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A MODEL-BASED FAULT DIAGNOSIS IN UNCERTAIN SYSTEMS

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ABSTRACT

This paper deals with the fault diagnosis problem in uncertain linear systems having undermodelling, linearization errors and noise inputs. The new approach proposed in this paper uses an appropriate test variable and the difference between system parameters which are estimated by the least squares method to locate the fault. The singular value decomposion is used to decouple the correlation between the estimated system parameters and to observe the trend of parameter changes. Some simulations applied to aircraft engines show good allocation of the fault even though the system model has significant uncertainties. The feature of the approach is to diagnose the uncertain system through simple parameter operations and not to need complex calculations in the diagnosis procedure as compared with other methods.

I. INTRODUCTION

In recent years, as the complexity and scale of the systems have increased, some considerable attentions have been directed to the fault detection and diagnosis in order to improve the system performance and reliability. The problem of fault detection and diagnosis occurs when one wishes to determine, by processing available measurements, whether or not a system has been subjected to change, and if so, what is the most likely nature of that change. The problem has major economic and technical importance and, for this reason, has been the subject of extensive study [1,2].

A fault may be defined as an abnormal change in the characteristics of a system which gives rise to undesirable performance [6]. The purpose of the fault diagnosis of dynamic systems is to locate the source of the fault. There is a vast body of existing literature on fault diagnosis, for example, [1] to [5] and many methods based on modelling and estimation have been proposed such as the parity space approach, the innovation-based approach, the fault detection filter approach and the parameter estimation approach [1].

In most existing methods, the models used are assumed to accurately describe the relationships between the various measurements with the primary source of uncertainty being the measurement noise. This assumption leads to the conclusion that arbitrary small faults can be detected provided the system is observed for long enough [4]. In addition, the assumption of perfect modelling impacts upon the form of the individual fault signatures used to distinguish between faulty and nonfaulty conditions.

In practical situation, however, all mathmatical models are only approximate descriptions of real systems and the major sources of uncertainties are undermodelling effects and linearization errors besides the measurement noise. Thus any method which has been developed based on the hypothesis of exact modelling may give misleading results when applied to practical systems having significant undermodelling or linearization errors. There is therefore a strong motivation to develop the robust fault detection and diagnosis method with respect to modelling errors in uncertain systems [1,7].

In recent papers, several approaches to improve the robustness of the fault detection schemes have been suggested, for example, [7] to [10]. On the other hand, there are only a few literatures on the robust fault diagnosis problem [1,11].

In the current paper, we propose a robust fault diagnosis scheme based on the fault detection method suggested by Kwon and Goodwin [7,12], which accounts for undermodelling effects, linearization errors and the measurement noise and shows a marked improvement over those obtained with traditional methods. The key idea in the current paper is to adopt a nominal model with its denominator fixed, which is believed to increase the parameter sensitivity with respect to the fault and enables us to take the test variable for fault detection also as an index for fault diagnosis.

The organization of this paper is as follows. In

Section II, the problem formulation is given. The fault diagnosis scheme is presented in Section III. Section IV presents simulation results applied to aircraft engines. Conclusions are given in Section V.

II. PROBLEM FORMULATION

The model uncertainty is characterized in a number of ways. For the transfer function model, uncertainty in the real system may be modelled as the additive unstructured form [13]. The major sources of uncertainty are undermodelling, linearization errors, and measurement noise. Thus the model uncertainty in linear discrete-time systems can be represented by the following system description based on the Taylor series expansion of the input-output relationship :

$$y(k)= G(q^{-1})u(k)+G_{\Delta}(q^{-1})u(k)+G_n(q^{-1})u^2(k)\mathrm{sign}(u(k)) + v(k) \qquad (2.1)$$

where q^{-1} denotes the delay operator, G is the nominal model, G_{Δ} and G_n denote the mismatched model due to undermodelling and linearization error respectively, v is the measurement noise, and sign(·) is the sign function. This system description is depicted by Fig. 2.1.

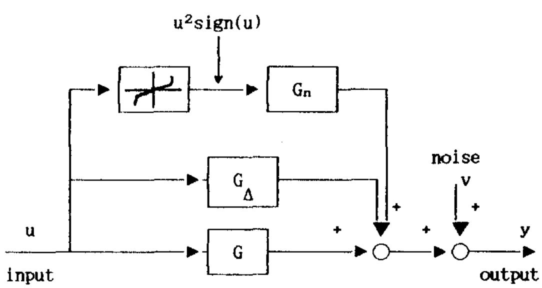

Fig.2.1 System description

The expansion given in Eq. (2.1) can be justified either in terms of linearization about an operating point or via the Hammerstein model description of nonlinear systems in which the nonlinearity is represented as a static element on the input side [14].

It is assumed that G, G_{Δ} and G_n are stable and causal and that the measurement noise v is a zero mean white noise with variance σ_v^2 . The nominal model is taken to be

$$G(z^{-1},\theta) = \frac{B(z^{-1},\theta,n_B)}{F(z^{-1},n_F)} \qquad (2.2)$$

where $F(z^{-1},n_F)$ is a predetermined denominator and

$$B(z^{-1},\theta,n_B) \equiv b_1z^{-1} + b_2z^{-2} + \cdot\cdot\cdot\cdot + b_{n_B}z^{-n_B}$$

$$F(z^{-1},n_F) \equiv 1 + f_1z^{-1} + f_2z^{-2} + \cdot\cdot\cdot + f_{n_B}z^{-n_F}$$

$$\theta \equiv [\, b_1\ b_2\ ,\ \cdot\cdot\cdot\cdot\ ,\ b_{n_B}]^T.$$

The denominator $F(z^{-1},n_F)$ can be determined from *a priori* information about the system, e.g., appropriate values of dominant poles or by some prior experiments for the parameter estimation of the system. Note that any linear stable system can be always approximated by the nominal model by adjusting the orders n_B and n_F. Basically, errors in the denominator polynomial are corrected by adjustments to the numerator polynomial.

Using the system description (2.1), the system output has the following form

$$y(k) = B(q^{-1},\theta,n_B)u_F(k) + \eta(k) \qquad (2.3)$$

where

$$u_F(k) = \frac{1}{F(q^{-1},n_B)}u(k)$$

$$\eta(k) \equiv G_{\Delta}(q^{-1})u(k) + G_n(q^{-1})u^2(k)\mathrm{sign}(u(k)) + v(k).$$

Eq.(2.3) can be represented in standard linear regression form as

$$y(k) = \phi^T(k)\theta + \eta(k) \qquad (2.4)$$

where

$$\phi(k) \equiv [\ u_F(k-1)\ u_F(k-2),\cdot\cdot\cdot,\ u_F(k-n_B)\ ]^T. \quad (2.5)$$

We define the parameter estimate using least squares as

$$\hat{\theta} \equiv \arg\min\{\frac{1}{N}\sum_{k=1}^{N}[y(k)-B(q^{-1},\theta,n_B)u_F(k)]^2\} \qquad (2.6)$$

where N is the number of data available. Note that Eq.(2.6) corresponds to an output error minimization problem. However, the ordinary least squares algorithm can be used to solve this problem due to the special form of Eq.(2.2), which is one of the advantages of the representation (2.2).

The least squares method then gives the estimated parameters as

$$\hat{\theta} = [\Phi^T\Phi]^{-1}\Phi^TY \qquad (2.7)$$

where

$$\Phi \equiv [\phi(1)\ \phi(2),\cdot\cdot\cdot\cdot,\ \phi(N)]^T \qquad (2.8)$$

$$Y \equiv [y(1)\ y(2),\cdot\cdot\cdot\cdot,\ y(N)]^T.$$

From Eqs.(2.4) and (2.7) we can also derive the following expression for the estimation error :

$$\tilde{\theta} \equiv \hat{\theta} - \theta = [\Phi^T\Phi]^{-1}\Phi^TS \qquad (2.9)$$

where

$$S \equiv [\eta(1)\ \eta(2),\cdot\cdot\cdot\cdot,\ \eta(N)].$$

Denoting the impulse responses of G_{Δ} and G_n as $\{h(\cdot)\}$ and $\{h_n(\cdot)\}$, $\eta(k)$ can be expressed as

$$\eta(k) = \sum_{i=0}^{k}h(i)u(k-i)+\sum_{i=0}^{k}h_n(i)u^2(k-i)\mathrm{sign}(u(k-i))+v(k)$$

assuming that $u(k)=0$ for $k\leq 0$ and $h(k)=h_n(k)=0$ for $k<0$. Then we obtain the following relationship :

$$S = \Psi H + \Psi_n H_n + V \tag{2.10}$$

where

$$\Psi \equiv \begin{bmatrix} u(1) & 0 & & \\ u(2) & u(1) & & 0 \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ u(N) & u(N-1) & \cdots & u(1) \end{bmatrix} \tag{2.11}$$

$$H \equiv [h(0)\ h(1)\ \cdots\ h(N-1)]^T$$

$$V \equiv [v(1)\ v(2)\ \cdots\ v(N)]^T$$

$$\Psi_n \equiv \begin{bmatrix} u^2(1)sign(u(1)) & 0 & & \\ u^2(2)sign(u(2)) & u^2(1)sign(u(1)) & & 0 \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ u^2(N)sign(u(N)) & \cdot & \cdot & u^2(1)sign(u(1)) \end{bmatrix} \tag{2.12}$$

$$H_n \equiv [h_n(0)\ h_n(1)\ \cdots\ h_n(N-1)].$$

Note that $\{h(\cdot)\}$ and $\{h_n(\cdot)\}$ are taken as stochastic processes here based on the stochastic embedding principle [18]. Given *a priori* information about the second order statistics of h, h_n and v, we can then evaluate the expected value of the estimation error, $E\{\tilde{\theta}\tilde{\theta}^T\}$, which will be the basis of the fault diagnosis method to be described later.

III. FAULT DIAGNOSIS MATHOD

In the fault diagnosis procedure, we shall use the test variable based on the covariance of the estimation error between two experiments. Thus in the sequel we assume that there are two sets of data I_n and I_f, where I_n corresponds to the nonfaulty data and I_f corresponds to the suspected faulty data. The estimated parameter by Eq.(2.7) may take different values in each experiment :

$$\hat{\theta} = \begin{cases} \hat{\theta}_n, & \text{for data set } I_n \\ \hat{\theta}_f, & \text{for data set } I_f \end{cases} \tag{3.1}$$

Given two sets of estimated parameters as in Eq. (3.1) for the uncertain system of Fig. 2.1, we can formulate an appropriate test variable as follows [7] :

$$T = [\hat{\theta}_n-\hat{\theta}_f]^T C^{-1}[\hat{\theta}_n-\hat{\theta}_f] \tag{3.2}$$

$$C = Cov(\theta_n-\theta_f) = E\{[\hat{\theta}_n-\hat{\theta}_f][\hat{\theta}_n-\hat{\theta}_f]^T\}$$

$$= [Q_n-Q_f]R[Q_n-Q_f]^T + [Q_{nn}-Q_{nf}]R_n[Q_{nn}-Q_{nf}]^T + [P_n+P_f]\sigma_v^2 \tag{3.3}$$

where

$$Q_i \equiv P_i\Phi_i^T\Psi_i, \quad Q_{ni} = P_i\Phi_i^T\Psi_{ni}$$

$$P_i \equiv [\Phi_i^T\Phi_i]^{-1}, \ i = n,f$$

$$R \equiv E\{HH^T\}, \quad R_n \equiv E\{H_nH_n^T\}$$

where E denotes the expectation with respect to the underlying probability space and Φ, Ψ and Ψ_n are as in (2.8), (2.11) and (2.12).

The first and second terms on the right side of Eq. (3.3) account for the effects of undermodelling, nonlinearities and the difference in input signals for the two experiments. Note that if there is neither undermodelling nor nonlinearity or if the inputs are identical, these terms vanish. The third term on the right side of Eq. (3.3) corresponds to the measurement noise.

The stochastic assumptions corresponding to $\{h(\cdot)\}$ and $\{h_n(\cdot)\}$ would be to assume

$$E\{h(k)h(j)\} = r(k)\delta_{ij}$$

$$E\{h_n(k)h_n(j)\} = r_n(k)\delta_{ij}$$

where

$$r(k) = \sigma_o^2 e^{-\beta k}, \tag{3.4}$$

$$r_n(k) = \sigma_n^2 e^{-\beta_n k}, \quad k = 0, 1, \cdots.$$

σ_o^2, σ_n^2, β and β_n can be estimated from a sequence of prior experiments on nonfaulty systems. The above assumptions give

$$R = \sigma_o^2 \ diag[1\ e^{-\beta}\ \cdots\ e^{-\beta(N-1)}]$$

$$R_n = \sigma_n^2 \ diag[1\ e^{-\beta_n}\ \cdots\ e^{-\beta_n(N-1)}].$$

It is possible to categorize several faults by the test variable T of Eq. (3.2) though it is used as the index to detect a fault. With a fault detected, we can locate the fault by investigating the magnitude of the test variable and the change of estimated parameters. It is, however, difficult to diagnose the fault directly with the parameters if the parameters are correlated with each other. In that case, we make a singular value decomposition on C of Eq. (3.3) to decouple the correlation between parameters as follows :

$$C = U\Sigma U^T \tag{3.5}$$

where U is orthonormal and Σ is diagonal.

For the purpose of the fault diagnosis, if necessary, we can transform the parameter difference between the two experiments to a diagnostic variable defined as follows :

$$\tilde{\alpha} \equiv U^T(\hat{\theta}_n-\hat{\theta}_f). \tag{3.6}$$

Note that the covariance of $\tilde{\alpha}$ is the diagonal matrix Σ as shown below

$$E\{\tilde{\alpha}\tilde{\alpha}^T\} = E\{U^T(\hat{\theta}_n-\hat{\theta}_f)(\hat{\theta}_n-\hat{\theta}_f)^T U\}$$
$$= U^T C U = U^T[U\Sigma U^T]U = \Sigma$$

and tha $\tilde{\alpha}$ has no correlation between each other component, i.e.,

$$E\{\tilde{\alpha}_i \tilde{\alpha}_j\} = \sigma_i \delta_{ij}$$

where

$$\tilde{\alpha} = [\tilde{\alpha}_1 \; \tilde{\alpha}_2 \; \cdots \; \tilde{\alpha}_{n_B}]^T$$
$$\Sigma = \mathrm{diag}[\sigma_1 \; \sigma_2 \; \cdots \; \sigma_{n_B}]^T.$$

Hence the parameter $\tilde{\alpha}$ of Eq. (3.6) can be utilized as the signature for the fault diagnosis.

IV. SIMULATION

In order to illustrate the feature of the proposed method, we present some simulations for a military turbofan gas turbine engine (F404-GE-400). Exact models of aircraft engines are highly nonlinear [15] and thus simplified linearized models are usually employed [16,17]. For example, taking the engine fuel flow W_F as the input and the fan spool speed N_L as the output, an appropriate linearized nominal model is given as follows :

Table 4.1 Data Sets from Aircraft Engine

Name	Fault type	Remark
CLF6	No-fault	Different operating point
CLF61	No-fault	
BLC	2% Compressor bleed	The same operating point as that of CLF6
FEF	-2% Change in fan efficiency	
CEF	-2% Change in compressor efficiency	
BEF	-2% Change in burner efficiency	
HTEF	-2% Change in high press. turb. eff.	
LTEF	-2% Change in low press. turb. eff.	
PEX	100 HP compressor spool speed power extract	
A8D	2% Decrease in final nozzle area	
A8I	2% Increase in final nozzle area	
BEF1	-2% Change in burner efficiency	Different operating point from CLF6
BEF2	2% Change in burner efficiency	
BEF3	5% Change in burner efficiency	

$$\Delta N_L(t) = \frac{b_{c1}p + b_{0c}q}{p^2 + f_{1c}p + f_{0c}} \Delta W_F(t) \tag{4.1}$$

where p denotes the differential operator.

Taking noise and linearization errors into consideration, we can describe the underlying system by the following discretized model similar to Eq. (2.1) :

$$\Delta N_L(k) = G(q^{-1})\Delta W_F(k) + G_n(q^{-1})[\Delta W_F(k)]^2 + v(k) \tag{4.2}$$

where

$$G(q^{-1}) = \frac{B(q^{-1},\theta,n_B)}{F(q^{-1},n_F)} = \frac{b_1q^{-1} + b_2q^{-2}}{1 + f_1q^{-1} + f_2q^{-2}}$$

Noise-free nonfaulty data sets (CLF6 and CLF61) and faulty data sets with various small changes were chosen for the study as shown in Table 4.1. We have applied the fault diagnosis method suggested in Section III to this problem, as shown in Table 4.2. The following constants were chosen : Sampling time T_s = 0.02, N = 350, n_B = 2, $\sigma_v^2 = 0.15^2$ and the input ΔW_F was assumed to be corrupted by white noise with variance $\sigma_u^2 = 0.003^2$. The fixed denominator was taken by prior experiments as $f_1 = -1.8238$ and $f_2 = 0.8294$ and the values of β_n and σ_n^2 as $\beta_n = 0.0837$ and $\sigma_n^2 = 0.0818$. Typical data sets used in this simulation are shown in Fig. 4.1.

Table 4.2 Simulation Cases in Aircraft Engine Fault Diagnosis

Case No.	Experiment n	Experiment f
NF1	CLF6	CLF6
NF2	CLF6	CLF61
F3	CLF6	BLC
F4	CLF6	FEF
F5	CLF6	CEF
F6	CLF6	BEF
F7	CLF6	HTEF
F8	CLF6	LTEF
F9	CLF6	PEX
F10	CLF6	A8D
F11	CLF6	A8I
F12	CLF6	BEF1
F13	CLF6	BEF2
F14	CLF6	BEF3

The simulation results are shown in Table 4.3 and Table 4.4. Table 4.3 shows values of the test variable and the diagnostic variable in each case of Table 4.2. The fault types were classified by the test variable T at first and then the more specific classification of the faults was processed by the decomposed estimation error $\tilde{\alpha}$. Note that although the four fault groups have two undistinct elements, Group 1 (BEF1 and BEF2) has the faults from the same source, burner efficiency and Group 5 (BLC and CEF) also from another same source, compressor.

Another diagnosis method based on the output error estimation requiring complex calculations was applied to the same problem by Smed et. al [17] and showed the diagnostic performance to classify faults into 5 groups. Simulation results here show that the proposed

Table 4.3 Signatures for Fault Diagnosis.

Case	T×10	$\tilde{\alpha}_1 \times 10^{-2}$	$\tilde{\alpha}_2 \times 10^{-2}$
NF1	0.20± 0.30	-0.46±9.39	-0.01±0.05
NF2	0.18± 0.13	0.55±9.02	0.10±0.07
F3	460.00±20.00	134.17±9.38	-2.27±0.05
F4	220.00±10.00	82.85±9.52	-1.56±0.05
F5	480.00±20.00	129.41±9.52	-2.33±0.05
F6	140.00± 8.00	53.65±9.67	-1.27±0.04
F7	620.00±20.00	144.21±9.20	-2.64±0.05
F8	150.00±10.00	71.12±9.20	-1.28±0.05
F9	90.00±10.00	64.29±9.21	-0.97±0.05
F10	210.00±10.00	78.44±9.21	-1.53±0.05
F11	200.00±10.00	-71.47±9.23	1.50±0.05
F12	12.60± 1.10	52.15±9.25	-1.16±0.05
F13	17.10± 1.40	53.60±9.22	-1.20±0.05
F14	26.00± 1.60	120.39±9.38	-2.62±0.04

method has such a good performance that it isolates faults into 9 groups occurred in an aircraft engine which is known as a highly nonlinear system having significant model uncertainties.

Note that the proposed method is based on the ordinary least squares and that its computational burden is much less than that of [17] based on the output error method. In addition, note that there is no false alarm in the fault detection using the method proposed here.

Table 4.4 Classification of Faults

	by T	by $\tilde{\alpha}$	Group No.
Non-faulty Groups	CLF6 CLF61	CLF6 CLF61	0
Faulty Groups	BEF1 BEF2 BEF3	BEF1 BEF2	1
		BEF3	2
	FEF A8D A8I	FEF A8D	3
		A8I	4
	BLC CEF	BLC CEF	5
	BEF LTEF	BEF LTEF	6
	HTEF	HTEF	7
	PEX	PEX	8

V. CONCLUSIONS

A fault diagnosis method for uncertain systems having undermodelling, linearization errors and noise has been proposed. The key feature of this method is that it accounts for the effects of model mismatch and linearization errors besides noise and that it requires simple parameter operations. Some simulations applied to aircraft engines show that the proposed method works well and outperform existing methods. This improvement is a consequence of the fact that the fault diagnosis method proposed here explicitly accounts for the effects of model uncertainties.

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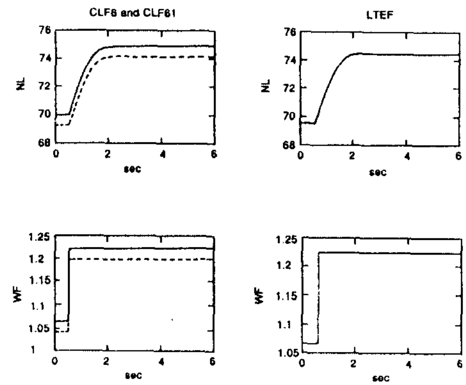

Fig. 4.1 Nonfaulty Data Sets and Faulty Data Set in Aircraft Engine.