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A STUDY ON THE OPTIMAL REDUNDANCY RESOLUTION OF A KINEMATICALLY REDUNDANT MANIPULATOR

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ABSTRACT

This paper proposes an optimal redundancy resolution of a kinematically redundant manipulator while considering homotopy classes. The necessary condition derived by minimizing an integral cost criterion results in a second-order differential equation. Also boundary conditions as well as the necessary condition are required to uniquely specify the solution. In the case of a cyclic task, we reformulate the periodic boundary value problem as a two point boundary value problem to find an initial joint velocity as many dimensions as the degrees of redundancy for given initial configuration. Initial conditions which provide desirable solutions are obtained by using the basis of the null projection operator. Finally, we show that the method can be used as a topological lifting method of nonhomotopic extremal solutions and also show the optimal solution with considering the manipulator dynamics.

1. INTRODUCTION

A kinematically redundant manipulator possesses more degrees of freedom (DOF) than that required for performing a specified task. Redundancy by adding redundant DOF to a manipulator which has a minimum number of DOF required to accomplish given task yields increased dexterity and versatility for performing the task due to the infinite number of inverse kinematic solutions. Therefore, the resolution of a redundant manipulator for reconfiguring the arm without affecting the end-effector position has been discussed in the framework of how to optimize some performance measure while carrying out its given task.

Because of these significant advantages, redundant manipulators have been the subject of considerable research, and a number of control schemes for determining joint trajectories have been developed by using both global and local optimization methods. The local redundancy resolution schemes determine joint trajectories required to achieve a desired end-effector trajectory while performing the local optimization of given performance criterion [1-6].

The global redundancy resolution schemes, on the other hand, determine the joint trajectory from a complete description of the desired end-effector trajectory, which are based on the global optimization with an integral cost criterion. Therefore, global optimization methods are preferable to local optimization methods. Recently, *Configuration Control* [7] and a *Cartesian Control* [8] are presented. The globally optimal redundancy control is considered by Nakamura and Hanafusa [9] strictly using *Pontryagin's maximum principle.* And Martin *et al.* [10] suggest a reduced order form equivalent to a second-order differential equation with n variables obtained by solving necessary conditions for optimality using the Euler-Lagrange equations. For a cyclic task, they show two different joint trajectories that satisfy the Euler-Lagrange equations and periodicity. However, they have been exhibited numerical solutions for the periodic boundary value problem.

In this paper, we discuss an optimal redundancy control problem of a redundant manipulator while considering homotopy classes. The necessary condition derived by minimizing an integral cost criterion results in a second-order differential equation with n variables. In order to uniquely specify the optimal solution, one must consider the boundary conditions as well as the necessary condition. For a cyclic task, the boundary conditions become periodic. We refine the periodic boundary condition problem to a two point boundary problem to find the initial joint velocity $\dot{\theta}(t_0)$ using the final time configuration error $\theta(t_1)-\theta(t_0)$, where t_0 and t_1 are the initial and final time, respectively.

The problem of a globally optimal redundancy resolution is solved by using Euler-Lagrange equations, and we show that the convergence of numerical search for the solution which satisfies periodic boundary conditions is very difficult to achieve because an integral cost is very sensitive to the initial condition. In order to determine $\dot{\theta}(t_0)$, minimal value searching must be performed in a space of as many dimensions as the number of degrees of redundancy for any integral cost criteria. Finally, we compare the optimal solutions obtained using the basis of a null projection operator of the Jacobian matrix with locally optimal solutions at the same initial configuration. And we show that the proposed method can be used as a topological lifting method of nonhomotopic extremal solutions through a three DOF planar manipulator for cyclic tasks with considering the dynamics of the manipulator.

2. NECESSARY CONDITIONS

For a global optimal resolution, one considers an integral type performance index subject to the kinematic constraints. In this paper, we first consider the following general integral cost criterion

$$r = \int_{t_0}^{t_1} p(\theta, \dot{\theta}, t)\, dt \tag{1}$$

subject to the kinematic constraints

$$x(t) = f(\theta(t)) \tag{2}$$

where $\theta(t) \in R^n$ is the joint vector and $x(t) \in R^m$ represents the position of the end-effector. The problem is to find the joint trajectory $\theta(t)$ which minimizes the performance index (1) among joint trajectories tracking the desired end-effector trajectory described as (2). For convenience, throughout this paper, the argument t is sometimes omitted when no confusion is likely to arise.

Let's define the Lagrangian function

$$L(\theta, \dot{\theta}, \lambda, t) = p(\theta, \dot{\theta}, t) + \lambda^T(x - f(\theta)). \tag{3}$$

Using the Lagrangian function, necessary conditions for optimality of (1) and (2) are given by the Euler-Lagrange equations.

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0, \tag{4}$$

$$\frac{\partial L}{\partial \lambda} = 0. \tag{5}$$

For a reasonable candidate for $p(\theta, \dot{\theta}, t)$, we choose the following function.

$$p(\theta, \dot{\theta}, t) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + \rho g(\theta) \tag{6}$$

where ρ is a scalar value and $g(\theta)$ is some function of configuration such as the *measure of manipulability* [3] or a potential function that gives large values in the neighborhood of obstacles. If $M \in R^{n \times n}$ is a configuration-independent diagonal matrix, we can minimize the weighted norm of joint velocity. And if one wish to minimize kinetic energy, $M(\theta)$ must be an inertial matrix of the dynamic equation of a manipulator.

The necessary conditions (4) and (5) for the performance index (6) yield

$$M\ddot{\theta} + \dot{M}\dot{\theta} - \rho g_\theta - J^T\lambda - \frac{\partial}{\partial \theta}(\frac{1}{2}\dot{\theta}^T M \dot{\theta}) = 0, \tag{7}$$

$$\ddot{x} - \dot{J}\dot{\theta} - J\ddot{\theta} = 0 \tag{8}$$

where g_θ is the gradient of $g(\theta)$. Using equations (7) and (8), the trajectory which satisfies Euler-Lagrange equations is obtained by eliminating the Lagrangian multiplier as

$$\begin{aligned}\ddot{\theta} &= J_M^+(\ddot{x} - \dot{J}\dot{\theta}) \\ &\quad - (I - J_M^+ J)M^{-1}(\dot{M}\dot{\theta} - \frac{\partial}{\partial \theta}(\frac{1}{2}\dot{\theta}^T M \dot{\theta}) - \rho g_\theta)\end{aligned} \tag{9}$$

where $J_M^+ = M^{-1}J^T(JM^{-1}J^T)^{-1}$ is a weighted pseudoinverse of J and $I \in R^{n \times n}$ is an identity matrix.

In the Lagrangian formulation of manipulator dynamics, the behavior of system dynamics is described in terms of work and energy stored in the system using the generalized coordinates and $\dot{M}\dot{\theta} - \frac{\partial}{\partial \theta}(\frac{1}{2}\dot{\theta}^T M \dot{\theta})$ yields V_{cc} [8]. Therefore, if $M(\theta)$ in equation (6) is the inertial matrix, (9) may be represented as

$$\ddot{\theta} = J_M^+(\ddot{x} - \dot{J}\dot{\theta}) - (I - J_M^+ J)M^{-1}(V_{cc} - \rho g_\theta). \tag{10}$$

Since equation (10) is a second-order differential equation with n variables, the boundary conditions of $2n$ variables are required to uniquely specify the optimal solution. Concerning boundary conditions at the initial time, $t = t_0$, $\theta(t_0)$ and $\dot{\theta}(t_0)$ must satisfy $x(t_0) = f(\theta(t_0))$ and $\dot{x}(t_0) = J\dot{\theta}(t_0)$, respectively. Since the kinematic constraints must be satisfied to achieve a task, the self-evident boundary conditions are $x(t_0) = f(\theta(t_0))$ and $x(t_1) = f(\theta(t_1))$.

If the workspace trajectory is periodic $x(t_1) = x(t_0)$, we must seek to joint trajectories to minimize (1) subject to the periodic boundary conditions $\theta(t_1) = \theta(t_0)$ and $\dot{\theta}(t_1) = \dot{\theta}(t_0)$. These two constraints in conjunction with the Euler-Lagrange equations then determine the solution. However, it is not easy to obtain a solution according to periodic boundary conditions with $2n$ unspecified variables of $\theta(t_0)$ and $\dot{\theta}(t_0)$. Thus, we reformulate this problem as a two point boundary problem to find initial joint velocity $\dot{\theta}(t_0)$ for given initial configuration. The joint velocity is composed of two parts; one is the particular solution due to the task and the other is the homogeneous solution due to the performance criterion [6], therefore we need to find a homogeneous solution of $\dot{\theta}(t_0)$ as many dimensions as the degrees of redundancy which satisfies the periodic boundary conditions.

3. A NUMERICAL SEARCH METHOD

To compare with the examples which were demonstrated of Martin *et al.* [10], we set $M(\theta) = I$ and $g(\theta) = 0$ and assume that the degree of redundancy $(n-m)$ is equal to one. In this case, the integrand of the performance index is described by $p(\theta, \dot{\theta}, t) = \frac{1}{2}\|\dot{\theta}\|^2$, then (10) becomes

$$\ddot{\theta} = J^+(\ddot{x} - \dot{J}\dot{\theta}) \tag{11}$$

where $\ddot{\theta}$ is the solution to minimize the norm of joint velocity along the path.

To satisfy periodic boundary condition, we present numerical examples of an optimal redundancy resolution using (11) to find $\dot{\theta}(t_0)$ for given initial configuration. For redundant manipulators $(m < n)$, a general solution for $\dot{x} = J\dot{\theta}$ is

$$\dot{\theta}(t) = J^+\dot{x}(t) - \alpha(I - J^+J)y \tag{12}$$

where $J^+ = J^T(JJ^T)^{-1}$, the More-Penrose generalized pseudoinverse of J, and α is a scalar value and $y \in R^n$ is an arbitrary vector which represents some desired second criterion for the manipulator to perform [1-6].

If we denote the null projection operator $(I - J^+J)$ as P, P can be represented by N, where N is the basis of the null space of J [6,10]. Under the assumption of redundancy of one, (12) can be represented as

$$\dot{\theta} = J^+\dot{x} - \alpha N(NN^T)^{-1}N^T y. \tag{13}$$

Since the second term of the right hand side in (13) has the rank of one, it can be parameterized by a scalar value μ as

$$\alpha N(N^T N)^{-1} N^T y = \mu N, \tag{14}$$

$$\mu = \alpha (N^T N)^{-1} N^T y. \tag{15}$$

Using equations (14) and (15), (13) becomes

$$\dot{\theta} = J^+ \dot{x} - \mu N. \tag{16}$$

In (16), N is any vector whose column spans the null space of J. Indeed, in order to uniquely parameterize $\dot{\theta}(t_0)$ we have to specifically define N, not any null space vector. We choose N as the orthonormal vector Z obtained by the theorem of singular value decomposition (SVD) [6,12]. Replace N in (16) with Z, then (16) becomes

$$\dot{\theta} = J^+ \dot{x} - \mu Z \tag{17}$$

where $\|Z\| = 1$.

Therefore, the problem is to find a desired parameter μ which yields the initial velocity minimizing the performance index while satisfying periodic boundary conditions. In this case, a second-order differential equation with n variables must be uniquely specified by $2n$ initial conditions. So that if we find the parameter μ satisfying the periodic boundary conditions and minimizing the performance index, we can obtain the optimal trajectories over the entire task for given initial configuration $\theta(t_0)$. In this case, if $\theta(t_0) = \theta(t_1)$ is satisfied, then $\dot{\theta}(t_0) = \dot{\theta}(t_1)$ is preserved as long as $\dot{\theta}$ is governed by (11).

In the resolved rate control for redundant manipulators, y in (12) can be written by $\nabla H(\theta)$, then the modified control scheme becomes

$$\dot{\theta} = J^+ \dot{x} - \alpha (I - J^+ J) \nabla H(\theta) \tag{18}$$

where $H(\theta)$ is an optimization criterion to be minimized subject to the required end-effector velocity and α is a gain constant. In this paper, we use (18) iteratively at the initial time to obtain an initial joint velocity which satisfies periodic boundary conditions. Therefore, we define $H(\theta)$ as follows.

$$H(\theta) = \frac{1}{2} \|\theta(t_1) - \theta(t_0)\|^2 \tag{19}$$

where $\theta(t_0)$ is the given initial configuration and $\theta(t_1)$ is the final joint vector evaluated at $t = t_1$ after forward integration from $t = t_0$ to $t = t_1$ of (11). Our optimal problem is to find the initial joint velocity $\dot{\theta}(t_0)$ which minimizes (19) for given initial configuration $\theta(t_0)$. Then the solution fulfills the periodic boundary conditions at $t = t_1$.

The initial joint velocity to be obtained is parameterized by μ described in (17). Since we do not know the joint value at the final time $\theta(t_1)$ in advance from $\theta(t_0)$, we must have forward integration of (11). In order to fulfill periodic boundary conditions, we should use e defined as the joint error at the final time to minimize (19), where e is the gradient of $H(\theta)$ with respect to $\theta(t_1)$, such that

$$e = \theta(t_1) - \theta(t_0). \tag{20}$$

On every iteration, the initial joint velocity may be updated by the joint error. Since the degree of redundancy is one, the initial joint velocity on the first iteration is represented as $\dot{\theta}(t_0) = J^+ \dot{x}(t_0) - \mu_0 Z$, where μ_0 is the initial value of μ. And the initial joint velocity on the kth iteration is updated as follows.

$$\dot{\theta}_k(t_0) = J^+ \dot{x}(t_0) - \mu_k Z, \tag{21}$$

$$\mu_{k+1} = \mu_k + \sigma Z^T e_k \qquad \text{for } k = 0, 1, \cdots \tag{22}$$

where σ $(0 < \sigma < 1)$ is chosen to minimize (19) and e_k is the joint error defined as (20) on the kth iteration.

Once $\dot{\theta}_k(t_0)$ is obtained, it must be verified that $H(\theta_k) \le H(\theta_{k-1})$ so that $\dot{\theta}_k(t_0)$ is indeed an improvement. If the improvement is not achieved, σ must be reduced and a new $\dot{\theta}_k(t_0)$ found. In the practical sense, the joint velocity must be bounded, so the following condition should be satisfied

$$\lim_{k \to \infty} Z^T e_k = 0. \tag{23}$$

Thus, we terminate iteration if e_k resides within some specified error bound. From (23), we can find the parameterized initial velocity in terms of the obtained value μ_f which is $\dot{\theta}_f(t_0) = J^+ \dot{x}(t_0) - \mu_f Z$. We note that (23) provides two kinds of solution; one satisfies periodic boundary conditions, $e = 0$, and the other arrives at a local point which satisfies (23), but $e \neq 0$. In the latter, e is not zero but is in the null space of Z. Since μ_k in (22) is not updated, the initial joint velocity in (21) stays at a local minimum point. Thus, we can not obtain the initial velocity which satisfies periodic boundary conditions. However, this case can be avoided by appropriate choices of μ_0.

4. NUMERICAL EXAMPLES

In this section we discuss some features of optimal resolution through numerical examples for a three-link planar redundant manipulator. Consider the three-link planar manipulator in a horizontal plane shown in Fig. 1. We choose the position of the end-effector in 2-D space described in Cartesian coordinates, accordingly $x \in R^2$. The degree of redundancy at nonsingular points is equal to one. In the numerical examples, the cyclic tasks are described as

$$x = \begin{bmatrix} -R\cos(2\pi t) + C \\ -R\sin(2\pi t) \end{bmatrix}, \tag{24}$$

where R is the radius of the circle to be carried out and C is the x-axis position of the center of the circle. The task is to rotate the circle of R unit radius, centered at $(C, 0)$, in unit time, in a counterclockwise, thus the initial position is $(C-R, 0)$ and $t_0 = 0$ and $t_1 = 1$.

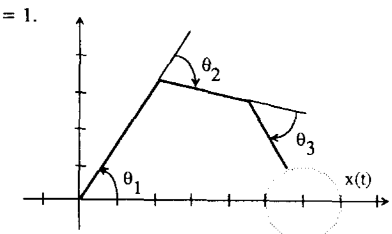

Fig. 1. Geometry of a manipulator and Task 1; $C = 0.0$, $R = 1.0$.

Nonhomotopic Extremal Solutions

Consider the task shown in Fig. 1 denoted as Task 1 where $R = 1$ and $C = 6$ units. In this task, we have initial configuration $\theta(t_0) = (0.7854, -0.8488, -1.3143)^T$ *radians*. The *measure of manipulability* can be expressed as the product of singular values in the theorem of SVD. For this initial configuration, the *measure of manipulability* denoted as M_m is 15.24 and the basis of the null projection operator $Z = (0.317, -0.644, 0.696)^T$ is obtained by the theorem of SVD.

First, we consider the norm of joint error E at the final time, which is defined as $E = \sum_{i=1}^{3} |\theta_i(t_1) - \theta_i(t_0)|$, after integration of (11) according to arbitrary initial velocity for given initial configuration. Since the volume of velocity ellipsoid is proportional to the *measure of manipulability*, we may only investigate the joint error for arbitrary value of μ which is bounded by $-M_m \le \mu \le M_m$.

Fig. 2 shows E with respect to arbitrary μ, and the performance index r, the norm of joint velocity, which is scaled down by 0.02. As shown in Fig. 2, we may find the nonhomotopic solutions around μ's such as 0, 5, 10.5, -5, and -10.5 and so on which fulfill periodic boundary conditions, but not the globally optimal solution. Therefore, we find solutions around several μ's, which are nonhomotopic and optimal in their homotopy classes, but not necessarily be the globally optimal. It is necessary that E be equal to zero, and r be minimized. So it is expected from Fig. 2 that the globally optimal trajectory can be obtained around $\mu = 0$, which yields the minimum norm solution.

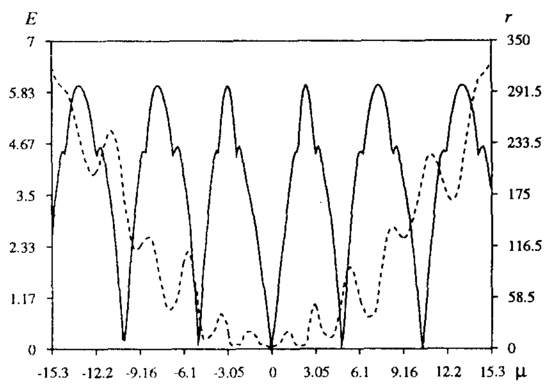

Fig. 2. Solid line E and dotted line r for Task 1 on $-M_m \le \mu \le M_m$.

Based on the inspection, the local minimum is searched by using (21) and (22) around $\mu_0 = 0$ with $\sigma = 0.5$, then it gives us that $\mu_f = -0.0345$ yields the optimal point. Optimal point means that one can obtain an initial condition $\dot{\theta}(t_0)$ which satisfies optimality conditions for given $\theta(t_0)$. If the joint trajectory generated by such $\dot{\theta}(t_0)$ and $\theta(t_0)$ has lower cost than all other locally optimal solutions, then it can be the globally optimal solution. Also if the joint velocity is searched around $\mu_0 = 5$ with $\sigma = 0.5$, then another local optimal solution which has $\mu_f = 4.88272$ will be obtained with the same initial configuration.

Fig. 3 shows, therefore, two different solutions for given initial configuration; one is denoted as solution A when $\mu_f = -0.0345$ and the other is denoted as solution B when $\mu_f = 4.88272$. We note that the homotopy classes are classified by a parameter μ which parameterizes the initial joint velocity shown in Fig. 2. In this example, we can find a solution which is the globally optimal in desirable homotopy class according to the performance index shown in Fig. 2, which is about $-3 \le \mu \le 3$.

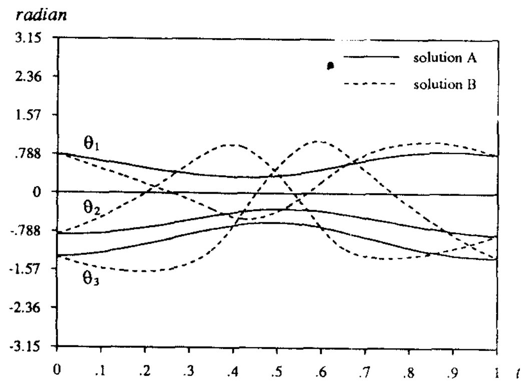

Fig. 3. Solutions for Task 1, $\theta(t_0) = (0.7854, -0.8488, -1.3143)^T$.

Fig. 4 shows $\theta_3 - \theta_2$ phase trajectories in order to compare two extremal solutions. The degree of redundancy in the three-link manipulator for accomplishing its given two-dimensional task is one, then we depict the set of joint values in $\theta_3 - \theta_2$ phase plane. The outer closed curve and inner closed curve are two sets of the inverse kinematic solutions at $t = t_0$ and $t = \frac{t_1}{2}$, respectively. Based on Fig. 4, the globally optimal solution is shorter trajectory than other locally trajectories for the same initial configuration $\theta(t_0)$. Also solution A is only in third quadrants, it means that the arm configuration does not changed. However, since solution B trace out about the inner closed curve and all quadrants, the arm configuration of θ_2 varies with respect to θ_3. Therefore, nonhomotopic extremal solutions can not be continuously transformed from one to the other.

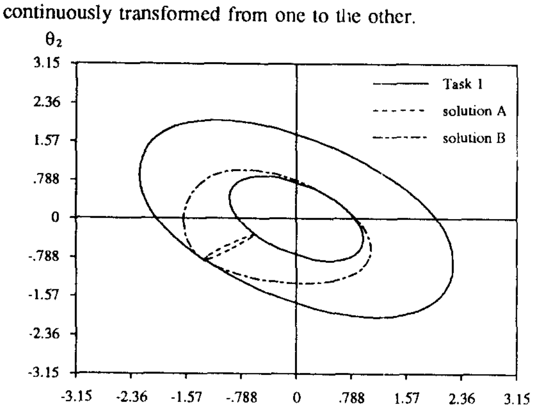

Fig. 4. $\theta_3 - \theta_2$ phase trajectories for Task 1.

If we want to carry out Task 1 at another initial configuration $\theta(t_0) = (-0.47124, 1.7875, -1.8734)^T$ *radians*, we should determine different initial joint velocity as shown in Fig. 3 for satisfying periodic boundary conditions. In this configuration, $Z = (0.4843, -0.5366, -0.6910)^T$ and $M_m = 9.85$. We note that the initial value of μ depends on the task not on initial configuration. Therefore the same initial values of μ_0 and σ are used as the case of solution A in Fig. 3. Then we can find the globally optimal solution at μ_f = -0.004345 which is shown as solution C in Fig. 5. And also another extremal solution is obtained at μ_f = -6.4765, which started the search around μ_0 = -6, denoted as solution D in Fig. 5. Solution D is equivalent to *Example 3* in Martin *et al* [10].

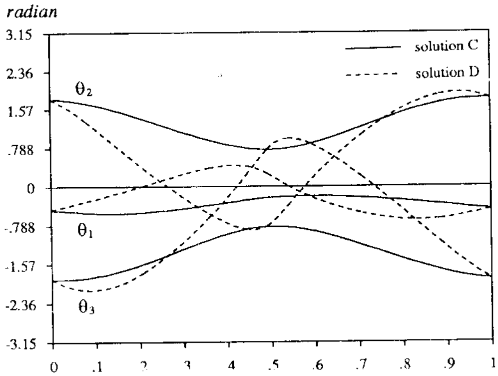

Fig. 5. Solutions for different initial configuration in Task 1, $\theta(t_0) = (-0.47124,\ 1.7875,\ -1.8734)^T$.

Topological Lifting of Extremal Solutions

The solutions from globally optimal resolution of a kinematically redundant manipulator are nonhomotopic and these solutions are classified by μ_0 for given initial configuration. Therefore, if one wish to distinguish true optimal solution among extremal solutions, the joint error at the final time and the performance index must be taken into account. Unfortunately, the true optimal trajectory among all feasible trajectories satisfying boundary conditions may be difficult to be found, because there exist multiple nonhomotopic extremal solutions even for the fixed initial configuration. In the practical sense, we suggest an algorithm to obtain a globally optimal solution in desirable homotopy class where the performance index is the norm of joint velocity for given initial configuration.

Step 1. Since the initial condition depends on the task and the performance criterion, μ_0 must be chosen according to the task. Considering the geometry of the manipulator, Task 1 is reasonable to be carried out. So, one may find μ_f which produces an optimal trajectory around minimum norm joint velocity. Therefore, we do not need to find joint errors at final time for arbitrary μ. If the circle to be traced out has large radius for example, one must increase μ_0 and then obtain an optimal solution.

Step 2. If the parameter μ_f which produces an optimal trajectory is found from μ_0 by the numerical search method, μ_0 can be used to find other μ_f which gives an optimal trajectory for other initial configuration of the same task. So, we can obtain the optimal solution for given initial configuration by using μ_0, and that is in desirable homotopy class.

As shown previous examples, solution C in Fig. 5 can be obtained by using the same parameter given in solution A in Fig. 3. This procedure can be used as a topological lifting method of trajectories for any initial configuration of the same task.

Consideration of Dynamics

So far, we have discussed the optimal redundancy resolution considering only the kinematics of a manipulator. However, when a trajectory is required to consider the dynamic response, the dynamics of a manipulator must be taken into consideration. For the optimal resolution, we may consider the kinetic energy minimization problem, such as $p(\theta, \dot{\theta}, t) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta}$, subject to the kinematic constraints, where $M(\theta)$ is the inertial matrix of the dynamic equation. For this performance index p, the necessary condition becomes (11) with ρ = 0. The inertial matrix $M(\theta)$ of which links modeled by a point mass at the distal end of each link [12].

By the numerical method discussed in Section 3, we can obtain the initial joint velocity which uniquely defines the joint trajectory for given initial configuration. For comparison, we consider the case of solution A for Task 1. The optimal solution is searched around minimum norm solution when μ_0 = 0, then we obtain a solution shown in Fig. 6 at μ_f = 0.00971. In this case the performance index given by the kinetic energy is 35.088, and that of solution A of Task 1 is 36.824. Therefore, the minimization of the norm of joint velocity vector is approximately as same as the minimization of the kinetic energy. As shown in Fig. 6, we note that the motions of joint one have moved much less than that of solution A of Task 1, then the kinetic energy is minimized with considering dynamics of the manipulator.

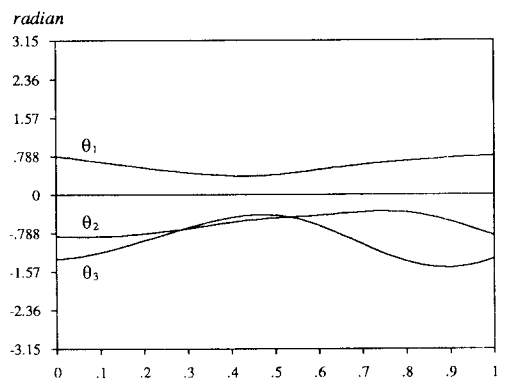

Fig. 6. Solution with consideration of dynamics of the manipulator

In order to better understand the dynamic response, we depict the motions of the manipulator in Fig. 7. In this case, minimizing the total manipulator kinetic energy integrated over the entire trajectory may lead to a uniform resolution in joint torques and a corresponding uniform increase in dynamic response. By investigating the motions of the manipulator, we can better understand the motions of the manipulator between solution A of Task 1 and the solution in Fig. 6. The motions of joint one in Fig. 7(b) have moved less than that in Fig. 7(a), therefore, the kinetic energy with the inertial matrix of which modeled by a point mass at the distal end of each link is minimized.

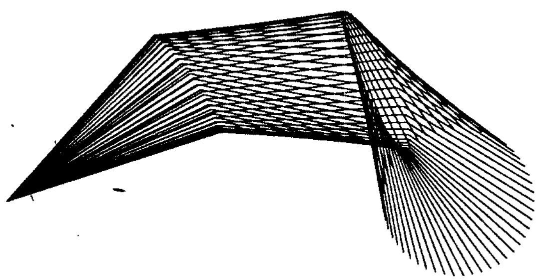

Fig. 7(a). Motions of solution A in Task 1 when the kinematics is considered only.

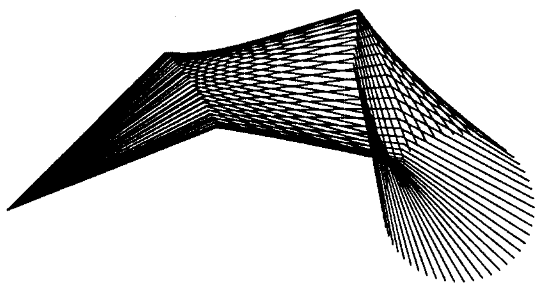

Fig. 7(b). Motions of solution in Task 1 when the dynamics is taken into account.

5. CONCLUSIONS

We have solved the problem of the optimal redundancy resolution for the integral performance criterion and obtained the necessary condition from the Euler-Lagrange equations in a second-order differential equation. In a cyclic task, we have considered periodic boundary conditions and then reformulated the periodic boundary value problem as a two point boundary value problem to find the initial joint velocity for given initial configuration.

In order to satisfy necessary condition as well as periodic boundary conditions, we presented a numerical method to find an optimal trajectory. The initial joint velocity which was obtained by using the joint error between the final and initial joint values at the final time produces an optimal solution for given initial configuration.

For a cyclic task, there exist nonhomotopic extremal solutions according to initial joint velocity for same initial configuration. Therefore, we suggested a topological lifting method of trajectories and demonstrated it. Also we graphically illustrated an optimal trajectory by considering the dynamics of a three-link planar redundant manipulator, which produces the improved dynamic response, and compared it when only kinematics was used.

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