < m_{\max}$, m <- m+1 and go to Step 2 ;
Else, conclude that the system cannot be diagnosed and stop ;
Step 11. End of diagnosis :
If F_t is empty, # of faults = t and stop ;
Else, J_t in F_t is faulty units set and stop ;

In the above t-fault diagnosis procedure, whenever the number of measurements is incremented, it needs to generate txt dimensional Gram matrix and its inversion. This computational burden, however, can be decreased if those matrices are evaluated recursively. To be specific, let a new measurement at $k = k_{m+1}$, denoted by $\Psi_J(k_{m+1}) = [\, \Phi^T(k_{m+1})e_{j_1} \cdots \Phi^T(k_{m+1})e_{j_t} \,]^T$, be given after computing the $G_J(s_m)$ as in eqn.(14). Also let s_{m+1} denote $s_m \cup \Phi(k_{m+1})$. Then we can use the well-known matrix inversion lemma [12] to compute the distance factor as follows.

$$G_J(s_{m+1}) = G_J(s_m) + \Psi_J(k_{m+1})\,\Psi_J(k_{m+1})^T \qquad (16)$$

$$H_J(s_{m+1}) = H_J(s_m) + [\, y(k_{m+1}) - \Phi^T(k_{m+1})\Theta_0 \,]\,\Psi_J(k_{m+1})^T \qquad (17)$$

$$P_J(s_{m+1}) = P_J(s_m) - P_J(s_m)\,\Psi_J(k_{m+1}) \qquad (18)$$

$$[\, \Psi_J(k_{m+1})^T P_J(s_m)\,\Psi_J(k_{m+1}) + 1]^{-1}\,\Psi_J(k_{m+1})^T P_J(s_m)$$

where $P_J(s_m) = G_J(s_m)^{-1}$.

$$d_J(s_{m+1}) = \| x_J^0(s_{m+1}) \|^2 = \| - H_J(s_{m+1})^T P_J(s_{m+1})\,\Delta_J(s_{m+1}) + \eta(s_{m+1}) \|^2 \qquad (19)$$

* Case when the number of fault units is unknown

When a system starts malfunctioning, we don't know in most cases how many faults have caused the system to malfunction. One way to deal with such a system is to begin with the single fault diagnosis procedure, with the understanding that the single fault is the most frequent case [13]. If it turns out that the system has more than one fault, then the proposed algorithm reveals that the system has more than one fault and we propose to keep applying the algorithm by increasing the number of assumed faulty units one at a time. Since the probability of simultaneous multiple faults decrease drastically as the number t gets larger, this is rather natural a method when the fault units is unknown in advance. The fault diagnosis procedure when the number of faulty units is unknown is depicted as a flow chart form in Fig. 1.

PRACTICAL APPLICATION

Suppose the system has M functional units and let P denote the M x 1 vector whose components correspond to the characteristic values of the units. Let the relationship between Θ and P is given by $\Theta = G(P)$. At present, no fault diagnosis is available for a general nonlinear relation $\Theta = G(P)$. However, for simple cases under a single fault assumption, we may use the proposed algorithm.

Let the nominal characteristic values of P be denoted by P_0 and the vector of their deviations due to fault by $\Delta P = [\, \delta p_1 \;\; \delta p_2 \;\; \cdots \;\; \delta p_M \,]^T$. Then we can write the vector $G(P) = G(P_0 + \Delta P)$ in the form of Taylor's expansion as follows:

$$G(P_0 + \Delta P) = G(P_0) + \delta p_1 \frac{\partial G}{\partial p_1}(P_0) + \delta p_2 \frac{\partial G}{\partial p_2}(P_0) + \cdots$$
$$+ \delta p_M \frac{\partial G}{\partial p_M}(P_0) + \frac{1}{2}\,\delta p_1^2 \frac{\partial^2 G}{\partial p_1^2}(P_0) + \cdots$$
$$+ \frac{1}{2}\,\delta p_1 \delta p_2 \frac{\partial^2 G}{\partial p_1 \partial p_2}(P_0) + \cdots \qquad (20)$$

Then, we can show that the proposed method can be applied for the system $y(k) = \Phi^T(k)\, G(P)$ if

1. There exists only single fault, and

2. For each $P \in R^M$, $\dfrac{\partial^n G(P)}{\partial p_i^n} = 0$

for i = 1, 2,..., M and $n \geq 2$

Then the system equation becomes of the same form as eqn.(7), if Θ_0 , e_i and α_i are replaced by $G(P_0)$, $\dfrac{\partial G(P)}{\partial p_i}$ at $P = P_0$ and δp_i, respectively, and thus we can apply the proposed algorithm.

Here two examples are provided ; the first one is considered to compare the proposed method with a typical parameter estimation scheme and the second example shows an application for a PID controller.

Example 1) Consider the system represented by following transfer function.

$$\frac{Y(z)}{U(z)} = \frac{1}{z^5 - 0.5z^4 - 0.1z^3 - 0.2z^2 + 0.05z + 0.1} \qquad (21)$$

The system is equivalent to

$$y(k) = \Phi^T(k)\,\Theta_0 \, , \qquad k = 1, 2, \ldots$$

where $\Phi(k) = [y(k-1)\ y(k-2)\ y(k-3)\ y(k-4)\ y(k-5)\ u(k-5)]^T$, and $\Theta_0 = [0.5\ 0.1\ 0.2\ -0.05\ -0.1\ 1]$. Suppose $u(k) = 2$, $k = 1, 2, \ldots$ was applied, when double faults occurs at $k = 12$ in such a way that Θ_0 is deviated to $\Theta_F = [0.8\ 0.1\ 0.2\ -0.55\ -0.1\ 1]$. The diagnosis algorithm is applied under the assumption that the number of faults is unknown. The result in Table 1 shows that the double faults are diagnosed in 4 sampling steps. Here, the algorithm went through a single fault diagnosis to conclude there exist double faults. The proposed algorithm may be compared with the method based on the parameter estimation. Estimated parameters obtained by the orthogonal projection method and the recursive least square method are shown in Table 2. By comparison, we may observe that the proposed method is more efficient in fault detection and location than the other methods.

Example 2) In the PID controller shown in Fig. 2, u_c, u_f and y denote the command input, fedback signal, control input to plant respectively. Suppose the controller contains 6 fault-prone units as follows:

$$P = [p_1\ p_2\ p_3\ p_4\ p_5\ p_6]^T = [K_c\ K_f\ K_p\ T_I\ K_D\ T_D]^T$$

To deal with the system in discrete domain, let the sampling time be T = 0.5sec. Then the discrete time dynamic equation of the PID controller obtained by Tustin's method [13] is $\Phi^T(k)\,\Theta = 0$ where $\Phi = [y(k)\ y(k-1)\ y(k-2)\ u_c(k)\ u_c(k-1)\ u_c(k-2)\ u_f(k)\ u_f(k-1)\ u_f(k-2)]^T$ and

$$\Theta = G(P) = \begin{bmatrix} -p_4(0.25+p_6) \\ 2p_4p_6 \\ p_4(0.25-p_6) \\ p_1\{p_3(0.25+p_4)(0.25+p_6)+p_4p_5p_6\} \\ p_1\{p_3(0.125-p_4p_6)-2p_4p_5p_6\} \\ p_1\{p_3(0.25-p_4)(0.25-p_6)+p_4p_5p_6\} \\ -p_2\{p_3(0.25+p_4)(0.25+p_6)+p_4p_5p_6\} \\ -p_2\{p_3(0.125-p_4p_6)-2p_4p_5p_6\} \\ -p_2\{p_3(0.25-p_4)(0.25-p_6)+p_4p_5p_6\} \end{bmatrix}$$

Thus, the mapping G is nonlinear and we can know that G(P) satisfies the assumption 2. Therefore, a single fault diagnosis is valid for the PID controller. When $u_c(k) = 900$, $u_f(k) = 60/61\ u_f(k-1) + 0.9/61\ y(k)$ and the normal characteristic values are P = [1.0 1.0 1.4 2.0 2.2 1.0], then it is found that for any single fault, the system is diagnosed exactly within as few as 2 sampling time-steps, as shown in Table 3.

CONCLUSION

An algorithm for multiple fault diagnosis of plants described by a linear difference model was proposed. It was shown that the system with t number of faults can be diagnosed within (t+1) sample-time units if the input-output measurements is rich. Also the proposed algorithm can be naturally extended to the case that the number of faults is unknown by beginning with a single fault diagnosis as a first step, and proceeding to multiple cases one by one. Specially, robustness property of the proposed algorithm against measurement noise were analyzed. We believe that this geometric approach will be effective for systems in which the number of units in simultaneous faulty state is less than the total number of units or for systems with large time constants and the sampling period is quite long.

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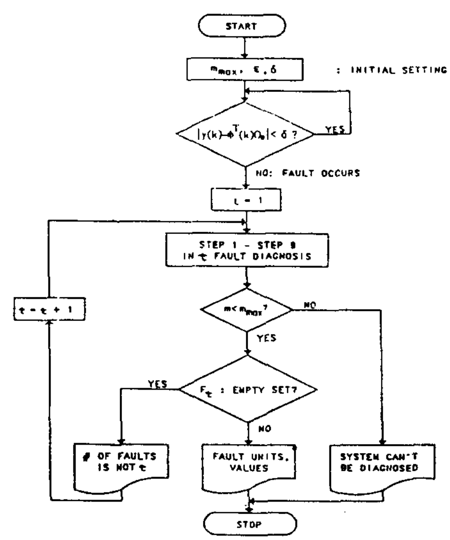

Fig. 1. Flow Diagram of Fault Diagnosis Procedure

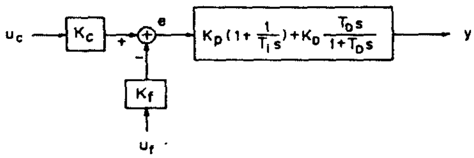

Fig. 2. PID Controller

normal = [0.5000 0.1000 0.2000 -0.0500 -0.1000 1.0000]
faulty = [0.8000 0.1000 0.2000 -0.5500 -0.1000 1.0000]
deviat = [0.3000 0.0000 0.0000 -0.5000 -0.0000 0.0000]

threshold=0.01

sampling step	Diagnosis Procedure
12	fault occures >> start of 1 -fault diagnosis
13	--fault candidates--distance factor-- 1 0.3012 2 0.2831 3 0.2837 4 0.2380 5 0.2371 6 0.3011
	RESULT : number of faults is not (1) >> start of 2 -fault diagnosis
14	--fault candidates--distance factor-- 1 , 2 0.0215 1 , 3 0.0106 1 , 4 0.0000 1 , 5 0.0680 1 , 6 0.2980 2 , 3 0.0008 2 , 4 0.0077 2 , 5 0.0802 2 , 6 0.2967 3 , 4 0.0135 3 , 5 0.1085 3 , 6 0.0088 4 , 5 0.2287 4 , 6 0.0535 5 , 6 0.0379
	RESULT : candidate not unique in diagnosis processing
15	--fault candidates--distance factor-- 1 , 4 0.0000 2 , 3 0.0384 2 , 4 0.0175
	RESULT : end of diagnosis faulty units and their values : (1 , 4) (0.3000 , -0.5000)

Table 1. Result of Diagnosis Procedure

PARAMETER ESTIMATED BY RECURSIVE LEAST SQUARE

sampling step	estimated deviations					
12	-0.0111	-0.0105	-0.0093	-0.0078	-0.0061	-0.0041
13	0.2512	0.0716	-0.1001	-0.2711	-0.2254	0.0815
14	0.2435	0.0590	-0.1448	-0.3676	-0.0845	0.1839
15	0.2275	0.1394	-0.1525	-0.4174	-0.0339	0.0842
16	0.1983	0.1073	-0.0051	-0.4877	-0.0473	0.0880
17	0.2941	0.0057	0.0004	-0.5001	-0.0028	0.0050

PARAMETER ESTIMATED BY ORTHOGONAL PROJECTION

sampling step	estimated deviations					
12	-0.0111	-0.0105	-0.0093	-0.0078	-0.0061	-0.0041
13	0.2512	0.0716	-0.1001	-0.2711	-0.2254	0.0815
14	0.2435	0.0590	-0.1440	-0.3678	-0.0845	0.1839
15	0.2275	0.1385	-0.1525	-0.4175	-0.0336	0.0842
16	0.1953	0.1073	-0.0049	-0.4978	-0.0473	0.0880
17	0.3000	0.0000	-0.0000	-0.5000	-0.0000	0.0000

Table 2. Result of Parameter Estimation

DIAGNOSIS OF FAULTY PID CONTROLLER

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [2.0000 1.0000 1.4000 2.0000 2.2000 1.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	0.0000	0.0000	0.0000
2	3.8906	11.0201	23.4201
3	0.5462	2.4220	6.5169
4	0.0182	0.0985	0.3208
5	2.9115	6.7469	11.2422
6	2.7529	7.4085	13.9725
unit having min. d.f.	1	1	1
estimated deviation	1.0000	1.0000	1.0000

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [1.0000 2.0000 1.4000 2.0000 2.2000 1.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	1.4387	7.7554	28.3140
2	-0.0000	-0.0000	0.0000
3	0.5559	2.4490	6.6194
4	0.0288	0.0919	0.1994
5	0.2077	0.9369	2.5330
6	0.6871	3.2844	9.6131
unit having min. d.f.	2	2	2
estimated deviation	1.0000	1.0000	1.0000

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [1.0000 1.0000 2.8000 2.0000 2.2000 1.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	1.4428	8.1904	29.2096
2	4.3174	12.8958	24.5456
3	0.	-0.0000	-0.0000
4	0.0002	0.0011	0.0036
5	4.5278	8.2947	10.4839
6	3.1793	5.7287	7.0024
unit having min. d.f.	3	3	3
estimated deviation	1.4000	1.4000	1.4000

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [1.0000 1.0000 1.4000 4.0000 2.2000 1.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	0.0056	0.0275	0.0848
2	0.0088	0.0294	0.0620
3	0.0001	0.0002	0.0004
4	-0.0000	0.	0.0000
5	0.0016	0.0065	0.0118
6	0.0005	0.0017	0.0034
unit having min. d.f.	4	4	4
estimated deviation	2.0000	2.0000	2.0000

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [1.0000 1.0000 1.4000 2.0000 4.4000 1.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	0.0020	0.0307	0.0780
2	0.0076	0.0528	0.0878
3	0.0000	0.0009	0.0019
4	0.0001	0.0011	0.0021
5	0.0000	0.	0.
6	0.0000	0.0002	0.0004
unit having min. d.f.	3,5,6	5	5
estimated deviation		2.2000	2.2000

normal = [1.0000 1.0000 1.4000 2.0000 2.2000 1.0000]
faulty = [1.0000 1.0000 1.4000 2.0000 2.2000 2.0000]

candidate	after 2-samples	after 3-samples	after 4-samples
1	2.7894	12.7175	37.8569
2	1.2632	4.9581	12.0869
3	0.4131	1.0485	1.5920
4	0.0047	0.0002	0.0108
5	0.8704	1.9479	2.8840
6	0.0000	0.0000	0.0000
unit having min. d.f.	6	6	6
estimated deviation	1.0000	1.0000	1.0000

Table 3. Fault Diagnosis of PID Controller