'90 KACC 1990. 10. 26~27

POLE ASSIGNMENT FOR THREE-DIMENSIONAL SYSTEMS USING TWO-DIMENSIONAL DYNAMIC COMPENSATORS

Atsushi Kawakami

Department of Electronics, Faculty of Engineering
Kanazawa Institute of Technology, Kanazawa, Japan

ABSTRACT

In this paper, we study the pole assignment problem for three-dimensional systems. We transform the denominator of transfer functions of the closed-loop system into the product of three stable one-dimensional polynomials, by performing two-dimensional dynamical feedback and input transformation on the given three-dimensional systems.

In the next, we consider the possibility that these two-dimensional dynamic compensators are realizable, thoroughly, and propose the counter-measure in case that they are not realizable. And , we obtain the conditions so that the closed-loop three-dimensional systems are stable.

Moreover, we calculate the dynamical dimension which is necessary for the pole assignment, and suggest the pole assignment method with the lowest dynamical dimension.

1. INTRODUCTION

In recent years, three-dimensional digital signal processing attracts attention. Because, recently, computer has high speed and large memorial capacity, it can act enormous computation needed in three-dimensional digital signal processing. Therefore, interest in the design of three-dimensional systems rises [1]-[9].

In this paper, we propose a method for assigning poles of transfer functions of such three-dimensional systems to arbitrary positions.

Some studies about the assignment problem of characteristic polynomials of two-dimensional systems have been reported [10]-[15]. They are very valuable, but it is very difficult to extend them to three-dimensional system.

In the first place, we define poles of three-dimensional systems. In this paper, the purpose is, by performing feedback and input transformation containing two-dimensional dynamic compensators on the given three-dimensional systems, we transform the denominator of transfer functions of the closed-loop systems into the product of three stable one-dimensional polynomials.

In this paper, to begin with, from the given three-dimensional systems, we calculate backward "first realization system" of three-dimensional transfer functions. First realization system is one-dimensional system over the field of two-dimensional rational functions. And, we perform two-dimensional feedback and input transformation on the first realization system for the desired pole assignment. Concretely, we regard these dynamic compensators as two-dimensional transfer functions and realize them with a minimal dimension in two steps respectively, so that we obtain the feedback system and input transformation system. As the dynamic compensators of input transformation is the separable-denominator form, it can be realized with a minimal dimension with respect to both of two variables.

In the next, in order that these dynamic compensators are realizable, they have to be proper with respect to both of two variables. We consider this possibility thoroughly, and propose the counter-measure in case that they are nonproper. And, we obtain the conditions so that the closed-loop three-dimensional systems are stable.

Moreover, we calculate the dynamical dimension which is necessary for the realization of these dynamic compensators, and suggest the pole assignment method with the lowest dynamical dimension.

Pole assignment problem for three-dimensional systems can be applied in case that distributed-parameter system over two-dimensional plane can be modelled by three-dimensional systems. As we define poles of three-dimensional systems as the above-mentioned ones, it is important in the stabilization or design of dynamical characteristics.

2. DESCRIPTION OF PROBLEM

We deal with the following three-dimensional systems of Roesser's type.

$$\begin{cases} \begin{bmatrix} x^h(i+1,j,k) \\ x^v(i,j+1,k) \\ x^w(i,j,k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x^h(i,j,k) \\ x^v(i,j,k) \\ x^w(i,j,k) \end{bmatrix} + [b_1^T \; b_2^T \; b_3^T]^T u(i,j,k) \\ y(i,j,k) = [C_1 \; C_2 \; C_3] \begin{bmatrix} x^h(i,j,k) \\ x^v(i,j,k) \\ x^w(i,j,k) \end{bmatrix} + d\,u(i,j,k) \end{cases} \quad (1)$$

where $x^h(i,j,k)$ is n_1th horizontal state vector $x^v(i,j,k)$ is n_2th vertical state vector, $x^w(i,j,k)$ is n_3th additional state vector, $u(i,j,k)$ is input, $y(i,j,k)$ is output and A_{11}, A_{12} --- are constant matrices.

If denominator polynomial of transfer functions can be expressed by

$$\hat{a}(z_1,z_2,z_3)=\prod_{i=1}^{n_1}(z_1-z_{1i})\prod_{j=1}^{n_2}(z_2-z_{2j})\prod_{k=1}^{n_3}(z_3-z_{3k}), \quad (2)$$

its poles are given by

$$\Lambda \triangleq \{(z_{1i},z_{2j},z_{3k})\,|\,(0,0,0)<(i,j,k)\leq(n_1,n_2,n_3)\}$$
$$(z_{10}=z_{20}=z_{30}=0) \quad (3)$$

where

$(i,j,k)\leq(l,m,n)$ iff $i\leq l,\ j\leq m,\ k\leq n$

$(i,j,k)=(l,m,n)$ iff $i=l,\ j=m,\ k=n$

$(i,j,k)<(l,m,n)$ iff $(i,j,k)\leq(l,m,n)$ and $(i,j,k)\neq(l,m,n)$

The purpose of this paper is, by performing the state feedback and input transformation containing two-dimensional dynamic compensators on (1), we transform the poles of the closed-loop systems into Λ designated arbitrarily in the unit circle.

We express $\hat{a}(z_1,z_2,z_3)$ in (2) by

$$\hat{a}(z_1,z_2,z_3)=\left(\sum_{i=0}^{n_1-1}\hat{a}_{1i}z_1^i+z_1^{n_1}\right)\cdot\left(\sum_{j=0}^{n_2-1}\hat{a}_{2j}z_2^j+z_2^{n_2}\right)\cdot\left(\sum_{k=0}^{n_3-1}\hat{a}_{3k}z_3^k+z_3^{n_3}\right) \quad (4)$$

$$\equiv\hat{a}_1(z_1)\cdot\hat{a}_2(z_2)\cdot\hat{a}_3(z_3) \quad . \quad (5)$$

3. A POLE ASSIGNMENT METHOD

We express transfer functions of the system in (1) by

$$F(z_1,z_2,z_3)=\frac{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3}b_{ijk}z_1^iz_2^jz_3^k}{\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}\sum_{k=0}^{n_3}a_{ijk}z_1^iz_2^jz_3^k} \quad (6)$$

$$\equiv\frac{b(z_1,z_2,z_3)}{a(z_1,z_2,z_3)} \quad (7)$$

where

$$a_{n_1n_2n_3}=1 \quad . \quad (8)$$

(I) Calculation of First Realization Systems

From the system in (1), we calculate backward

$$\begin{bmatrix}\mathbf{x}^R(i+1,z_2,z_3)\\ y(i,z_2,z_3)\end{bmatrix}=\begin{bmatrix}\mathbb{A}(z_2,z_3) & \mathbb{b}(z_2,z_3)\\ \mathbb{C}(z_2,z_3) & d(z_2,z_3)\end{bmatrix}\begin{bmatrix}\mathbf{x}^R(i,z_2,z_3)\\ u(i,z_2,z_3)\end{bmatrix} \quad (9)$$

We call the following system ((9) is rewritten)

$$\begin{cases}\mathbf{x}^R(i+1,z_2,z_3)=\mathbb{A}(z_2,z_3)\mathbf{x}^R(i,z_2,z_3)+\mathbb{b}(z_2,z_3)u(i,z_2,z_3)\\ y(i,z_2,z_3)=\mathbb{C}(z_2,z_3)\mathbf{x}^R(i,z_2,z_3)+d(z_2,z_3)u(i,z_2,z_3)\end{cases} \quad (10)$$

first realization system of $F(z_1,z_2,z_3)$ in (6).

(II) Calculation of Two-Dimensional Dynamic Compensators

In the system in (10), if $(\mathbb{A}(z_2,z_3),\mathbb{b}(z_2,z_3))$ is controllable over $\mathbb{R}^{n_1}(z_2,z_3)$, by performing the state feedback of $\mathbf{x}^R(i,z_2,z_3)$, whose coefficients are rational functions of z_2,z_3, characteristic polynomial of the closed-loop system can be designated to arbitrary one.

It is proved by the pole assignment theory in one-dimensional systems [16].

We perform

$$u(i,z_2,z_3)=\mathbb{f}(z_2,z_3)\mathbf{x}^R(i,z_2,z_3)+g(z_2,z_3)v(i,z_2,z_3) \quad (11)$$

on (10), so that characteristic polynomial of (11) is

$$\det\{z_1\mathbb{I}_{n_1}-\mathbb{A}(z_2,z_3)-\mathbb{b}(z_2,z_3)\mathbb{f}(z_2,z_3)\} \quad . \quad (12)$$

Now, assuming that $(\mathbb{A}(z_2,z_3),\mathbb{b}(z_2,z_3))$ is controllable over $\mathbb{R}^{n_1}(z_2,z_3)$ in (10), by performing (11), we let (12) coincide with $\hat{a}_1(z_1)$ in (5).

$\mathbb{f}(z_2,z_3)$ can be obtained by the design formula of the feedback in one-dimensional system [16], that is to say,

$$\mathbb{f}(z_2,z_3)=\left[\frac{a_{n_1-1}(z_2,z_3)}{a_{n_1}(z_2,z_3)}-\hat{a}_{1(n_1-1)}\ \cdots\ \frac{a_0(z_2,z_3)}{a_{n_1}(z_2,z_3)}-\hat{a}_{10}\right]\cdot\begin{bmatrix}1 & a_{n_1-1}(z_2,z_3)/a_{n_1}(z_2,z_3) & \cdots & a_1(z_2,z_3)/a_{n_1}(z_2,z_3)\\ 0 & 1 & \ddots & \vdots\\ \vdots & & \ddots & a_{n_1-1}(z_2,z_3)/a_{n_1}(z_2,z_3)\\ 0 & \mathbb{0} & & 1\end{bmatrix}^{-1}\cdot\left[\mathbb{b}(z_2,z_3),\ \mathbb{A}(z_2,z_3)\mathbb{b}(z_2,z_3),\ \cdots,\ \mathbb{A}^{n_1-1}(z_2,z_3)\mathbb{b}(z_2,z_3)\right]^{-1} \quad , \quad (13)$$

$g(z_2,z_3)$ is

$$g(z_2,z_3)=\frac{a_{n_1}(z_2,z_3)}{\hat{a}_2(z_2)\hat{a}_3(z_3)} \quad . \quad (14)$$

In (10), if $(\mathbb{A}(z_2,z_3),\mathbb{b}(z_2,z_3))$ is not controllable over $\mathbb{R}^{n_1}(z_2,z_3)$, we perform the state feedback of $\mathbf{x}^U(z_1,j,z_3)$ or $\mathbf{x}^W(z_1,z_2,k)$.

(III) Realization of Two-Dimensional Dynamic Compensators

Concretely, we regard $\mathbb{f}(z_2,z_3)$ in (11) as the transfer function row vector and regard one of z_2,z_3 as the main variable, and realize it with respect to the main variable.

Then, we calculate least common multiple of denominators of each entry of $\mathbb{f}(z_2,z_3)$ in (13), and realize $\mathbb{f}(z_2,z_3)$ with respect to the variable whose degree is higher.

Now, it is assumed that it is n_2'th about z_2, n_3'th about z_3.

Then, assuming that $n_2'>n_3'$, we realize $\mathbb{f}(z_2,z_3)$ by an observable companion form with respect to z_2. It is a minimal realization with respect to z_2.

$$\begin{cases}\hat{\mathbf{x}}^U(i,j+1,z_3)=\hat{\mathbb{A}}_f(z_3)\hat{\mathbf{x}}^U(i,j,z_3)+\hat{\mathbb{B}}_f(z_3)\mathbf{x}^R(i,j,z_3)\\ u(i,j,z_3)=\hat{\mathbb{C}}_f\hat{\mathbf{x}}^U(i,j,z_3)+\hat{d}_f(z_3)\mathbf{x}^R(i,j,z_3)\end{cases} \quad (15)$$

And, we express (15) by

$$\begin{bmatrix}\hat{\mathbf{x}}^U(i,j+1,z_3)\\ u(i,j,z_3)\end{bmatrix}=\begin{bmatrix}\hat{\mathbb{A}}_f(z_3) & \hat{\mathbb{B}}_f(z_3)\\ \hat{\mathbb{C}}_f & \hat{d}_f(z_3)\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}^U(i,j,z_3)\\ \mathbf{x}^R(i,j,z_3)\end{bmatrix} \quad (16)$$

and realize (16) with a minimal dimension.

$$\begin{cases}\hat{\mathbf{x}}^W(i,j,k+1)=\tilde{\mathbb{A}}_f\hat{\mathbf{x}}^W(i,j,k)+[\tilde{\mathbb{B}}_{f1}\ \tilde{\mathbb{B}}_{f2}]\begin{bmatrix}\hat{\mathbf{x}}^U(i,j,k)\\ \mathbf{x}^R(i,j,k)\end{bmatrix}\\ \begin{bmatrix}\hat{\mathbf{x}}^U(i,j+1,k)\\ u(i,j,k)\end{bmatrix}=\begin{bmatrix}\tilde{\mathbb{C}}_{f1}\\ \tilde{\mathbb{C}}_{f2}\end{bmatrix}\hat{\mathbf{x}}^W(i,j,k)+\begin{bmatrix}\tilde{\mathbb{D}}_{f11} & \tilde{\mathbb{D}}_{f12}\\ \tilde{d}_{f21} & \tilde{d}_{f22}\end{bmatrix}\cdot\left[\hat{\mathbf{x}}^U(i,j,k)^T\ \mathbf{x}^R(i,j,k)^T\right]^T\end{cases} \quad (17)$$

In the next, we realize $g(z_2,z_3)$.

In case of $g(z_2,z_3)$ in (14), as its denominator is separable, it can be realized with a minimal dimension with respect to both of two variables [17]-[21].

By realizing $g(z_2,z_3)$ in (14) by the method in [21], the following canonical form is obtained.

$$\begin{cases} \begin{bmatrix} \tilde{x}^v(i,j+1,k) \\ \tilde{x}^w(i,j,k+1) \end{bmatrix} = \begin{bmatrix} A_{g11} & 0 \\ A_{g21} & A_{g22} \end{bmatrix} \begin{bmatrix} \tilde{x}^v(i,j,k) \\ \tilde{x}^w(i,j,k) \end{bmatrix} + \begin{bmatrix} b_{g1} \\ b_{g2} \end{bmatrix} v(i,j,k) \\ u(i,j,k) = [C_{g1}\ C_{g2}] \begin{bmatrix} \tilde{x}^v(i,j,k) \\ \tilde{x}^w(i,j,k) \end{bmatrix} + d_g v(i,j,k) \end{cases} \tag{18}$$

Combining (1)-(17),(18), the system which has the desired poles ((3)) is obtained.

From the obtained system, we calculate backward three-dimensional transfer function.

$$F(z_1,z_2,z_3) = \frac{b(z_1,z_2,z_3)\hat{f}(z_2,z_3)f'_{n_2}(z_3)\hat{a}_2(z_2)\hat{a}_3(z_3)}{\hat{a}_1(z_1)\hat{a}_2(z_2)^2\hat{a}_3(z_3)^2\hat{f}(z_2,z_3)f'_{n_2}(z_3)} \tag{19}$$

$\hat{f}(z_2,z_3)$ in (19) is least common multiple of denominators of each entry of $F(z_2,z_3)$ in (11).

$f_{n_2'}(z_3)$ in (19) is coefficient polynomial of $z_2^{n_2'}$ in $\hat{f}(z_2,z_3)$.

Because, in the system in (17),(18),

$$\det\begin{bmatrix} z_2 I_{n_2'} - \hat{D}_{f11} & -\hat{C}_{f1} \\ -\hat{B}_{f1} & z_3 I_{n_3''} - \hat{A}_f \end{bmatrix} = \hat{f}(z_2,z_3) f_{n_2'}(z_3) \tag{20}$$

$$\det\begin{bmatrix} z_2 I_{n_2} - A_{g11} & 0 \\ -A_{g21} & z_3 I_{n_3} - A_{g22} \end{bmatrix} = \hat{a}_2(z_2)\hat{a}_3(z_3) . \tag{21}$$

n_3'' in (20) is the dimension of $\hat{x}^w(i,j,k)$ in (17).

4. EXISTENCE OF TWO-DIMENSIONAL DYNAMIC COMPENSATORS

In order that $F(z_2,z_3)$, $g(z_2,z_3)$ are realizable, its each entry has to be proper with respect to both of z_2 and z_3.

Practically, nonproper rational functions happen very rarely, in entries of $F(z_2,z_3)$ in (13).

In case that nonproper rational functions happen in entries of $F(z_2,z_3)$ in (13), by performing

$$u(i,j,k) = f_R x^R(i,j,k) + v(i,j,k) , \tag{22}$$

we let every row and every column of $A_{11}+b_1 F_R$ contain at least one nonzero entry.

In that case, from the closed-loop system, we calculate backward

$$\begin{bmatrix} x^R(i+1,z_2,z_3) \\ y(i,z_2,z_3) \end{bmatrix} = \begin{bmatrix} \hat{A}(z_2,z_3) & \hat{b}(z_2,z_3) \\ \hat{C}(z_2,z_3) & \hat{d}(z_2,z_3) \end{bmatrix} \begin{bmatrix} x^R(i,z_2,z_3) \\ v(i,z_2,z_3) \end{bmatrix} \tag{23}$$

Every row and every column of $\hat{A}(z_2,z_3)$ in (23) contain at least one rational function whose denominator and numerator are the same degree about both z_2 and z_3.

Then, if b_1 in (1) contains at least one nonzero entry, $\hat{b}(z_2,z_3)$ in (23) contains at least one rational function whose denominator and numerator are the same degree about both of z_2 and z_3.

In this case, $F(z_2,z_3)$ in (13) almost becomes a proper rational function row vector about both of z_2 and z_3.

Still more, nonproper rational functions happen in entries of $F(z_2,z_3)$ in (13), by changing one entry of F_R in (22) a little, it becomes a proper rational function row vector.

In (1), in case that one or all entries of b_1 are 0, that's why nonproper rational functions certainly happen in entries of $F(z_2,z_3)$ in (13), then, we must perform the state feedback of $x^v(z_1,j,z_3)$ or $x^w(z_1,z_2,k)$.

In order that $F(z_2,z_3)$ in (11) is realizable, each entry of transfer function matrix in (16) has to be a proper rational function, too.

If coefficient polynomial of $z_2^{n_2}$ of common denominator of each entry of $F(z_2,z_3)$ in (11) is n_3'th, (16) is a proper rational function matrix.

In case that nonproper rational functions happen in entries of (16), by performing (22), coefficient polynomial of $z_2^{n_2}$ of common denominator of each entry of $F(z_2,z_3)$ in (11) becomes n_3' th.

In the next, $g(z_2,z_3)$ in (14) is always proper rational function about both of z_2 and z_3.

And, we realize $g(z_2,z_3)$ in (14) by a controllable companion form with respect to z_2,

$$\begin{cases} \tilde{x}^v(i,j+1,z_3) = A_g \tilde{x}^v(i,j,z_3) + b_g v(i,j,z_3) \\ u(i,j,z_3) = C_g(z_3)\tilde{x}^v(i,j,z_3) + d_g(z_3) v(i,j,z_3) \end{cases} \tag{24}$$

We express (24) by

$$\begin{bmatrix} \tilde{x}^v(i,j+1,z_3) \\ u(i,j,z_3) \end{bmatrix} = \begin{bmatrix} A_g & b_g \\ C_g(z_3) & d_g(z_3) \end{bmatrix} \begin{bmatrix} \tilde{x}^v(i,j,z_3) \\ v(i,j,z_3) \end{bmatrix} . \tag{25}$$

(25) is a proper rational function matrix.

In order that the obtained three-dimensional system is stable, common denominator polynomial of each entry of $F(z_2,z_3)$ in (11) and its coefficient polynomial of $z_2^{n_2}$ have to be stable.

5. DYNAMICAL DIMENSION FOR POLE ASSIGNMENT

McMillan degree of (16) is

$$2n_3' \cdot \min\{n_2'+n_1, n_2'+1\} = 2n_3'(n_2'+1) \tag{26}$$

at the highest.

Therefore, total dimension which is necessary for the realization of $F(z_2,z_3)$ in (11) is

$$n_2' + 2n_3'(n_2'+1) \tag{27}$$

at the highest.

In $F(z_2,z_3)$ in (11), we regard z_3 as the main variable and realize it by an observable companion form with respect to z_3 formerly. Then, total dimension which is necessary for the realization is

$$n_3' + 2n_2'(n_3'+1) \tag{28}$$

at the highest

We subtract (28) from (27)

$$\{n_2'+2n_3'(n_2'+1)\} - \{n_3'+2n_2'(n_3'+1)\} = n_3' - n_2' , \tag{29}$$

if $n_2' > n_3'$, (28)<0. Then, realizing with respect to z_2 formerly, upper bound of total dimension is fewer.

That is to say, realizing with respect to the higher variable in least common multiple of denominator polynomial of each entry of $F(z_2,z_3)$ in (11) formerly, upper bound of realization dimension is fewer.

n_2', n_3' are

$$n_2' \leq n_1^2 n_2 \qquad n_3' \leq n_1^2 n_3 . \tag{30}$$

Therefore, upper bound of total dimension

which is necessary for the realization of $f(z_2,z_3)$ in (11) is

(in case that $n_2>n_3$)

$$n_1^2 n_2 + 2 n_1^2 n_3 (n_1^2 n_2 + 1) \tag{31}$$

(in case that $n_2<n_3$)

$$n_1^2 n_3 + 2 n_1^2 n_2 (n_1^2 n_3 + 1) \quad . \tag{32}$$

And, as $g(z_2,z_3)$ in (14) can be realized with a minimal dimension with respect to both of z_2 and z_3, realization dimension is

$$n_2 + n_3 \quad . \tag{33}$$

That is to say, dynamical dimension which is necessary for pole assignment is

(in case that $n_2>n_3$)

$$n_1^2 (n_2 + 2 n_1^2 n_2 n_3 + 2 n_3) + n_2 + n_3 \tag{34}$$

(in case that $n_2<n_3$)

$$n_1^2 (n_3 + 2 n_1^2 n_2 n_3 + 2 n_2) + n_2 + n_3 \quad . \tag{35}$$

By substituting the given n_1, n_2, n_3 into (34) or (35), pole assignment method with the lowest dynamical dimension can be found.

6. CONCLUSION

In this paper, we studied a pole assignment problem for three-dimensional systems by the feedback and input transformation containing two-dimensional dynamic compensators.

In the next, we considered the possibility that these two-dimensional dynamic compensators are realizable, thoroughly, and proposed the counter-measure in case that they are not realizable. And, we obtained the conditions so that the closed-loop three-dimensional systems are stable.

Moreover, we calculated the dynamical dimension which is necessary for the pole assignment and suggested the pole assignment method with the lowest dynamical dimension.

REFERENCES

[1] K.Hirano, M.Sakane and M.Mulk, "Design of three-dimensional recursive digital filters," IEEE Trans. Circuits & Syst., CAS-31, 6, pp.550-561, 1984.

[2] H.E.Mutluay and M.M.Fahmy, "Frequency-domain design of N-D digital filters," IEEE Trans. Circuits & Syst., CAS-32, pp.1226-1233, 1985.

[3] J.K.Pitas and A.N.Venetsanopoulos, "The use of symmetries in the design of multidimensional digital filters," IEEE Trans. Circuits & Syst., CAS-33, pp.863-873, 1986.

[4] L.T.Bruton and N.R.Bartley, "The design of highly selective adaptive three-dimensional recursive cone filters," IEEE Trans. Circuits & Syst., CAS-34, 7, pp.775-781, 1987.

[5] M.E.Zervakis and A.N.Venetsanopoulos, "Design of three-dimensional digital filters using two-dimensional rotated filters," IEEE Trans. Circuits & Syst., CAS-34, 12, pp.1452-1469, 1987.

[6] T.Hinamoto, A.Nakatsuji and S.Maekawa, "Design of three-dimensional recursive digital filters with symmetry," Trans. IEICE of Japan, 71-A, 6, pp.1234-1241, 1988.

[7] A.Kawakami, "A realization method of three-dimensional digital filters," Trans. IEICE of Japan, 72-A, 2, pp.331-340, 1989.

[8] M.Ooki, M.Kawamata and T.Higuchi, "Design of three-dimensional digital filters based on the outer product expansion of impulse response specifications," Trans. IEICE of Japan, 72-A, 4, pp.648-655, 1989.

[9] T.Sekiguchi and S.Takahashi, "Design of three-dimensional IIR digital filters with complex coefficients using spectral transformations," Trans. IEICE of Japan, 72-A, 10, pp.1528-1537, 1989.

[10] P.N.Paraskevopoulos and O.I.Kosmidou, "Eigenvalue assignment of two-dimensional systems using PID controllers," Int. J. Systems Sci., 12, 4, pp.407-422, 1981.

[11] T.Kaczorek, "Pole assignment problem in two-dimensional linear systems," Int. J. Control, 37, 1, pp.183-190, 1983.

[12] W.S.Lu and E.B.Lee, "Stability analysis for two-dimensional systems," IEEE Trans. Circuits & Syst., CAS-30, 7, pp.455-461, 1983.

[13] P.N.Paraskevopoulos, "Eigenvalue assignment of linear multivariable 2-dimensional systems," Proc. IEE, 126, 11, pp.1204-1208, 1983.

[14] T.Hinamoto, J.Shimonishi and S.Maekawa, "Pole assignment for two-dimensional systems using a multifunctional observer," Trans. IEICE of Japan, 67-A, 12, pp.1209-1216, 1984.

[15] J.Shimonishi, T.Hinamoto and S.Maekawa, "2-D exact model matching using 1-D dynamic compensators," Trans. SICE of Japan, 22, 2, pp.136-142, 1986.

[16] R.W.Bass and I.Gura, "Higher-order system design via state space considerations, "JACC, pp.311-318, 1965.

[17] S.Y.Kung, B.C.Levy, M.Morf and T.Kailath, "New results in 2-D systems theory, part II: 2-D state-space models——realization and the notions of controllability, observability, and minimality," Proc. IEEE, 65, 6, pp.945-961, 1977.

[18] E.D.Sontag, "On first order equations for multidimensional filters," IEEE Trans. Acoust., Speech & Signal Process., ASSP-26, 5, pp.480-482, 1978.

[19] T.Hinamoto, K.Sano and S.Maekawa, "Structure analysis for a class of two-dimensional systems and two-dimensional realization problems," Trans. IEICE of Japan, 62-A, 2, pp.143-150, 1979.

[20] A.Kawakami, J.Tajima and S.Takahashi, "Approximation and construction of the network containing variable elements," Trans. IEICE of Japan, 64-A, 4, pp.277-284, 1981.

[21] N.Haratani and S.Takahashi, "Minimally realizable two-dimensional transfer functions," Trans. IEICE of Japan, 65-A, 1, pp.54-61, 1982.