Regression Analysis and Recursive Identification of the Regression Model With Unknown Operational Parameter Variables, and Its Application to Sequential Design

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ABSTRACT

This paper offers the theory and method for regression analysis of the regression model with operational parameter variables based on the fundamentals of mathematical statistics. Regression coefficients are usually constants related to the problem of regression analysis. This paper considers that regression coefficients are not constants but the functions of some operational parameter variables. This is a kind of method of two-step fitting regression model. The second part of this paper considers the experimental step numbers as recursive variables, the recursive identification with unknown operational parameter variables, which includes two recursive variables, is deduced. Then the optimization and the recursive identification are combined to obtain the sequential experiment optimum design with operational parameter variables. This paper also offers a fast recursive algorithm for a large number of sequential experiments.

1. INTRODUCTION

To conserve energy and materials, people recently study the techniques for augmenting heat transfer in different heat transfer equipments. One kind of these equipment is spirally corrugated tubes, which is shown in Fig.1, a section of the four-start corrugated tubes is shown in Fig.2 [8]. The structure design of these tubes is a important scientific research subject. Though many studies have been conducted on these tubes, a lack of systematic study on the prediction of optimum tube geometrical structure and the development of unified correlation which can be examined for any type of these tubes.

For example, in the study of heat transfer characteristics on single- and multistart spirally corrugated tubes, the friction factor and heat transfer were considered.

Ganeshan, S. (1981) got the result of friction and heat transfer correlation as follows[6], which was from an experimental investigation of seven spirally corrugated tubes,

friction correlation:

$$R(h^+)[h/(p-w)]^{0.52}(N)^{0.24}(n')^{2.5} = 0.273\ln(h^+) + 0.127$$

heat transfer correlation:

$$\log[G(h^+, Pr)Pr^{-0.55}] = 2.576 - 1.707\log h^+ + 0.497(\log h^+)^2 - 0.0103(\log h^+)^3$$

Sethumadhavan, R. (1986) obtained the result of friction and heat transfer correlation in the following[7], which was from an experimental investigation of five spirally corrugated tubes,

friction correlation:

$$R(h^+)(h^2/pD_{(eq)})^{0.33} = 0.40(h^+)^{0.164} \qquad 3 < h^+ < 200$$

heat transfer correlation:

$$G(h^+, Pr)(Pr^{-0.55}) = 8.6(h^+)^{0.13} \qquad 25 < h^+ < 180$$

The above models were fitted in their own experimental area and the cost factor, besides, the operating conditions at the fluid (e.g. Reynolds number, etc.), which are defined as the operational parameter variables, were not taken into consideration, so it might influence the final selection of the tubes for use in heat exchangers.

This paper proposes to solve the above optimal design problem in another aspect. Firstly, as concerns as the spirally corrugated tubes, without losing generality, we will consider the relationship between the structure parameters[5], that is, $\alpha = \tan^{-1}(\pi D/p), H = p/N$, so the structure parameters of tubes can be concluded as concisely two independent dimensionless structure parameters $x_1 = h/D$ and $x_2 = H/h$.

The friction factor f and the heat transfer characteristics factor St (Stantom number) are considered as the quantitative indexes Y, which is the function of the structure parameters $\mathbf{X} = [x_1, x_2]$ and the operational parameter variables $\mathbf{Z} = [z_1] = Re$, respectively, that is,

$$\begin{aligned} St &= \mathcal{G}(h/D, H/h, Re) \\ f &= \mathcal{G}(h/D, H/h, Re) \end{aligned} \qquad (1.1)$$

This can be written as

$$Y = \mathcal{G}(x_1, x_2, z). \qquad (1.2)$$

The objective function Q is the function of model Y, $Q = J(Y)$. For example, Q is chosen as $Q = f/(Pr^3 St^3)$ in the optimal design of intensified heat transfer.

Nomenclature

h – height of corrugation (mm)
D – tube inside diameter (mm)
N – number of starts
p – pitch (mm)
H – effective pitch(mm),(p/N)
α – helix angle (degree)
n' – flow behavior index
f – friction factor
Pr – Prandtl number
St – Stantom number
h^+ – roughness Reynolds number $[(h/D_{eq})Re\sqrt{f}/2]$
Re – Reynolds number
$R(h^+)$ – momentum transfer roughness function
$G(h^+, Pr)$ – heat transfer roughness function
D_{eq} – equivalent diameter (mm), $(D-h)$

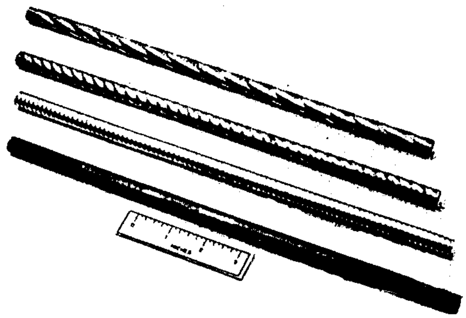

Fig. 1 Typical spirally corrugated tubes

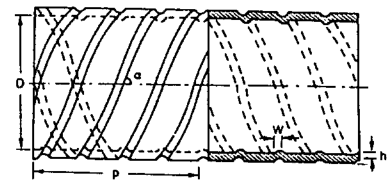

FIG. 2. Characteristic parameters of a spirally corrugated tube. h, height of corrugation, w, width of corrugation, p, pitch of corrugation, D, diameter of tube, α, helix angle.

Because the property of the object we want to study is not clearly known, and it is difficult to certain the physical characteristics model. So we can think of the object as a "black box" to build its regression model on the basis of mathematical statistics. The regression coefficients of the models mentioned above are not constants but the functions of the operational parameter variables. This is essentially different from present theories and methods of sequential experimental design and regression analysis. This will be discussed in Section 2.

Secondly, the model mentioned above is got in the certain size of the sample, that is the area of experiment, and it has the random error, so in order to search the optimal design parameters, it is necessary to effectively enlarge the size of the sample to modify the models with recursive identification, and to attain the goal of optimization. This is the concept of optimal sequential design originated from this paper. In addition, the recursive identification here includes two recursive variables, this paper offers a new kind of recursive identification. It is also obtained by two steps. It will be stated in Section 3.

In Section 4, we also give a fast recursive algorithm for a large number of sequential experiments.

2. REGRESSION ANALYSIS OF QUADRATIC MODEL WITH THE OPERATIONAL PARAMETER VARIABLES

2.1 Description of general mathematical form

Let $X=[x_1,\ldots,x_n]^*$ be n-dimensional vector denoting n structural variables, $Z=[z_1,\ldots,z_m]^*$ be m-dimensional vector denoting m operational parameter variables, and $\mathbf{Y}$ (quantitative index) be the quadratic form with the operational parameter variables, that is,

$$\mathbf{Y} = \beta_0(z_1,\ldots,z_m) + \sum_{i=1}^{n}\beta_i(z_1,\ldots,z_m)x_i + \sum_{i=1}^{n}\beta_{ii}(z_1,\ldots,z_m)x_i^2 + \sum_{i<j}^{n}\beta_{ij}(z_1,\ldots,z_m)x_ix_j + e. \tag{2.1.1}$$

where e is the random error.

To fit the equation (2.1.1), we do the following experiments, the experiment scheme is shown in the Table. 1 (See the Appendix).

Hence, we obtain,

$$\begin{aligned} y_u(v) &= \beta_0(v) + \sum_{i=1}^{n}\beta_i(v)x_{iu} + \sum_{i=1}^{n}\beta_{ii}(v)x_{iu}^2 \\ &\quad + \sum_{i<j}^{n}\beta_{ij}(v)x_{iu}x_{ju} + e_u(v) \\ &\quad u=1,\ldots,N; v=1,\ldots,M. \end{aligned} \tag{2.1.2}$$

Where

$$\begin{aligned} &\beta_0(v)\triangleq\beta_0(z_{1v},\ldots,z_{mv}), && \beta_i(v)\triangleq\beta_i(z_{1v},\ldots,z_{mv}), \\ &\beta_{ii}(v)\triangleq\beta_{ii}(z_{1v},\ldots,z_{mv}), && \beta_{ij}(v)\triangleq\beta_{ij}(z_{1v},\ldots,z_{mv}), \\ &e_u(v)\triangleq e_u(z_{1v},\ldots,z_{mv}), && y_u(v)\triangleq y_u(v,\ldots,v). \end{aligned}$$

The random errors $\{e_u(v), u=1,\ldots,N; v=1,\ldots,M\}$ satisfy the four assumptions: (1). independence; (2). unbiasedness, $E[e_u(v)]=0$; (3). equality in variance, $Var[e_u(v)]=\sigma^2$; (4). normality, $N(0,\sigma^2)$.

The model is fitted by two steps.

2.2 First step fitting models – the quantitative variable versus the structural variables

2.2.1. The least square identification

For any fixed v, that is any given set of operational parameters, after N experiments, the least square estimators $\hat{\overline{\mathbf{B}}}_N(v)$ of the coefficients $\overline{\mathbf{B}}_N(v)$ in the equation (2.1.2) is the following form,

$$\hat{\overline{\mathbf{B}}}_N(v) = (\overline{\mathbf{X}}_N^* \overline{\mathbf{X}}_N)^{-1} \overline{\mathbf{X}}_N^* \overline{\mathbf{Y}}_N(v). \qquad v = 1, \cdots, M. \tag{2.2.1}$$

where,

$$\overline{\mathbf{Y}}_N(v) = [y_1(v), \ldots, y_N(v)]^*$$

$$\hat{\overline{\mathbf{B}}}_N(v) = [b_{o,N}(v), b_{1,N}(v), \ldots, b_{n,N}(v), b_{11,N}(v), \ldots, b_{nn,N}(v), b_{12,N}(v), \ldots, b_{(n-1,n),N}(v)]^*$$

$$\overline{\mathbf{B}}_N(v) = [\beta_0(v), \beta_1(v), \ldots, \beta_n(v), \beta_{11}(v), \ldots, \beta_{nn}(v), \beta_{12}(v), \ldots, \beta_{n-1,n}(v)]^*$$

When $v = 1, \cdots, M$, we have M sets of $\hat{\overline{\mathbf{B}}}_N(v)$. The equation got from least square estimators $\hat{\overline{\mathbf{B}}}_N(v)$ is called initial regression equation, which can be written as the form of matrix,

$$\hat{\overline{\mathbf{Y}}}(v) = \overline{\mathbf{X}}_N \hat{\overline{\mathbf{B}}}_N(v) \tag{2.2.2}$$

where

$$\hat{\overline{\mathbf{Y}}}(v) = [\hat{Y}_1(v), \ldots, \hat{Y}_N(v)]^*$$

the symbol " $*$ " represents transpose.

2.2.2 Variance analysis

[THEOREM] For any fixed v, the variance sum of squares of equation (2.2.2) can be written as follows,

$$\sum_{u=1}^{N}[y_u(v) - \overline{y}(v)]^2 = \sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v)]^2 + \sum_{u=1}^{N}[\hat{Y}_u(v) - \overline{y}(v)]^2 \tag{2.2.3}$$

Where,

$$\overline{y}(v) = \frac{1}{N}\sum_{u=1}^{N} y_u(v)$$

$y_u(v)$ and $\hat{Y}_u(v)$ are represented in the equation (2.1.2) and (2.2.2), respectively.

The proof is omitted here.

We may give the analysis of variance as usual in Table 2(See the Appendix). Where the F-ratio is as follows,

$$\mathbf{F}_{N_d}^{N_{d1}} = \frac{N_d}{N_{d1}} \frac{\sum_{u=1}^{N}[\hat{Y}_u(v) - \overline{y}(v)]^2}{\sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v)]^2}. \tag{2.2.4}$$

$$N_d = N - [2n + n(n-1)/2] - 1, \qquad N_{d1} = 2n + n(n-1)/2$$

2.2.3 Regression analysis

Now we make use of the table of variance analysis to get the significance testing of the model (2.2.2).

Since the estimators of regression coefficients $b_0(v), b_i(v), b_{ii}(v), b_{ij}(v)$, $(i, j = 1, \cdots, n, i < j)$ obey the normal distributions $N(\beta_0(v), c_{00}\sigma^2)$, $N(\beta_i(v), c_{ii}\sigma^2)$, $N(\beta_{ii}(v), c_{\underline{iiii}}\sigma^2)$, $N(\beta_{ij}(v), c_{\underline{ijij}}\sigma^2)$, respectively, and $S_1(v)/\sigma^2$ follows $\mathcal{X}^2$ - distribution with the degree of freedom N_d, where $S_1(v) = \sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v)]^2$, therefore

$$\frac{(b_i(v) - \beta_i(v))\Big/\sqrt{c_{\underline{ii}}\sigma^2}}{(1/\sqrt{N_d})\sqrt{S_1(v)/\sigma^2}} = t_{N_d}$$

follows "Student" distribution with $d.f.N_d$, and we can test the following hypotheses: $\beta_i = 0, i = 1, \cdots, n$.

We can also get the same conclusions about $\beta_0(v), \beta_{ii}(v), \beta_{ij}(v)$, if we change the note (i) to (0), (ii) or (ij).

2.3 Second step fitting models – the regression coefficients versus the operational parameter variables

The coefficients $\beta_0(z_1, \ldots, z_m)$, $\beta_i(z_1, \ldots, z_m)$, $\beta_{ii}(z_1, \ldots, z_m)$, $\beta_{ij}(z_1, \ldots, z_m)$, with the operational parameter variables $(z_1, \ldots, z_m)$ can be expended with Talor series. We use a quadratic form to denote the relationship between $\beta_i(z_1, \ldots, z_m)$ and $(z_1, \ldots, z_m)$ as follows,

$$\beta_i = \alpha_0^{(i)} + \sum_{e=1}^{m} \alpha_e^{(i)} z_e + \sum_{e=1}^{m} \alpha_{ee}^{(i)} z_e^2 + \sum_{e<e'}^{m} \alpha_{ee'}^{(i)} z_e z_{e'} + e^{(i)}. \tag{2.3.1}$$

$$(i = 1, \cdots, n)$$

$$b_i(v) = \alpha_0^{(i)} + \sum_{e=1}^{m} \alpha_e^{(i)} z_{ev} + \sum_{e=1}^{m} \alpha_{ee}^{(i)} z_{ev}^2 + \sum_{e<e'}^{m} \alpha_{ee'}^{(i)} z_{ev} z_{e'v} + e_v^{(i)} \tag{2.3.2}$$

$$(v = 1, \cdots, M)$$

From the assumptions in section 2.1, $\{e_v^{(i)}, v = 1, \ldots, M, i = 1, \ldots, n\}$ satisfy the four conclusions: (1) independence; (2) unbiasedness: $E[e_v^{(i)}] = 0$; (3) equality in variance: $Var[e_v^{(i)}] = c_{\underline{ii}}\sigma^2$; (4) normality: $N(0, c_{\underline{ii}}\sigma^2)$.

The least square estimators $\hat{\overline{\mathbf{A}}}_{M,N}^{(i)}$ of the coefficients $\overline{\mathbf{A}}_{M,N}^{(i)}$ in the equation (2.3.2) can be obtained through the same procedures as section 2.2, so do the variance analysis and the regression analysis, that is,

$$\hat{\overline{\mathbf{A}}}_{M,N}^{(i)} = (\overline{\mathbf{Z}}_M^* \overline{\mathbf{Z}}_M)^{-1} \overline{\mathbf{Z}}_M^* \overline{\mathbf{b}}_{M,N}^{(i)}. \tag{2.3.3}$$

where,

$$\hat{\overline{\mathbf{A}}}_{M,N}^{(i)} = [a_0^{(i)}, a_1^{(i)}, \cdots, a_m^{(i)}, a_{11}^{(i)}, \cdots, a_{mm}^{(i)}, a_{12}^{(i)}, \cdots, a_{m-1,m}^{(i)}]^*$$

$$\overline{\mathbf{A}}_{M,N}^{(i)} = [\alpha_0^{(i)}, \alpha_1^{(i)}, \cdots, \alpha_m^{(i)}, \alpha_{11}^{(i)}, \cdots, \alpha_{mm}^{(i)}, \alpha_{12}^{(i)}, \cdots, \alpha_{m-1,m}^{(i)}]^*$$

$$\overline{\mathbf{b}}_{M,N}^{(i)} = (b_i(1), \cdots, b_i(M))^*$$

Now the regression model can be written as follows,

$$\hat{\beta}_i = a_0^{(i)} + \sum_{e=1}^{m} a_e^{(i)} z_e + \sum_{e=1}^{m} a_{ee}^{(i)} z_e^2 + \sum_{e<e'}^{m} a_{ee'}^{(i)} z_e z_{e'}. \tag{2.3.4}$$

$$(i = 1, \cdots, n)$$

We can also obtain the same conclusions about $\hat{\overline{\mathbf{A}}}_{M,N}^{(0)}$, $\hat{\overline{\mathbf{A}}}_{M,N}^{(ii)}$, $\hat{\overline{\mathbf{A}}}_{M,N}^{(ij)}$ if we change the superscript (i) to (0), (ii) or (ij).

2.4 Total regression model and total variance analysis

Now we yield the total regression equation:

$$\begin{aligned}\hat{Y} &= \hat{\beta}_o + \sum_{i=1}^{n}\hat{\beta}_i x_i + \sum_{i=1}^{n}\hat{\beta}_{ii}x_i^2 + \sum_{i<j}^{n}\hat{\beta}_{ij}x_ix_j \\ &= \Big(a_0^{(0)} + \sum_{e=1}^{m} a_e^{(0)}z_e + \sum_{e=1}^{m} a_{ee}^{(0)}z_e^2 + \sum_{e<e'}^{m} a_{ee'}^{(0)}z_ez_{e'}\Big)x_0 \\ &+ \sum_{i=1}^{n}\Big(a_0^{(i)} + \sum_{e=1}^{m} a_e^{(i)}z_e + \sum_{e=1}^{m} a_{ee}^{(i)}z_e^2 + \sum_{e<e'}^{m} a_{ee'}^{(i)}z_ez_{e'}\Big)x_i \\ &+ \sum_{i=1}^{n}\Big(a_0^{(ii)} + \sum_{e=1}^{m} a_e^{(ii)}z_e + \sum_{e=1}^{m} a_{ee}^{(ii)}z_e^2 + \sum_{e<e'}^{m} a_{ee'}^{(ii)}z_ez_{e'}\Big)x_i^2 \\ &+ \sum_{i<j}^{n}\Big(a_0^{(ij)} + \sum_{e=1}^{m} a_e^{(ij)}z_e + \sum_{e=1}^{m} a_{ee}^{(ij)}z_e^2 \\ &\quad + \sum_{e<e'}^{m} a_{ee'}^{(ij)}z_ez_{e'}\Big)x_ix_j\end{aligned} \tag{2.4.1}$$

From the equation (2.2.3), we consider the total variance sum of squares be,

$$\begin{aligned}&\sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \overline{y}(v)]^2 \\ &= \sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v) + \hat{Y}_u(v) - \hat{\hat{Y}}_u(v) + \hat{\hat{Y}}_u(v) - \overline{y}(v)]^2 \\ &= \sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v)]^2 + \sum_{v=1}^{M}\sum_{u=1}^{N}[\hat{Y}_u(v) - \hat{\hat{Y}}_u(v)]^2 \\ &\quad + \sum_{v=1}^{M}\sum_{u=1}^{N}[\hat{\hat{Y}}_u(v) - \overline{y}(v)]^2 \\ &+ 2\sum_{v=1}^{M}\sum_{u=1}^{N}[\hat{Y}_u(v) - \hat{\hat{Y}}_u(v)][\hat{\hat{Y}}_u(v) - \overline{y}(v)].\end{aligned} \tag{2.4.2}$$

Besides, the variance sum of squares in the first step fitting, that is, the equation (2.2.3) can be written as follows,

$$\begin{aligned}&\sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \overline{y}(v)]^2 = \\ &\sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \hat{Y}_u(v)]^2 + \sum_{v=1}^{M}\sum_{u=1}^{N}[\hat{Y}_u(v) - \overline{y}(v)]^2\end{aligned} \tag{2.4.3}$$

For comparison with two equations, the sum of 3rd and 4th terms in the right side of the equation (2.4.2) is less than the sum of 2nd term in the right side of the equation (2.4.3), and we desire that they are approaching nearly.

Hence we have,

$$\sum_{v=1}^{M}\sum_{u=1}^{N}[\hat{Y}_u(v) - \hat{\hat{Y}}_u(v)]^2 < \varepsilon(>0), \varepsilon \quad \text{is given.}$$

When having $\sum_{v=1}^{M}\sum_{u=1}^{N}[y_u(v) - \hat{\hat{Y}}_u(v)]^2$ as a measurement, we express the relative error form as follows,

$$\frac{\sum_{v=1}^{M}\sum_{u=1}^{N}\{[\hat{Y}_u(v) - \hat{\hat{Y}}_u(v)]\big/\hat{Y}_u(v)\}^2}{\sum_{v=1}^{M}\sum_{u=1}^{N}\{[y_u(v) - \hat{\hat{Y}}_u(v)]\big/y_u(v)\}^2} < \delta_1(>0),$$

$$\delta_1 \quad \text{is given} \tag{2.4.4}$$

If the equation (2.4.4) does not satisfy, then we should increase such order of the regression equation in the second step fitting procedure that the equation (2.4.4) satisfies.

Up to now, we have finished the fit of initial models through the pre-experiments and statistical analysis. The initial models determine the structure of the process models.

3. SEQUENTIAL DESIGN AND RECURSIVE IDENTIFICATION WITH THE OPERATIONAL PARAMETER VARIABLES

3.1. The resource of the problem

Suppose the objective function of optimization be Q. The relationship between Q and the structure parameters $\mathbf{X}$ and the operational parameters $\mathbf{Z}$ can be written as follows,

$$Q = f(\mathbf{X}, \mathbf{Z}). \tag{3.1.1}$$

We want to search such optimal treatment combination $(\tilde{\mathbf{X}}, \tilde{\mathbf{Z}})$ that Q arrives at the extremum. That is

$$f(\tilde{\mathbf{X}}, \tilde{\mathbf{Z}}) = \max_{\mathbf{X},\mathbf{Z}} f(\mathbf{X}, \mathbf{Z}). \tag{3.1.2}$$

The equation of (3.1.2) can be solved by a general non-linear multi-variables optimization, e.g. method of optimal steepest ascents.

Because the function $f(\mathbf{X}, \mathbf{Z})$ is got in the certain size of the sample , and it has the random error, we take the following measures to search the optimal design parameters.

3.2 The selection of the sequential experiment point

We can find a better treatment combination in the process of pre-experiment design, and take it as the start point $(\mathbf{X}^{(k)}, \mathbf{Z}^{(k)})$ for the optimization of the equation (3.1.2).

The new experiment point, that is the new experiment treatment combination, can be obtained,

$$\begin{aligned}\mathbf{X}^{(k+1)} &= \mathbf{X}^{(k)} + h^{(k)}\frac{\nabla f(\mathbf{X}^{(k)})}{\|\nabla f(\mathbf{X}^{(k)}, \mathbf{Z}^{(k)})\|} \\ \mathbf{Z}^{(k+1)} &= \mathbf{Z}^{(k)} + h^{(k)}\frac{\nabla f(\mathbf{Z}^{(k)})}{\|\nabla f(\mathbf{X}^{(k)}, \mathbf{Z}^{(k)})\|}.\end{aligned} \tag{3.2.1}$$

where $h^{(k)}$ is the optimal step in the process of one-dimensional search technology.

The new experiment observing values can be obtained on the point of $(\mathbf{X}^{(k+1)}, \mathbf{Z}^{(k+1)})$ to modify the model with recursive identification.

3.3 Recursive identification of regression model with operational parameter variables

The recursive model here differs from the general recursive identification algorithm, because it has the factors of the operational parameter variables $\mathbf{Z}$.

The first step fitting models are called $\mathbf{X}$-model, the second step fitting models $\mathbf{Z}$- model, and the total model $\mathbf{XZ}$-model here.

The total model (2.4.1) is obtained by N sets of $\{(x_{1u}, \cdots, x_{nu}), u = 1, \cdots, N\}$ and M sets of $\{(z_{1v}, \cdots, z_{mv}), v = 1, \cdots, M\}$. If we want to modify the **XZ**-model through the sequential experiment point $(x_1^*, \cdots, x_n^*, z_1^*, \cdots, z_m*)$, that is, (N+1)th $(x_{1,N+1}, \cdots, x_{n,N+1})$ and (M+1)th $(z_{1,M+1}, \cdots, z_{m,M+1})$. The experiment scheme is listed in Table 3(See the Appendix).

From the Table 3, we can see that there are two experimental step numbers to be increased in the sequential experiment design, they are N and M. So there are two recursive variables in the recursive identification of the total regression model shown in the equation (2.4.1). It increases both the number of regression models and the estimated parameters. This is different from the general recursive identification.

So we consider the total recursive identification yielded by two steps.

Firstly to find the recursive form of the **X**−model, suppose N be the variable parameter, the recursive formulae of the equation (2.2.1) is,

$$\begin{aligned}
&\hat{\overline{\mathbf{B}}}_{N+1}(v) = \hat{\overline{\mathbf{B}}}_N(v) + \overline{\mathcal{K}}_{N+1}[y_{N+1}(v) - \overline{\mathcal{P}}_{N+1}\hat{\overline{\mathbf{B}}}_N(v)] \\
&\overline{\mathcal{K}}_{N+1} = \overline{\mathcal{R}}_N \overline{\mathcal{P}}^*_{N+1}(1 + \overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_N\overline{\mathcal{P}}^*_{N+1})^{-1} \\
&\overline{\mathcal{R}}_{N+1} = \overline{\mathcal{R}}_N - \overline{\mathcal{R}}_N\overline{\mathcal{P}}^*_{N+1}(1 + \overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_N\overline{\mathcal{P}}^*_{N+1})^{-1}\overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_N \\
&v = 1, \cdots, M. \qquad (3.3.1)
\end{aligned}$$

Where,

$$\begin{aligned}
\hat{\overline{\mathbf{B}}}_{N+1}(v) = [&b_{0,N+1}(v), b_{1,N+1}(v), \ldots, b_{n,N+1}(v), \\
&b_{11,N+1}(v), \ldots, b_{nn,N+1}(v), \\
&b_{12,N+1}(v), \ldots, b_{(n-1,n),N+1}(v)]^*
\end{aligned}$$

$\overline{\mathcal{P}}_{N+1}$ is the last line of the matrix $\overline{\mathbf{X}}_{N+1}$, that is,

$$\begin{aligned}
\overline{\mathcal{P}}_{N+1} = [&1, x_{1,N+1}, \cdots, x_{n,N+1}, x^2_{1,N+1}, \cdots, x^2_{n,N+1}, \\
&x_{1,N+1}x_{2,N+1}, \cdots, x_{n-1,N+1}x_{n,N+1}] \\
\overline{\mathcal{R}}_{N+1} = (&\overline{\mathbf{X}}^*_{N+1}\overline{\mathbf{X}}_{N+1})^{-1}
\end{aligned}$$

For $v = M + 1$, the number of **X**-model is increased, that is,

$$\begin{aligned}
y_u(M+1) = \beta_{0,N+1}(M+1) &+ \sum_{i=1}^{n} \beta_{i,N+1}(M+1)x_i \\
&+ \sum_{i=1}^{n} \beta_{ii,N+1}(M+1)x_i^2 \\
&+ \sum_{i<j}^{n} \beta_{ij,N+1}(M+1)x_ix_j + e_u(M+1). \\
u = 1, \cdots, N, N+1. &\qquad (3.3.2)
\end{aligned}$$

The least square estimators $\hat{\overline{\mathbf{B}}}_{N+1}(M+1)$ of the $\overline{\mathbf{B}}_{N+1}(M+1)$ is,

$$\begin{aligned}
&\hat{\overline{\mathbf{B}}}_{N+1}(M+1) \\
&= (\overline{\mathbf{X}}^*_{N+1}\overline{\mathbf{X}}_{N+1})^{-1}\overline{\mathbf{X}}^*_{N+1}\overline{\mathbf{Y}}_{N+1}(M+1). \qquad (3.3.3)
\end{aligned}$$

where,

$$\begin{aligned}
&\overline{\mathbf{B}}_{N+1}(M+1) = [\beta_{0,N+1}(M+1), \beta_{1,N+1}(M+1), \ldots, \\
&\beta_{n,N+1}(M+1), \beta_{11,N+1}(M+1), \ldots, \beta_{nn,N+1}(M+1), \\
&\beta_{12,N+1}(M+1), \ldots, \beta_{(n-1,n),N+1}(M+1)]^*
\end{aligned}$$

$$\begin{aligned}
&\hat{\overline{\mathbf{B}}}_{N+1}(M+1) = [b_{0,N+1}(M+1), b_{1,N+1}(M+1), \ldots, \\
&b_{n,N+1}(M+1), b_{11,N+1}(M+1), \ldots, b_{nn,N+1}(M+1), \\
&b_{12,N+1}(M+1), \ldots, b_{(n-1,n),N+1}(M+1)]^* \\
&\qquad \overline{\mathbf{Y}}_{N+1}(M+1) = [y_1(M+1), \cdots, y_N(M+1), \\
&\qquad\qquad y_{N+1}(M+1)]^*
\end{aligned}$$

Secondly to find the recursive form of the **Z**-model, and let M be the variable parameter.

The recursive form of $\hat{\overline{\mathbf{A}}}^{(i)}_{M,N}$ is concerned with two variable parameters, one is the recursive of M, the other is the recursive of N in respect to the recursive model (3.3.1) through the term $\overline{\mathbf{b}}^{(i)}_{M,N}$.

The $\hat{\overline{\mathbf{A}}}^{(i)}_{M+1,N+1}$ can be the following form,

$$\hat{\overline{\mathbf{A}}}^{(i)}_{M+1,N+1} = (\overline{\mathbf{Z}}^*_{M+1}\overline{\mathbf{Z}}_{M+1})^{-1}\overline{\mathbf{Z}}^*_{M+1}\overline{\mathbf{b}}^{(i)}_{M+1,N+1}. \qquad (3.3.4)$$

Where, the element in $\overline{\mathbf{b}}^{(i)}_{M+1,N+1}$ is from $\hat{\overline{\mathbf{B}}}_{N+1}(1), \cdots, \hat{\overline{\mathbf{B}}}_{N+1}(M), \hat{\overline{\mathbf{B}}}_{N+1}(M+1)$ in the equation (3.3.1) and (3.3.3), that is,

$$\begin{aligned}
&\overline{\mathbf{b}}^{(i)}_{M+1,N+1} = [b_{i,N+1}(1), \cdots, b_{i,N+1}(M), b_{i,N+1}(M+1)]^* \\
&\overline{\mathbf{b}}^{(i)}_{M,N+1} \triangleq [b_{i,N+1}(1), \cdots, b_{i,N+1}(M)]^*
\end{aligned}$$

We have a theorem about the recursive form of the equation (3.3.4).

[THEOREM]

$$\begin{aligned}
\hat{\overline{\mathbf{A}}}^{(i)}_{M+1,N+1} = &\overline{\mathbf{E}}_{M+1}\hat{\overline{\mathbf{A}}}^{(i)}_{M,N} + \overline{\mathbf{F}}_{M+1,N+1}[\overline{\mathbf{D}}_M\overline{\mathbf{Y}}_{N+1}(M) \\
&- p_{i,N+1}\hat{\overline{\mathbf{A}}}^{(i)}_{M,N}] + \overline{\mathcal{L}}_{M+1}b_{i,N+1}(M+1) \\
\overline{\mathcal{L}}_{M+1} = &\overline{\mathcal{T}}_M\overline{\mathcal{Q}}^*_{M+1}(1 + \overline{\mathcal{Q}}_{M+1}\overline{\mathcal{T}}_M\overline{\mathcal{Q}}^*_{M+1})^{-1} \\
\overline{\mathcal{T}}_{M+1} = &\overline{\mathcal{T}}_M - \overline{\mathcal{T}}_M\overline{\mathcal{Q}}^*_{M+1}(1 + \overline{\mathcal{Q}}_{M+1}\overline{\mathcal{T}}_M\overline{\mathcal{Q}}^*_{M+1})^{-1}\overline{\mathcal{Q}}_{M+1}\overline{\mathcal{T}}_M \\
i = 1, \cdots, n. &\qquad (3.3.5)
\end{aligned}$$

where,

$$\begin{aligned}
\overline{\mathbf{E}}_{M+1} &= \mathbf{I}_{M_{d1}} - \overline{\mathcal{L}}_{M+1} \cdot \overline{\mathcal{Q}}_{M+1} \\
\overline{\mathbf{F}}_{M+1,N+1} &= k_{i,N+1}\overline{\mathbf{E}}_{M+1} \\
M_{d1} &= 2m + m(m-1)/2 + 1 \\
\overline{\mathcal{T}}_M = (\overline{\mathbf{Z}}^*_M\overline{\mathbf{Z}}_M)^{-1}&; \qquad \overline{\mathbf{D}}_M = \overline{\mathcal{T}}_M\overline{\mathbf{Z}}^*_M
\end{aligned}$$

$\overline{\mathcal{Q}}_{M+1}$ is the last line of matrix $\overline{\mathbf{Z}}_{M+1}$, that is,

$$\begin{aligned}
\overline{\mathcal{Q}}_{M+1} = [&1, z_{1,M+1}, \ldots, z_{m,M+1}, z^2_{1,M+1}, \ldots, z^2_{m,M+1}, \\
&z_{1,M+1}z_{2,M+1}, \ldots, z_{m-1,M+1}z_{m,M+1}]
\end{aligned}$$

$\overline{\mathcal{K}}_{N+1}$ and $\overline{\mathcal{P}}_{N+1}$ are in the equation (3.3.1), we note,

$$\begin{aligned}
\overline{\mathcal{K}}_{N+1} \triangleq [&k_{0,N+1}, k_{1,N+1} \cdots, k_{n,N+1}, k_{11,N+1}, \cdots, k_{nn,N+1}, \\
&k_{12,N+1}, \cdots, k_{(n-1,n),N+1}]^*
\end{aligned}$$

$$\begin{aligned}
\overline{\mathcal{P}}_{N+1} \triangleq [&p_{0,N+1}, p_{1,N+1} \cdots, p_{n,N+1}, p_{11,N+1}, \cdots, p_{nn,N+1}, \\
&p_{12,N+1}, \cdots, p_{(n-1,n),N+1}] \\
&\overline{\mathbf{Y}}_{N+1}(M) = [y_{N+1}(1), \cdots, y_{N+1}(M)]^*
\end{aligned}$$

There are the same results for $\hat{\overline{\mathbf{A}}}^{(0)}_{M+1,N+1}$, $\hat{\overline{\mathbf{A}}}^{(ii)}_{M+1,N+1}$, $\hat{\overline{\mathbf{A}}}^{(ij)}_{M+1,N+1}$ if the superscript (i) is changed to (0),(ii) or (ij).

Here, the recursive estimators $\hat{\overline{\mathbf{B}}}_{N+1}(M+1)$, $\hat{\overline{\mathbf{A}}}_{M+1,N+1}$ can be used to modify the total **XZ**-model with operational parameter variables. The modified model is forming the objective function again, the new experiment point is got again according to the optimal sequential design, the new observing values are obtained on the new experiment point again to modify the model again. The iterative process continues until the optimal design parameters are established.

4. Fast algorithm for a large number of sequential experiments

When the number of sequential experiments is increased, let be $N+l$ and $M+l,(l>1)$, the equation (3.3.3) in the recursive identification will be as follows,

$$\hat{\overline{\mathbf{B}}}_{N+l}(M+l) = (\overline{\mathbf{X}}^{*}_{N+l}\overline{\mathbf{X}}_{N+l})^{-1}\overline{\mathbf{X}}^{*}_{N+l}\mathbf{Y}_{N+l}(M+l)$$

The larger the matrix $\overline{\mathbf{X}}_{N+l}$ is, the more difficult the recursive identification algorithm is.

To get the simple calculation, we can do as follows,

First, calculate

$$\hat{\overline{\mathbf{B}}}_{N}(M+l) = (\overline{\mathbf{X}}^{*}_{N}\overline{\mathbf{X}}_{N})^{-1}\overline{\mathbf{X}}^{*}_{N}\overline{\mathbf{Y}}_{N}(M+l)$$

then calculate the $\hat{\overline{\mathbf{B}}}_{N+1}(M+l)$ with the recursive form, that is

$$\hat{\overline{\mathbf{B}}}_{N+1}(M+l) = \hat{\overline{\mathbf{B}}}_{N}(M+l) + \overline{\mathcal{K}}_{N+1}[y_{N+1}(M+l) - \overline{\mathcal{P}}_{N+1}\hat{\overline{\mathbf{B}}}_{N}(M+l)]$$

$$\overline{\mathcal{K}}_{N+1} = \overline{\mathcal{R}}_{N}\overline{\mathcal{P}}^{*}_{N+1}(1+\overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_{N}\overline{\mathcal{P}}^{*}_{N+1})^{-1}$$

$$\overline{\mathcal{R}}_{N+1} = \overline{\mathcal{R}}_{N} - \overline{\mathcal{R}}_{N}\overline{\mathcal{P}}^{*}_{N+1}(1+\overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_{N}\overline{\mathcal{P}}^{*}_{N+1})^{-1}\overline{\mathcal{P}}_{N+1}\overline{\mathcal{R}}_{N}$$

With l iterations, $\hat{\overline{\mathbf{B}}}_{N+l}(M+l)$ can be obtained.

Hence, we can make use of the previous results while avoiding complicated and repeated calculations.

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Appendix

Table 1: Experimental Scheme

No. of experiment	term					
	Levels of Structural variables			Observing value Y — Levels of operational parameter variables		
	x_1	$\cdots$	x_n	$(z_{11},\ldots,z_{m1})$	$\cdots$	$(z_{1,M},\ldots,z_{m,M})$
1	$x_{1,1}$	$\cdots$	$x_{n,1}$	$y_1(1,\ldots,1)$	$\cdots$	$y_1(M,\ldots,M)$
2	$x_{1,2}$	$\cdots$	$x_{n,2}$	$y_2(1,\ldots,1)$	$\cdots$	$y_2(M,\ldots,M)$
$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
N	$x_{1,N}$	$\cdots$	$x_{n,N}$	$y_N(1,\ldots,1)$	$\cdots$	$y_N(M,\ldots,M)$

Table 2: Variance Analysis

Source of difference	Squares	Degrees of freedom	F-ratio
Effect on variable	$\sum_{u=1}^{N}[\hat{Y}_u(v)-\overline{y}(v)]^2$	N_{d1}	$F_{N_d}^{N_{d1}}$
Regression variance	$\sum_{u=1}^{N}[y_u(v)-\hat{Y}_u(v)]^2$	N_d	
Total	$\sum_{u=1}^{N}[y_u(v)-\overline{y}(v)]^2$	$N-1$	

Table 3: Experimental Scheme

No. of experiment	term						
	Levels of Structural variables			Observing value Y — Levels of operational parameter variables			
	x_1	$\cdots$	x_n	$(z_{11},\ldots,z_{m1})$	$\cdots$	$(z_{1,M},\ldots,z_{m,M})$	$(z_{1,M+1},\ldots,z_{m,M+1})$
1	$x_{1,1}$	$\cdots$	$x_{n,1}$	$y_1(1,\ldots,1)$	$\cdots$	$y_1(M,\ldots,M)$	$y_1(M+1,\ldots,M+1)$
2	$x_{1,2}$	$\cdots$	$x_{n,2}$	$y_2(1,\ldots,1)$	$\cdots$	$y_2(M,\ldots,M)$	$y_2(M+1,\ldots,M+1)$
$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	$x_{1,N}$	$\cdots$	$x_{n,N}$	$y_N(1,\ldots,1)$	$\cdots$	$y_N(M,\ldots,M)$	$y_N(M+1,\ldots,M+1)$
N+1	$x_{1,N+1}$	$\cdots$	$x_{n,N+1}$	$y_{N+1}(1,\ldots,1)$	$\cdots$	$y_{N+1}(M,\ldots,M)$	$y_{N+1}(M+1,\ldots,M+1)$