Exerted Force Minimization for Weak Points in Cooperating Multiple Robot Arms

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ABSTRACT

This paper discusses a force distribution scheme which minimizes the weighted norm of the forces/torques applied on weak points of cooperating multiple robot arms. The scheme is proposed to avoid the damage or unwanted motion of any weak point of robots or object stemming from excessive forces/torques. Since the proposed scheme can be used for either the joint torque minimization or the exerted force minimization on the object, it can be regarded as a unified force minimization method for multiple robot arms. The computational complexity in this scheme is analyzed using the properties of Jacobian. Simulation of two identical PUMA robots held an object is carried out to illustrate the proposed scheme. By the proper choice of the weighting matrix in the performance index, we show that force minimization for a weak point can be achieved, and that the exerted force minimization on the object can be changed to the joint torque minimization.

NOMENCLATURE

- r number of robots
- n number of joints for each robot
- l number of weak points for each robot
- q_i generalized coordinates of the ith robot in R^n
- q extended generalized coordinates for all robots in R^{rn}, $q^t = \left[q_1^t, q_2^t, \cdots, q_r^t\right]$
- τ_i joint torque vector for q_i in R^n
- τ extended joint torque vector for q in R^{rn}, $\tau^t = \left[\tau_1^t, \tau_2^t, \cdots, \tau_r^t\right]$
- x_o position vector of an object with respect to the world coordinates in R^6
- F_o force vector of an object in R^6
- x_{ei} position vector of the ith end-effector in R^6
- x_e extended position vector of all end-effectors in R^{6r}, $x_e^t = \left[x_{e1}^t, x_{e2}^t, \cdots, x_{er}^t\right]$
- F_{ei} force vector for x_{ei} in R^6
- F_e extended force vector for x_e in R^{6r}, $F_e^t = \left[F_{e1}^t, F_{e2}^t, \cdots, F_{er}^t\right]$
- x_{wij} position vector of the jth weak point on the ith robot in R^6
- x_{wi} extended position vector of the weak points for the ith robot in R^{6l}, $x_{wi}^t = \left[x_{wi1}^t, x_{wi2}^t, \cdots, x_{wir}^t\right]$
- x_w extended position vector of all weak points in R^{6rl}, $x_w^t = \left[x_{w1}^t, x_{w2}^t, \cdots, x_{wr}^t\right]$
- F_{wij} weak point force vector for x_{wij} in R^6
- F_{wi} extended weak point force vector for x_{wi}, $F_{wi}^t = \left[F_{wi1}^t, F_{wi2}^t, \cdots, F_{wil}^t\right]$ in R^{6l}
- F_w extended force vector for x_w in R^{6rl}, $F_w^t = \left[F_{w1}^t, F_{w2}^t, \cdots, F_{wr}^t\right]$
- J_{ai} Jacobian matrix between the end-effector and joint for the ith robot in $R^{6\times n}$
- J_a extended Jacobian matrix consisting of diagonal submatrices of J_{ai} in $R^{6r\times nr}$
- J_{ij} Jacobian matrix between the end-effector and the jth weak point on the ith robot in $R^{6\times 6}$
- J extended Jacobian matrix consisting of submatrices of J_{ij} in $R^{6rl\times 6r}$
- $\hat{J}_i$ Jacobian matrix between the object and the ith end effector in $R^{6\times 6}$
- $\hat{J}$ extended Jacobian matrix in $R^{6\times 6r}$, $\hat{J} = \left[\hat{J}_1, \hat{J}_2, \cdots, \hat{J}_r\right]$
- R rotation matrix in $R^{3\times 3}$
- T homogeneous transformation matrix in $R^{4\times 4}$

1. Introduction

Recently, multiple robot arms which provide greater lifting capability, manipulability, and flexibility in automated manufacturing has been focused. However, the coordination of multiple robot arms causes many problems such as load distribution, trajectory planning, and tracking control. Among them, load distribution becomes very important problem, especially, when multiple robot arms handle a single object simultaneously. The load that imposes on the end-effectors can in general be represented by three dimensional forces and torques. If the equal amount of force/torque which warrants the desired motion of the object are generated by cooperating multiple robot arms, the object follows the given trajectory. However, if the total number of degrees of freedom of multiple robot arms are greater than six, then the applied joint forces/torques of each robot for the required motion of the object is not uniquely determined. Since only 6 independent forces are required to describe the object motion, the remaining forces may contribute to generate the internal force which cancel each other end-effector forces. Therefore, the cancelled forces can be regarded as redundant forces. The redundancy in forces may be used to optimize a certain kind of performance criteria.

During recent years, load distribution among multiple robot arms has been the subject of considerable research, and several approaches have been suggested. One approach is based on joint torque minimization[1][2], which minimizes the norm of joint torques, and it is also equivalent to the least energy consumption. In [1], computation for joint torque minimization involves time consuming matrix multiplications and inversions. Carignan and Akin[2] proposed a reduced order form for two planar arms and obtained the optimal torque distribution using simple equations for real time application. Since their algorithm states in the case of two planar arms with two links only, it is not applicable to general multiple robot arms. Pittelkau[3] proposed an adaptive scheme of the load distribution for two robot arms, and the end-effector forces of two robot arms are chosen as $F_{e1} = \alpha F_o$ and $F_{e2} = (1 - \alpha) F_o$ with $0 \le \alpha \le 1$ respectively, and the norm of joint torques is minimized by using an adaptive rule of α. Since the end-effector forces are assumed to have the same direction with the required object forces, it does not give true minimum joint torques in general. Walker *et al*[4] suggested a null space variation scheme, where a null vector is chosen to reduce the torque requirement. In these joint torque minimization approaches, the exerted joint torque, *i.e*, exerted joint energy, can be minimized. However, the problem stemming from the excessive exerted forces to the object is still remaining. We interpret the *exerted force* as the force exerted on a point which has 6 components of forces/torques.

Another approach is based on the exerted end-effector force minimization which prevents the damage of the object as well as the end-effector holding the object[1][2][5][6][7]. A large amount of exerted forces may be harmful to both the object and end-effectors. The minimization of exerted forces to the object leads to an alternative optimal algorithm. Hayati[6] proposed the weighted exerted end-effector force minimization. In rigid grasp of an object, the multiplications of a constant matrix to the desired object force gives the optimal end-effector forces. And Zheng[1] introduced a scalar parameter α of $F_{e1} = \alpha F_o$, $F_{e2} = (1 - \alpha) F_o$ where $0 \le \alpha \le 1$, and α was obtained by the minimization of the norm of F_{ei}. But for real time application, half or even distribution was proposed. Hsu[5] proposed a method to choose a weighting matrix in the performance index of weighted exerted end-effector force minimization. The weighting matrix was chosen by considering on the structural characteristics of the object and grip position of end-effectors. On the other hands, these approaches, which minimize exerted end-effector forces on the object, do not guarantee the minimization of energy to joint torques. And the consideration of joint torque limits for exerted force minimization is very difficult because the weighting corresponds to the exerted force, and joint torques are not directly related to the weighting. Therefore, the joint torques may exceed its limit. And, if there is a weak or fragile point on the object, then the tuning of the elements in the weighting matrix for the particular point is very difficult because the optimized forces for the end-effectors, not for the weak points.

From the previous works, we observe that there are limitations in both approaches. It is required to find a unified approach for multiple robot arms, which minimizes the forces at points of interest. In this context, we define a *weak point* as follows : A point on the link of a robot or on the object which can be damaged structurally due to the excessive force or a point which may give the unwanted motion due to the joint torque limits. Therefore, the exerted forces on the weak point should be reduced enough to avoid unwanted effects.

In this paper, a force distribution scheme which minimizes the weighted norm of force/torque exerted on weak points in multiple robot arms is proposed. The maximum allowable forces on weak points can be chosen as the inverse of diagonal elements of a weighting matrix. Since joint torque limits are the maximum driving torques of joints, these limits are the maximum allowable forces for the joints. Therefore, the limits of joint torques for the joints can be generalized by the concept of the maximum allowable forces for arbitrary points including the joints. In this generalized concept, joint torque minimization can be treated as a subset of force minimization of the weak points. In this case, joints become weak points. Also, exerted force minimization can be solved by treating the end-effector of each robot as a weak point from the concept of maximum allowable forces. The proposed scheme, therefore, generalizes the joint torque minimization problem and/or the exerted force minimization problem as a force minimization problem at weak points.

2. Force relationship in a single link

Weak point forces in multiple robot arms can be represented as a function of the forces exerted from the object as well as from the robot motion. In this section, we describe the force relation between two points in a rigid link. In Fig.1, the origins of two coordinate frames are established at points **a** and **b**, and the coordinate frames are denoted as O_a and O_b respectively. A 3×3 submatrix R represents a rotational relation from the coordinate frame O_b to O_a. And p is defined as a 3×1 position vector from the point a to the point b with respect to the coordinates frame O_a. Therefore the notation p is different from the conventional 3×1 vector which describes the position from the origin of coordinates frame O_a to O_b. F_b is an exerting force from an external object to the point b with respect to the coordinate frame O_b. And F_a is an induced force to the point a with respect to the coordinates frame O_a.

If a point to be considered, a, does not a contact with other objects but is an arbitrary point on a link, we can not generally define the force at that point, since the force is distributed as the form of the density of force. In a rigid link or an object, therefore, we consider the smallest area crossing the point to be considered. If we interpret the force integrated on the smallest area as the force at that point[8], the link can be divided into two sublinks as shown in Fig.1.

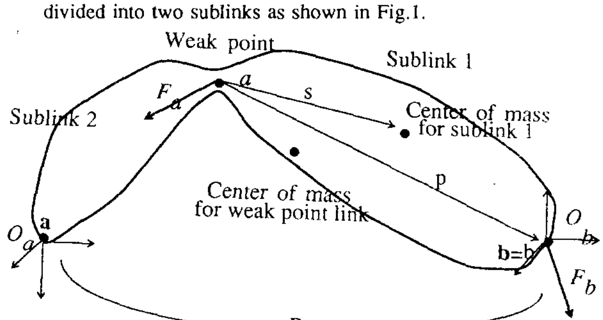

Fig. 1. Force relation between arbitrary two points.

For further discussion, we define a *weak link* as a link which has a weak point. Then, a weak sublink is defined as a sublink between the weak point a and the contact point. Assume that the point a in Fig.1 is a weak point. Then the force relation for sublink 1 can be expressed by

$$F_a = J_{ab} F_b + F_{1s} \tag{1}$$

where J_{ab} is Jacobian matrix for two points a and b, and it can be obtained using coordinate transformation matrix[11].

$$J_{ab} = \begin{bmatrix} R & 0 \\ PR & R \end{bmatrix}$$

where P is defined as follows:

$$P \equiv \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}$$

which satisfies the following properties:

$$(PR)v = p \times (Rv) \quad \text{and} \quad P^t = -P$$

And F_{1s} is the force induced by the motion of the link, and can be described as

$$F_{1s} = \begin{bmatrix} m_1 \alpha_1 \\ s_1 \times (m_1 \alpha_1) + I_1 \dot{w} + w \times I_1 w \end{bmatrix}$$

and m_1 is the total mass of sublink 1, and s_1 is the position vector from the weak point to the center of mass for sublink 1 with respect to the coordinate frame O_a. α_1 is the acceleration vector of the center of mass point for sublink 1, and can be obtained from the acceleration vector of the center of mass point of the original link. The angular velocity and angular acceleration w_1 and $\dot{w}_1$ are the same as w and $\dot{w}$ of the original link. To calculate F_{1s}, m_1, p_1 and I_1 must be properly predetermined from the structure of the original link.

3. Force minimization at weak points

The weak point forces can be divided into two types of forces: the end-effector forces and the forces induced by robot motion. The forces due to robot motion can be determined if a trajectory for the object is given. The remaining unknown forces are due to exerted force from the end-effectors. If we can determine the force induced by robot motion from the trajectory, the weak point force is a function of the end-effector force only.

In multiple robot arms, because of the redundancy, there exist many solutions for end-effector forces which generate the desired object forces. Therefore, by the proper choice of end-effector forces satisfying the constraints, the weak point forces can be reduced properly and protected from the excessive forces.

At weak points, weak point forces are described by $F_w = J F_e + F_s$ where F_s is a $6rl \times 1$ force vector induced from robot motion. In this case, the force minimization problem can be formulated as follows:

Problem I

Determine the weak point forces F_w to minimize the quadratic performance index

$$\frac{1}{2} F_w^t Q F_w \tag{2}$$

subject to $F_o = \hat{J} F_e$ and $F_w = J F_e + F_s$

Although the weak point forces F_w is minimized, the force is not need to know because the required forces to control or to simulate are joint torques or end-effector forces. In this paper, therefore, we will solve the problem as the end-effector forces which minimizes the performance index.

To solve the problem, we introduce a Lagrange multiplier, λ, and augment the constraints $F_o = \hat{J} F_e$ to the original problem. Then, the force minimization problem can be reformulated as unconstrained optimization problem.

Problem II

Determine the end-effector forces F_e to minimize the quadratic performance index

$$\frac{1}{2} (F_e^t J^t + F_s^t) Q (J F_e + F_s) + \lambda^t (F_o - \hat{J} F_e) \tag{3}$$

J^t is the transpose of $6r \times 6rl$ Jacobian matrix and expressed as follows:

$$J^t = \begin{bmatrix} J_{11}^t & \cdots & J_{1l}^t & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & J_{21}^t & \cdots & J_{2l}^t & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & J_{r1}^t & \cdots & J_{rl}^t \end{bmatrix} \tag{4}$$

where J_{ij} is the Jacobian matrix, which describes the force relationship between the jth weak point and the end-effector of the ith robot. And the weighting matrix Q in (3) is a $6rl \times 6rl$ weighting matrix for weak points as follows:

$$Q = diag\{Q_{11}, \cdots, Q_{1l}, Q_{21}, \cdots, Q_{2l}, \cdots, Q_{r1}, \cdots, Q_{rl}\}, \tag{5}$$

$$Q_{ij} = diag\{\{Q_{ij}\}_{11}, \cdots \{Q_{ij}\}_{66}\}$$

where Q_{ij} are the weighting matrix corresponding to the weak point force F_{wij}.

In this formulation, the weak point forces are minimized by a weighted minimization method. The weighting matrix Q of (3) gives the relative weakness among weak points. For instance, if the jth weak point on the ith robot is relatively weaker than the others, then the forces exerted on the weakest point can be reduced by increasing the norm of the corresponding submatrix Q_{ij}. Since the forces exerted on a weak point are composed of six independent terms of forces and torques, the weighting submatrices Q_{ij} corresponding to the jth weak point of the ith robot can be represented by a 6×6 diagonal matrix if the cross weighting terms are ignored. Also, the elements of Q_{ij} can be assigned depending on the relative weakness among force directions of the weak point. For example, if x-direction force for the weak point, *i.e.* f_{ijx}, is relatively weaker than y-direction force, f_{ijy}, then we can choose a large value for $\{Q_{ij}\}_{11}$ and small value for $\{Q_{ij}\}_{22}$. Conversely, if the force direction of a weak point along partial coordinate is relatively stronger than the others, then the value of the weighting matrix corresponding to that direction can be reduced.

The end-effector force which minimizes the performance index becomes

$$\begin{aligned} F_e = {} & (J^t Q J)^{-1} \hat{J}^t (\hat{J} (J^t Q J)^{-1} \hat{J}^t)^{-1} F_o + \\ & (J^t Q J)^{-1} \hat{J}^t (\hat{J} (J^t Q J)^{-1} \hat{J}^t)^{-1} \hat{J} (J^t Q J)^{-1} J^t Q F_s \\ & - (J^t Q J)^{-1} J^t Q F_s \end{aligned} \tag{6}$$

The above equation seems to be very complex because the inversion of $(J^t\,Q\,J)^{-1}$ are required. However, if we make some assumptions, (6) can be solved.

For r robots and l weak points for each robot, the calculation of $(J^t\,Q\,J)^{-1}$ requires a matrix inversion of dimension $6rl \times 6rl$. For the case of two cooperating robot arms which have eight weak points for each robot, the inversion of a 96×96 matrix is required.

In this case, the matrix inversion can be simplified if the characteristics of a robot is used. In $(J^t\,Q\,J)^{-1}$, J and Q are defined in (4) and (5) respectively. To calculate the matrix $(J^t\,Q\,J)^{-1}$, we consider the properties of J_{ij} which are submatrices of J. By the definition, J_{ij} can be described as follows:

$$J_{ij} \equiv \begin{bmatrix} R & 0 \\ PR & R \end{bmatrix} \tag{7}$$

where R and P are submatrices of the coordinate transformation matrix, and the subscripts i and j of R and P are ignored for brevity. Using the orthonormal properties of a rotational matrix, $R^{-1} = R^t$, the inversion becomes

$$J_{ij}^{-1} = \begin{bmatrix} R^{-1} & 0 \\ -R^{-1}P & R^{-1} \end{bmatrix} = \begin{bmatrix} R^t & 0 \\ (PR)^t & R^t \end{bmatrix} \tag{8}$$

Also, the transpose and its inverse for J_{ij}^t are defined respectively as follows:

$$J_{ij}^t = \begin{bmatrix} R^t & (PR)^t \\ 0 & R^t \end{bmatrix} \quad , \quad J_{ij}^{-t} = \begin{bmatrix} R & PR \\ 0 & R \end{bmatrix} \tag{9}$$

Inverse of the Jacobian matrix, J_{ij}^{-1}, can be obtained by simple operations such as the transpose of submatrices. Also, if Q_{ij} are assumed as diagonal matrices, the weighting matrix Q becomes a $6rl \times 6rl$ diagonal matrix.

After simple operations for submatrices, $J^t\,Q\,J$ becomes

$$J^t\,Q\,J = diag\,[\sum_{i=1}^{l} J_{1i}^t Q_{1i} J_{1i}\ ,\ \sum_{i=1}^{l} J_{2i}^t Q_{2i} J_{2i}\ , \tag{10}$$
$$\cdots\ ,\ \sum_{i=1}^{l} J_{ri}^t Q_{ri} J_{ri}\,]$$

Since this matrix is composed of block diagonal submatrices, the inverse of $J^t\,Q\,J$ can be determined as follows:

$$(J^t\,Q\,J)^{-1} = diag\,[(\sum_{i=1}^{l} J_{1i}^t Q_{1i} J_{1i})^{-1}\ ,\ (\sum_{i=1}^{l} J_{2i}^t Q_{2i} J_{2i})^{-1}\ , \tag{11}$$
$$\cdots\ ,\ (\sum_{i=1}^{l} J_{ri}^t Q_{ri} J_{ri})^{-1}]$$

In the case of $l = 1$, *i.e.* one weak point for each robot, then

$$(\sum_{j=1}^{l} J_{ij}^t Q_{ij} J_{ij})^{-1} = (J_i^t Q_i J_i)^{-1} = J_i^{-1} Q_i^{-1} J_i^{-t} \tag{12}$$

where Q_{ij}^{-1} are also diagonal matrices, and the diagonal elements of Q_{ij}^{-1} are replaced by the inverse of the diagonal elements of Q_{ij}. J_{ij}^{-1} and J_{ij}^{-t} can be obtained from (8) and (9). If $l \neq 1$, *i.e.* multiple weak points for each robot, then the inversion of a 6x6 matrix is required r times to calculate $(J^t\,Q\,J)^{-1}$ as much as the number of robots.

The 6×6 matrix inversion for $(\hat{J}\,(J^t\,Q\,J)^{-1}\hat{J}^t)^{-1}$ can be done similarly as follows:

$$(\hat{J}\,(J^t\,Q\,J)^{-1}\hat{J}^t)^{-1} = (\sum_{i=1}^{r} \hat{J}\,(\sum_{j=1}^{l} J_{ij}^t Q_{ij} J_{ij})^{-1} \hat{J}_i^t)^{-1} \tag{13}$$

The total number of the matrix inversion to calculate F_e can be summarized as follows:

if $l = 1$, then only one 6×6 matrix inversion is required.

if $l \neq 1$, then $(r+1)$ times

4. Simulation of force distribution for two PUMA560 robots

In this section, it is shown that the proposed force distribution scheme can be used in joint torque minimization or exerted force minimization as well as weak point force minimization. First, the exerted force minimization turned into the joint torque minimization by the variation of weighting values. Next, the weak point force minimization is carried out to show the force reduction when the paticular joint or paticular point in link of each robot are defined as the weak points.

As shown in Fig.2, environments for simulations, we show that pertaining to two identical PUMA 560 robots are summarized. the base coordinates of robot 1 and robot 2 are established at O_{b1} and O_{b2}, respectively. The base coordinates of each robot and world coordinates, O_w, are placed with distance $0.75m$, and the relation among them are represented by homogenous transformation matrices as follows:

$${}^{w}\mathrm{T}_{b1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.75 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad {}^{w}\mathrm{T}_{b2} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0.75 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

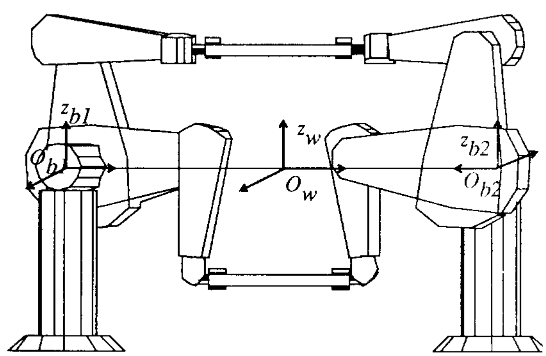

Fig. 2. Coordinate systems of robots and world

And the rigid object held by two robots is chosen as the hexahedronal bar. It is assumed that the object is uniformly distributed and its length, depth, and width are $0.4m$, $0.1m$, and $0.1m$, respectively. The total mass of the object is assumed to be 12Kg. And the inertia matrix of the object for the center of mass is given by

$$\mathbf{I} = \begin{bmatrix} 0.68 & -0.12 & -0.03 \\ -0.12 & 0.08 & -0.12 \\ -0.03 & -0.12 & 0.68 \end{bmatrix}$$

And the grip position of both robots are expressed as the following matrices with respect to the object coordinates.

$${}^{o}\mathrm{T}_{e1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad {}^{o}\mathrm{T}_{e2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let the initial and final positions of the center of mass of the object be

$$x_{oi} = (0, 0, -0.4, 0, 0, 0)^t \text{ and } x_{of} = (0, 0, 0.4, 0, 0, 0)^t$$

where the robots are moving along z-direction with respect to world coordinates in 2sec as shown in Fig.2. The desired trajectory of the center of mass of the object with respect to the world coordinates can be obtained by (14)-(17).

$$x_{oe} = x_{of} - x_{oi} \tag{14}$$

$$x_o = x_{oi} + \frac{x_{oe}}{t_f}\left(t - \frac{t_f}{2\pi}\sin\left(\frac{2\pi t}{t_f}\right)\right) \tag{15}$$

$$\dot{x}_o = \frac{x_{oe}}{t_f}\left(1 - \cos\left(\frac{2\pi t}{t_f}\right)\right) \tag{16}$$

$$\ddot{x}_o = \frac{2\pi x_{oe}}{t_f^2}\sin\left(\frac{2\pi t}{t_f}\right) \tag{17}$$

where x_{oe} is the total distance from the initial to the final position of the object, and t_f is the total traveling time under the assumption $t_0 = 0$.

Using the trajectory of the object obtained through (14)-(17), the joint trajectories of robots are obtained in order to determine the forces, F_s, due to robots motion. The object trajectory and the required load force, F_o, are illustrated in Figs. 3.

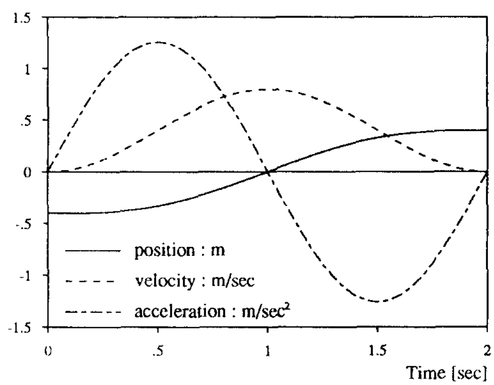

Fig. 3. Trajectories of the object.

A. Joint force minimization *vs* exerted force minimization

Using the concept of the weak point force, we show that the exerted force minimization and the joint torque minimization can be treated as the same procedures. Weak points are chosen as each joint and each end-effector, and the joint coordinates and end-effector coordinates become the weak point coordinates. Then, seven weak points are assigned to each robot. Initially, the diagonal elements of the weighting matrix corresponding to the end-effectors are set to large values. And the other diagonal elements corresponding to joints are set to small values. In this situation, the weak point force minimization problem is nearly equivalent to an exerted force minimization problem. As the weighting values for joints increase, and those for the end-effectors decrease, the weak point force minimization problem becomes a joint torque minimization.

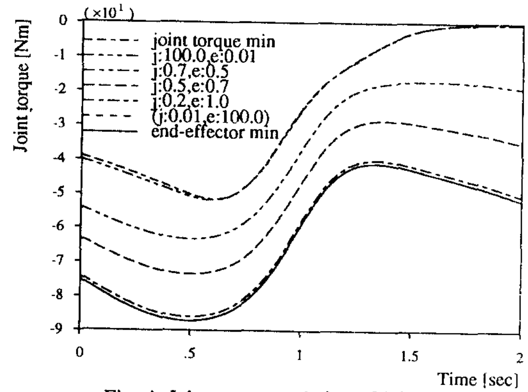

Fig. 4. Joint torque variation of joint 2

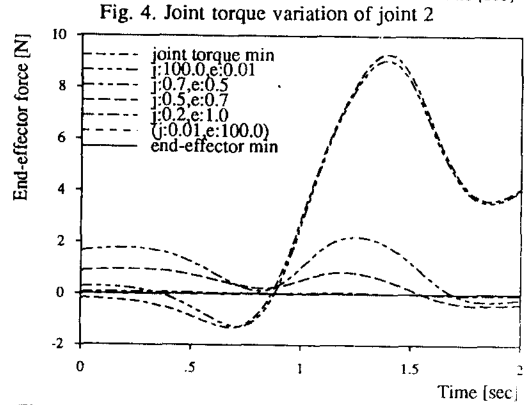

Fig. 5. Exerted force variation of x-axis of the end-effector

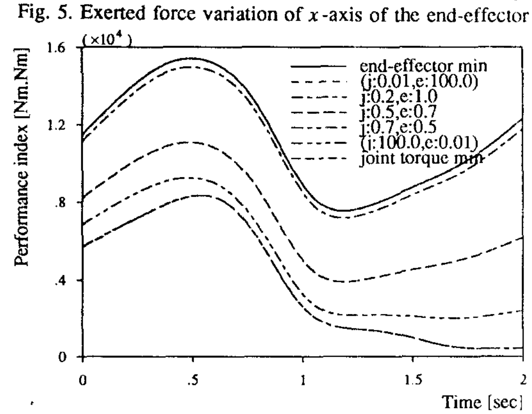

Fig. 6. The variation of performance index of $\tau^t\tau$.

In Fig. 4, joint torque of the second joint of the first robot is illustrated for the variation of diagonal values of the weighting matrix. Since the robots are assumed to have identical structures and to grasp the object symmetrically, the joint torques for each robot are equal if same weighting values are assigned. When the weighting values correspond to joint are 0.01, and the weighting values correspond to the end-effectors are 100.0, the weak point force minimization is nearly equal to exerted force minimization. On the other hand, in the case of 0.01 for the end-effectors and 100.0 for joints, it is nearly equal to the joint torque minimization. By adjusting the weighting values, the curve resulted from exerted force minimization transfers to the curve of joint torque minimization, and the magnitude of joint torques are reduced. In Fig. 5, the exerted force variation of x-axis of the end-effector is illustrated. When the weighting matrix is adjusted, we notice that the magnitude of the exerted force increase as the magnitude of the joint torques are reduced. In this example, all forces except f_{ey} generate internal force. Therefore, f_{ex} is cancelled each other when the exerted force is minimized. In Fig. 6, P.I. = $\tau^t \tau$ is illustrated for the resultant joint torque. As shown in these figures, the weak point force minimization method can be regarded as the generalized approach which include the joint torque minimization and the exerted force minimization for end-effectors.

B. Weak point force minimization for the specific joint.

Assume that a joint is weaker than the other joints. Let the fourth joint (wrist joint) be the weakest point. To show the force reduction at the weak point, we apply the weak point force minimization approach in this example. The weak points are chosen as same as in the first example. Let the weighting values be chosen as 1.0 for all diagonal elements except f_{y3} terms of both robots. And the weighting values corresponding to the f_{y3} are varied from 1.0 to 5.0. Fig. 7 shows that the force reduction, and Fig. 9 shows the force variation of the other joint to reduce the force of the weakest point, f_{y3}, is done significantly.

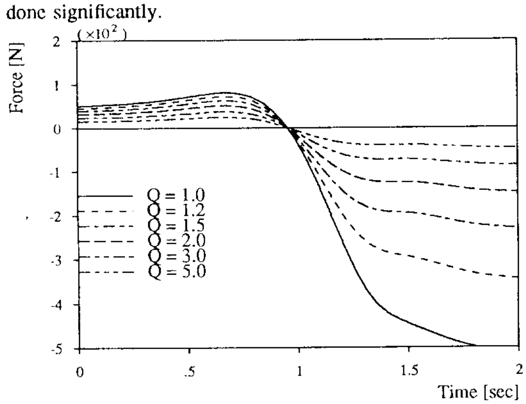

Fig. 7. Trends of f_{y3} according to the variation of weighting.

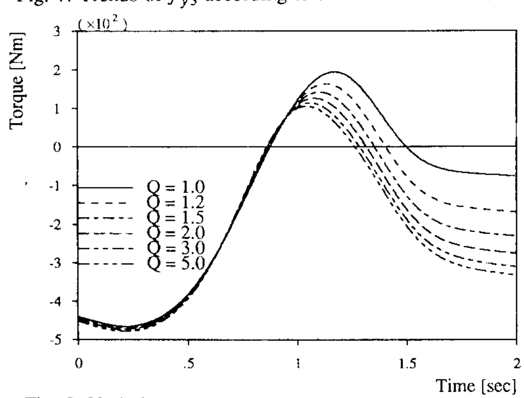

Fig. 8. Variation of n_{x4} to reduce the weakest point force f_{y3}

5. Conclusion

A force distribution scheme which minimizes the weighted norm of force/torque exerted on the weak points in multiple robot arms is proposed. There exist two method of a joint torque minimization and an exerted force minimization for load distribution, and we propose the generalized method as one-combined approach using the concept of a weak point. By choosing the weak points to the joint coordinates for all robots and properly assigning the weighting, joint torque minimization can be achieved. Also, by defining the end-effectors for all robots as the weak points, the exerted force minimization for an object can be achieved. The computational complexity of this algorithm is significantly reduced, which seems to prevent from real time application, using the properties of the Jacobian matrix. Simulation results of two identical PUMA robot are given to illustrate the proposed scheme. From the variation of the weighting, transfer action from exerted force minimization on the object to joint torque minimization is simulated to show that the proposed algorithm is generalized approach for force minimization. And weak point force minimization for a point, which is weak especially, is simulated to show the force reduction to avoid a damage. From the comparison of the exerted force minimization and joint torque minimization, this algorithm reveals the feasibility and applicability.

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