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DECOUPLING OF MULTI-INPUT MULTI-OUTPUT TWO-DIMENSIONAL SYSTEMS

Atsushi Kawakami

Department of Electronics, Faculty of Engineering
Kanazawa Institute of Technology, Kanazawa, Japan

ABSTRACT

In this paper, we propose a method to decouple the multi-input multi-output two-dimensional system. Then, we analyze the realization dimension of the feedback, feedforward given to decouple. Moreover, we consider the possibility of the reduction of the dynamical dimension needed to decouple. Besides, in order to stabilize the decoupled two-dimensional system, we suggest a method to assign the poles of each entry of the transfer function matrix to the desired positions.

1. INTRODUCTION

The decoupling of the system is to let the transfer function matrix of the multi-input multi-output system be the invertible diagonal matrix by giving the feedback. That is to say, it is to let the inputs and outputs correspond one to one. This decoupling is the practical theory which was proposed in order to make easily to control the tarboprop, boiler and many researches have been done since 1960's [1]-[3].

The consideration about the decoupling problem of the multivariable control system with time delays was reported, too [4].

In this paper, we propose a method to decouple the general multi-input multi-output two-dimensional system, which include the system in the special form like the system with time delays. We regard the one-dimensional transfer function matrix calculated backward from the given two-dimensional system as the variable system. Then, this variable system is the realization of the two-dimensional transfer function matrix which is the outer description of the given two-dimensional system, with respect to one variable. In the stage of this variable system, we obtain the dynamical feedback, feedforward given to decouple. Then, we analyze the realization dimension of the dynamical feedback, feedforward.

Moreover, we consider the possibility of the reduction of the dynamical dimension needed to decouple, by using the freedom in the coefficients matrix of the dynamical feedback and the similar transformation of the variable system.

Besides, we suggest a method to assign the poles of each entry of the transfer function matrix to the desired positions, in order to stabilize the decoupled two-dimensional system.

Methods proposed in our paper can be applied to the decoupling of two-dimensional digital system, delay-differential system, the system of mixed lumped and distributed networks, the networks containing variable elements, etc.

2. A DECOUPLING METHOD OF THE TWO-DIMENSIONAL SYSTEM

We decouple the two-dimensional system in the next equation.

$$\begin{cases} \begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \mathbb{Z}_{2j} \end{bmatrix} = \left[\begin{array}{c|c} \mathbb{A}_{11} & \mathbb{A}_{12} \\ \hline \mathbb{A}_{21} & \mathbb{A}_{22} \end{array}\right] \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{V}_{2j} \end{bmatrix} + \begin{bmatrix} \mathbb{B}_1 \\ \hline \mathbb{B}_2 \end{bmatrix} \mathbb{U} \\ \mathbb{Y} = [\mathbb{C}_1 \mid \mathbb{C}_2][\mathbb{V}_{1i}^T \mid \mathbb{V}_{2j}^T]^T + \mathbb{D}\,\mathbb{U} \end{cases} \tag{1}$$

$$\left(\begin{array}{l} \mathbb{Z}_{1i} = [\mathbb{Z}_{11}\ \mathbb{Z}_{12} \cdots \mathbb{Z}_{1(n_1p)}]^T,\ \mathbb{V}_{1i} = [\mathbb{V}_{11}\ \mathbb{V}_{12} \cdots \mathbb{V}_{1(n_1p)}]^T \\ \mathbb{Z}_{1i} = s_1 \mathbb{V}_{1i} \quad (i = 1, 2, \cdots, n_1 p) \\ \mathbb{Z}_{2j} = [\mathbb{Z}_{21}\ \mathbb{Z}_{22} \cdots \mathbb{Z}_{2n_2'}]^T,\ \mathbb{V}_{2j} = [\mathbb{V}_{21}\ \mathbb{V}_{22} \cdots \mathbb{V}_{2n_2'}]^T \\ \mathbb{Z}_{2j} = s_2 \mathbb{V}_{2j} \quad (j = 1, 2, \cdots, n_2') \\ \mathbb{Y} = [y_1\ y_2 \cdots y_p]^T,\ \mathbb{U} = [u_1\ u_2 \cdots u_p]^T \end{array}\right)$$

(1) has the outer description described by the transfer function matrix as in the next equation.

$$\frac{1}{\sum_{k=0}^{n_1} f_k(s_2) s_1^k} \begin{bmatrix} \sum_{k=0}^{n_1} g_{11}^{(k)}(s_2) s_1^k & \cdots & \sum_{k=0}^{n_1} g_{1p}^{(k)}(s_2) s_1^k \\ \vdots & & \vdots \\ \sum_{k=0}^{n_1} g_{p1}^{(k)}(s_2) s_1^k & \cdots & \sum_{k=0}^{n_1} g_{pp}^{(k)}(s_2) s_1^k \end{bmatrix} \tag{2}$$

where $f_k(s_2)$, $g_{ij}^{(k)}(s_2)$ $(i,j=1,2,\cdots,p \quad k=0,1,\cdots,n_1)$ are n_2 degree polynomial.

In case of (1), $n_2 p \le n_2' \le 2n_2 p$.

We rewrite (1) as in the next equation.

$$\begin{cases} \mathbb{Z}_{2j} = \mathbb{A}_{22}\mathbb{V}_{2j} + [\mathbb{A}_{21} \mid \mathbb{B}_2][\mathbb{V}_{1i}^T \mid \mathbb{U}^T]^T \\ \begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \mathbb{Y} \end{bmatrix} = \begin{bmatrix} \mathbb{A}_{12} \\ \hline \mathbb{C}_2 \end{bmatrix} \mathbb{V}_{2j} + \left[\begin{array}{c|c} \mathbb{A}_{11} & \mathbb{B}_1 \\ \hline \mathbb{C}_1 & \mathbb{D} \end{array}\right] \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{U} \end{bmatrix} \end{cases} \tag{3}$$

From (3), we calculate the transfer function matrix as in the next equation.

$$\begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \mathbb{Y} \end{bmatrix} = \left\{ \begin{bmatrix} \mathbb{A}_{12} \\ \hline \mathbb{C}_2 \end{bmatrix} (s_2 \mathbb{I}_{n_2'} - \mathbb{A}_{22})^{-1} [\mathbb{A}_{21} \mid \mathbb{B}_2] + \left[\begin{array}{c|c} \mathbb{A}_{11} & \mathbb{B}_1 \\ \hline \mathbb{C}_1 & \mathbb{D} \end{array}\right] \right\} \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{U} \end{bmatrix} \tag{4}$$

$$\equiv \begin{bmatrix} g_{11}(s_2)/f_{11}(s_2) & \cdots & g_{1p}(s_2)/f_{1p}(s_2) \\ \vdots & & \vdots \\ g_{p1}(s_2)/f_{p1}(s_2) & \cdots & g_{pp}(s_2)/f_{pp}(s_2) \end{bmatrix} \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{U} \end{bmatrix} \tag{5}$$

We describe (5) as in the next equation.

$$\begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \mathbb{Y} \end{bmatrix} = \left[\begin{array}{c|c} \mathbb{A}(s_2) & \mathbb{B}(s_2) \\ \hline \mathbb{C}(s_2) & \mathbb{D}(s_2) \end{array}\right] \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{U} \end{bmatrix} \tag{6}$$

We regard (6) as the variable system in the next equation.

$$\begin{cases} \mathbb{Z}_{1i} = \mathbb{A}(s_2)\mathbb{V}_{1i} + \mathbb{B}(s_2)\mathbb{U} \\ \mathbb{Y} = \mathbb{C}(s_2)\mathbb{V}_{1i} + \mathbb{D}(s_2)\mathbb{U} \end{cases} \tag{7}$$

From (7), we calculate the two-dimensional transfer function matrix

$$\mathbb{C}(s_2)\{s_1\mathbb{I}_{(n_1p)} - \mathbb{A}(s_2)\}^{-1}\mathbb{B}(s_2) + \mathbb{D}(s_2) . \tag{8}$$

(8) is equal to (2).

Consequently, it can be understood that (7) is a realization of (2) with respect to s_1.

We call (7) the first realization and call (1) the second realization.

To begin with, in case that (7) is the realization by the controllable companion form, each system matrix is

$$\mathbb{A}(s_2) = \begin{bmatrix} \mathbb{O} & \mathbb{I}_{(n_1-1)p} \\ -\frac{f_0(s_2)}{f_{n_1}(s_2)}\mathbb{I}_p & \cdots & -\frac{f_{n_1-1}(s_2)}{f_{n_1}(s_2)}\mathbb{I}_p \end{bmatrix} \quad \mathbb{B} = \begin{bmatrix} \mathbb{O} \\ \mathbb{I}_p \end{bmatrix}$$

$$\mathbb{C}(s_2) = \frac{1}{f_{n_1}^2(s_2)} \begin{bmatrix} h_{11}^{(0)}(s_2) & \cdots & h_{1p}^{(0)}(s_2) & \cdots & h_{11}^{(n_1-1)}(s_2) & \cdots & h_{1p}^{(n_1-1)}(s_2) \\ \vdots & & \vdots & & \vdots & & \vdots \\ h_{p1}^{(0)}(s_2) & \cdots & h_{pp}^{(0)}(s_2) & \cdots & h_{p1}^{(n_1-1)}(s_2) & \cdots & h_{pp}^{(n_1-1)}(s_2) \end{bmatrix}$$

$$(h_{ij}^{(k)}(s_2) = g_{ij}^{(k)}(s_2) f_{n_1}(s_2) - g_{ij}^{(n_1)}(s_2) f_k(s_2)$$

$$(i,j = 1,2,\cdots,p \quad k = 0,1,\cdots,n_1-1)$$

$$\mathbb{D}(s_2) = \frac{1}{f_{n_1}(s_2)} \begin{bmatrix} g_{11}^{(n_1)}(s_2) & \cdots & g_{1p}^{(n_1)}(s_2) \\ \vdots & & \vdots \\ g_{p1}^{(n_1)}(s_2) & \cdots & g_{pp}^{(n_1)}(s_2) \end{bmatrix} . \tag{9}$$

We consider the sufficient condition so that (1) can be decoupled.

It is necessary that (7) can be decoupled so that (1) can be decoupled.

In the case of (7), the decoupling indices in the next equation

$$d_i = \begin{cases} \min\{j : \mathbb{C}_i(s_2)\mathbb{A}^j(s_2)\mathbb{B} \neq \mathbb{O} \quad j = 0,1,\cdots,n_1p-1 \\ n_1p-1 : \mathbb{C}_i(s_2)\mathbb{A}^j(s_2)\mathbb{B} = \mathbb{O} \quad \forall j \end{cases} \tag{10}$$

are all 0, unless that among $f_k(s_2)$, $g_{ij}^{(k)}(s_2)$ $(i,j = 1,2,\cdots,p \quad k=0,1,\cdots,n_1)$ in (2), very special relation exist.

We restrict ourselves the system, in which

$$d_1 = d_2 = \cdots\cdots = d_p = 0 \tag{11}$$

is satisfied.

In this case,

(Condition 1)

$$[h_{i1}^{(n_1-1)}(s_2)\ h_{i2}^{(n_1-1)}(s_2) \cdots h_{ip}^{(n_1-1)}(s_2)] \quad (i = 1,\cdots,p) \tag{12}$$

are independent.

is the sufficient condition so that (1) can be decoupled.

From now on, we decouple the system which satisfies (Condition 1).

We give the dynamical feedback

$$\mathbb{U} = \mathbb{F}(s_2)\mathbb{V}_{1i} + \mathbb{G}(s_2)\mathbb{V} \tag{13}$$

to (1). In (13),

$$\mathbb{G}(s_2) = \begin{bmatrix} \mathbb{C}_1(s_2)\mathbb{A}^{d_1}(s_2)\mathbb{B} \\ \vdots \\ \mathbb{C}_p(s_2)\mathbb{A}^{d_p}(s_2)\mathbb{B} \end{bmatrix}^{-1} \tag{14}$$

$$\equiv \frac{f_{n_1}^2(s_2)}{\hat{f}(s_2)} \begin{bmatrix} \hat{h}_{11}(s_2) & \cdots & \hat{h}_{1p}(s_2) \\ \vdots & & \vdots \\ \hat{h}_{p1}(s_2) & \cdots & \hat{h}_{pp}(s_2) \end{bmatrix} \tag{15}$$

$$\mathbb{F}(s_2) = -\mathbb{G}(s_2)\mathbb{C}(s_2)\mathbb{A}(s_2) \tag{16}$$

$$\equiv \frac{1}{\hat{f}(s_2)\cdot f_{n_1}(s_2)} \begin{bmatrix} g_{11}(s_2) & \cdots & g_{1(n_1p)}(s_2) \\ \vdots & & \vdots \\ g_{p1}(s_2) & \cdots & g_{p(n_1p)}(s_2) \end{bmatrix} . \tag{17}$$

We give the dynamical feedforward

$$\mathbb{Y} = \mathbb{Z} + \mathbb{P}(s_2)\mathbb{V}_{1i} + \mathbb{H}(s_2)\mathbb{V} \tag{18}$$

to (1). In (18),

$$\mathbb{P}(s_2) = \mathbb{D}(s_2)\mathbb{F}(s_2) \tag{19}$$

$$\equiv \frac{1}{\hat{f}(s_2)\cdot f_{n_1}^2(s_2)} \begin{bmatrix} \hat{g}_{11}(s_2) & \cdots & \hat{g}_{1(n_1p)}(s_2) \\ \vdots & & \vdots \\ \hat{g}_{p1}(s_2) & \cdots & \hat{g}_{p(n_1p)}(s_2) \end{bmatrix} \tag{20}$$

$$\mathbb{H}(s_2) = \mathbb{D}(s_2)\mathbb{G}(s_2) \tag{21}$$

$$\equiv \frac{f_{n_1}(s_2)}{\hat{f}(s_2)} \begin{bmatrix} \tilde{g}_{11}(s_2) & \cdots & \tilde{g}_{1p}(s_2) \\ \vdots & & \vdots \\ \tilde{g}_{p1}(s_2) & \cdots & \tilde{g}_{pp}(s_2) \end{bmatrix} . \tag{22}$$

Concretely, we rewrite (13)

$$\mathbb{U} = [\mathbb{F}(s_2) \vdots \mathbb{G}(s_2)][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \tag{23}$$

and realize (23).

In $\mathbb{G}(s_2)$ in (15), $\hat{f}(s_2)$ is at most $2n_2p$ degree polynomial. Numerator is at most $2n_2p$ degree polynomial. Therefore, $\mathbb{G}(s_2)$ is a proper rational function matrix.

In the next, $\mathbb{F}(s_2)$ in (17) is proper. Its denominator is at most $n_2(2p+1)$ degree polynomial.

Consequently, (23) is a proper transfer function matrix and its McMillan degree is at most $n_2(2p+1)p$.

We realize (23) with the minimal dimension.

$$\begin{cases} \hat{\mathbb{Z}}_{2j} = \hat{\mathbb{A}}\hat{\mathbb{V}}_{2j} + [\hat{\mathbb{B}}_1 \vdots \hat{\mathbb{B}}_2][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \\ \mathbb{U} = \hat{\mathbb{C}}\hat{\mathbb{V}}_{2j} + [\hat{\mathbb{D}}_1 \vdots \hat{\mathbb{D}}_2][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \end{cases} \tag{24}$$

In the next, we rewrite (18).

$$\mathbb{Y} - \mathbb{Z} = [\mathbb{P}(s_2) \vdots \mathbb{H}(s_2)][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \tag{25}$$

$\mathbb{P}(s_2)$ in (20), $\mathbb{H}(s_2)$ in (22) are proper, therefore, (25) is proper.

Then, the McMillan degree of (25) is at most $2n_2(p+1)p$ degree.

We realize (25) with the minimal dimension.

$$\begin{cases} \tilde{\mathbb{Z}}_{2j} = \tilde{\mathbb{A}}\tilde{\mathbb{V}}_{2j} + [\tilde{\mathbb{B}}_1 \vdots \tilde{\mathbb{B}}_2][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \\ \mathbb{Y} - \mathbb{Z} = \tilde{\mathbb{C}}\tilde{\mathbb{V}}_{2j} + [\tilde{\mathbb{D}}_1 \vdots \tilde{\mathbb{D}}_2][\mathbb{V}_{1i}^T \vdots \mathbb{V}^T]^T \end{cases} \tag{26}$$

By giving (24) as the feedback, (26) as the feedforward, to (1), the system in the next equation is obtained.

$$\begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \hat{\mathbb{Z}}_{2j} \\ \hline \tilde{\mathbb{Z}}_{2j} \end{bmatrix} = \left[\begin{array}{c|ccc} \mathbb{A}_{11} + \mathbb{B}_1\hat{\mathbb{D}}_1 & \mathbb{A}_{12} & \mathbb{B}_1\hat{\mathbb{C}} & \mathbb{O} \\ \hline \mathbb{A}_{21} + \mathbb{B}_2\hat{\mathbb{D}}_1 & \mathbb{A}_{22} & \mathbb{B}_2\hat{\mathbb{C}} & \mathbb{O} \\ \hline \hat{\mathbb{B}}_1 & \mathbb{O} & \hat{\mathbb{A}} & \mathbb{O} \\ \hat{\mathbb{B}}_2 & \mathbb{O} & \mathbb{O} & \tilde{\mathbb{A}} \end{array}\right] \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \mathbb{V}_{2j} \\ \hline \hat{\mathbb{V}}_{2j} \\ \hline \tilde{\mathbb{V}}_{2j} \end{bmatrix}$$

$$\begin{cases} +\left[(\mathbb{B}_1\hat{\mathbb{D}}_2)^T \,\vdots\, (\mathbb{B}_2\hat{\mathbb{D}}_2)^T \quad \hat{\mathbb{B}}_2^T \quad \tilde{\mathbb{B}}_2^T\right]^T \\ \mathbb{Z} = \left[\mathbb{C}_1 + \mathbb{D}\hat{\mathbb{D}}_1 - \tilde{\mathbb{D}}_1 \,\vdots\, \mathbb{C}_2 \quad \mathbb{D}\tilde{\mathbb{C}} \quad -\tilde{\mathbb{C}}\right]^T \\ \quad \cdot \left[\mathbb{V}_{1i}^T \,\vdots\, \mathbb{V}_{2j}^T \,\vdots\, \hat{\mathbb{V}}_{2j}^T \,\vdots\, \tilde{\mathbb{V}}_{2j}^T\right]^T + (\mathbb{D}\hat{\mathbb{D}}_2 - \tilde{\mathbb{D}}_2)\mathbb{V} \end{cases} \tag{27}$$

We express (27) as in the next equation.

$$\begin{cases} \begin{bmatrix} \mathbb{Z}_{1i} \\ \hline \mathbb{Z}_{2j} \end{bmatrix} = \begin{bmatrix} \bar{\mathbb{A}}_{11} & \bar{\mathbb{A}}_{12} \\ \hline \bar{\mathbb{A}}_{21} & \bar{\mathbb{A}}_{22} \end{bmatrix} \begin{bmatrix} \mathbb{V}_{1i} \\ \hline \bar{\mathbb{V}}_{2j} \end{bmatrix} + \begin{bmatrix} \bar{\mathbb{B}}_1 \\ \hline \bar{\mathbb{B}}_2 \end{bmatrix} \mathbb{V} \\ \mathbb{Z} = \left[\bar{\mathbb{C}}_1 \,\vdots\, \bar{\mathbb{C}}_2\right]\left[\mathbb{V}_{1i}^T \,\vdots\, \bar{\mathbb{V}}_{2j}^T\right]^T + \bar{\mathbb{D}}\mathbb{V} \end{cases} \tag{28}$$

From (28), we calculate the transfer function like

$$\left[\bar{\mathbb{C}}_1 \,\vdots\, \bar{\mathbb{C}}_2\right] \begin{bmatrix} s_1\mathbb{I}_{n_1p} - \bar{\mathbb{A}}_{11} & -\bar{\mathbb{A}}_{12} \\ \hline -\bar{\mathbb{A}}_{21} & s_2\mathbb{I}_{\{n_2p(4p+3)+n_2'\}} - \bar{\mathbb{A}}_{22} \end{bmatrix}^{-1} \begin{bmatrix} \bar{\mathbb{B}}_1 \\ \hline \bar{\mathbb{B}}_2 \end{bmatrix} + \bar{\mathbb{D}} \tag{29}$$

The next equation is obtained.

$$\begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} = \begin{bmatrix} 1/s_1 & & \mathbb{O} \\ & \ddots & \\ \mathbb{O} & & 1/s_1 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \tag{30}$$

In the next, we consider the case that (7) is a realization by the observable companion form.

In this case, each system matrix in (7) is

$$\mathbb{A}(s_2) = \begin{bmatrix} \mathbb{O} & \{-f_0(s_2)/f_{n_1}(s_2)\}\mathbb{I}_p \\ \mathbb{I}_{(n_1-1)p} & \{-f_{(n_1-1)}(s_2)/f_{n_1}(s_2)\}\mathbb{I}_p \end{bmatrix}$$

$$\mathbb{B}(s_2) = \frac{1}{f_{n_1}^2(s_2)} \begin{bmatrix} r_{11}^{(0)}(s_2) & \cdots & r_{1p}^{(0)}(s_2) \\ \vdots & & \vdots \\ r_{p1}^{(0)}(s_2) & \cdots & r_{pp}^{(0)}(s_2) \\ \vdots & & \vdots \\ r_{11}^{(n_1-1)}(s_2) & \cdots & r_{1p}^{(n_1-1)}(s_2) \\ \vdots & & \vdots \\ r_{p1}^{(n_1-1)}(s_2) & \cdots & r_{pp}^{(n_1-1)}(s_2) \end{bmatrix}, \quad \mathbb{C} = \left[\mathbb{O} \,\vdots\, \mathbb{I}_p\right] \tag{31}$$

$$\mathbb{D}(s_2) = \frac{1}{f_{n_1}(s_2)} \begin{bmatrix} g_{11}^{(n_1)}(s_2) & \cdots & g_{1p}^{(n_1)}(s_2) \\ \vdots & & \vdots \\ g_{p1}^{(n_1)}(s_2) & \cdots & g_{pp}^{(n_1)}(s_2) \end{bmatrix}.$$

In this case, (Condition 1) is a sufficient condition so that (1) can be decoupled, too.

We decouple the system which satisfies (Condition 1), too.

$\mathbb{G}(s_2)$ in (13) is the same with (15).

$\mathbb{F}(s_2)$ in (13) is

$$\mathbb{F}(s_2) = -\mathbb{G}(s_2)\mathbb{C}\mathbb{A}(s_2) \tag{32}$$

$$\equiv \frac{f_{n_1}(s_2)}{\hat{f}(s_2)} \begin{bmatrix} r_{11}(s_2) & \cdots & r_{1(n_1p)}(s_2) \\ \vdots & & \vdots \\ r_{p1}(s_2) & \cdots & r_{p(n_1p)}(s_2) \end{bmatrix}. \tag{33}$$

$\mathbb{P}(s_2)$ in (18) is

$$\mathbb{P}(s_2) = \mathbb{D}(s_2)\mathbb{F}(s_2) \tag{34}$$

$$\equiv \frac{1}{\hat{f}(s_2)} \begin{bmatrix} \hat{r}_{11}(s_2) & \cdots & \hat{r}_{1(n_1p)}(s_2) \\ \vdots & & \vdots \\ \hat{r}_{p1}(s_2) & \cdots & \hat{r}_{p(n_1p)}(s_2) \end{bmatrix}. \tag{35}$$

$\mathbb{H}(s_2)$ in (18) is the same with (22).

Consequently, (23) is proper and its McMillan degree is at most $2n_2p^2$.

(25) is proper, too and its McMillan degree is at most $2n_2p^2$.

From now on, we decouple alike the case that (7) is a realization by the controllable companion form.

3. REDUCTION OF THE DIMENSION NEEDED TO DECOUPLE

We consider the system whose decoupling indices of the first realization are all 0.

In the first place, we consider the possibility of the reduction of the dimension needed to decouple, compared with the case that the first realization is the controllable companion form.

We give the similar transformation to (7) by the nonsingular matrix $\mathbb{T}(s_2)$

$$\begin{cases} \hat{\mathbb{Z}}_{1i} = \mathbb{T}^{-1}(s_2)\mathbb{A}(s_2)\mathbb{T}(s_2)\hat{\mathbb{V}}_{1i} + \mathbb{T}^{-1}(s_2)\mathbb{B}\mathbb{U} \\ \mathbb{Y} = \mathbb{C}(s_2)\mathbb{T}(s_2)\hat{\mathbb{V}}_{1i} + \mathbb{D}(s_2)\mathbb{U} \end{cases} \tag{36}$$

$$(\hat{\mathbb{Z}}_{1i} = [\hat{z}_{11}\ \hat{z}_{12} \cdots \hat{z}_{1(n_1p)}]^T, \hat{\mathbb{V}}_{1i} = [\hat{v}_{11}\ \hat{v}_{12} \cdots \hat{v}_{1(n_1p)}]^T$$

$$\hat{z}_{1i} = s_1\hat{v}_{1i} \quad (i = 1, 2, \cdots, n_1p)$$

We decouple the two-dimensional system whose first realization is (36).

If (7) can be decoupled, (36) can be decoupled, too.

$\mathbb{G}(s_2)$ in (13) can be expressed

$$\mathbb{G}(s_2) = (\mathbb{I}4) \times \mathbb{\Lambda}(s_2) \tag{37}$$

generally. $\mathbb{\Lambda}(s_2)$ in (37) is an arbitrary diagonal matrix.

If (11) is satisfied, (37) can be expressed by

$$\mathbb{G}(s_2) = \frac{f_{n_1}^2(s_2)}{\hat{f}(s_2)} \begin{bmatrix} \hat{r}_{11}(s_2) & \cdots & \hat{r}_{1p}(s_2) \\ \vdots & & \vdots \\ \hat{r}_{p1}(s_2) & \cdots & \hat{r}_{pp}(s_2) \end{bmatrix}. \tag{38}$$

Then, we designate

$$\mathbb{\Lambda}(s_2) = \{\delta(s_2)/f_{n_1}^2(s_2)\}\mathbb{I}_p \tag{39}$$

where $\delta(s_2)$ is a divisor of $\hat{f}(s_2)$ in (15).

We substitute (39) into (37),

$$\mathbb{G}(s_2) = \frac{1}{\hat{\delta}(s_2)} \begin{bmatrix} \hat{r}_{11}(s_2) & \cdots & \hat{r}_{1p}(s_2) \\ \vdots & & \vdots \\ \hat{r}_{p1}(s_2) & \cdots & \hat{r}_{pp}(s_2) \end{bmatrix} \tag{40}$$

where $\hat{\delta}(s_2)$ is $\hat{f}(s_2)/\delta(s_2)$.

$\delta(s_2)$ is supposed to be n_3 degree polynomial, then $\mathbb{G}(s_2)$ is at most $(2n_2p-n_3)$ degree.

In case that $n_3 \leq n_2$, we designate

$$\mathbb{\Lambda}(s_2) = \{\delta(s_2)/f_{n_1}(s_2)\}\mathbb{I}_p, \tag{41}$$

then, $\mathbb{G}(s_2)$ is

$$\mathbb{G}(s_2) = \frac{f_{n_1}(s_2)}{\hat{\delta}(s_2)} \begin{bmatrix} \hat{r}_{11}(s_2) & \cdots & \hat{r}_{1p}(s_2) \\ \vdots & & \vdots \\ \hat{r}_{p1}(s_2) & \cdots & \hat{r}_{pp}(s_2) \end{bmatrix} \tag{42}$$

and it is proper.

$\mathbb{F}(s_2)$ in (13) is

$$\mathbb{F}(s_2) = -(\mathbb{I}4) \times \mathbb{C}(s_2)\mathbb{T}(s_2)\mathbb{T}^{-1}(s_2)\mathbb{A}(s_2)\mathbb{T}(s_2) \tag{43}$$

$$= -(\mathbb{I}4) \times \mathbb{C}(s_2)\mathbb{A}(s_2)\mathbb{T}(s_2). \tag{44}$$

We put as in the next equation

$$-(\mathbb{I}4) \times \mathbb{C}(s_2)\mathbb{A}(s_2) \equiv [1/\{\hat{f}(s_2) \cdot f_{n_1}(s_2)\}]\hat{\mathbb{F}}(s_2) \tag{45}$$

and $\hat{\mathbb{F}}(s_2)$ is supposed to be decomposed into

$$\hat{\mathbb{F}}(s_2) = \hat{\mathbb{F}}_1(s_2) \cdot \hat{\mathbb{F}}_2(s_2) \tag{46}$$

where $\hat{\mathbb{F}}_1(s_2)$ is $p \times n_1p$ at most $(2n_2p-n_3)$ degree polynomial matrix and $\hat{\mathbb{F}}_2(s_2)$ is $n_1p \times n_1p$ at most (n_2+n_3) degree polynomial matrix.

We let $\mathbb{T}(s_2)$ in (36) be

$$\mathbb{T}(s_2) = \delta(s_2) \cdot f_{n_1}(s_2) \cdot \hat{\mathbb{F}}_2^{-1}(s_2), \tag{47}$$

then, $\mathbb{F}(s_2)$ is

$$\mathbb{F}(s_2)=\{1/\hat{\delta}(s_2)\}\hat{\mathbb{F}}_1(s_2) \tag{48}$$

and it is proper.

(23) in this case is proper and its McMillan degree is at most $(2n_2p-n_3)p$. It is reduced by $(n_2+n_3)p$.

As $\mathbb{P}(s_2)$ in (18) is the product of $\mathbb{D}(s_2)$ and $\mathbb{F}(s_2)$, $\mathbb{P}(s_2)$ is proper.

And, as $\mathbb{H}(s_2)$ in (18) is the product of $\mathbb{D}(s_2)$ and $\mathbb{G}(s_2)$, $\mathbb{H}(s_2)$ is proper, too.

Consequently, (25) in this case is proper and its McMillan degree is at most $(2n_2p-n_3+n_2)p$. It is reduced by $(n_2+n_3)p$, too.

In the next, we analyze the possibility of reduction of the dynamical dimension needed to decouple, compared with the two-dimensional system whose first realization is the observable companion form.

In this case, we decouple the two-dimensional system whose first realization is the system given the similar transformation by $\mathbb{T}(s_2)$, too.

$\mathbb{G}(s_2)$ in (13) is (40) or (42).

As $\mathbb{F}(s_2)$ in (13) is expressed by

$$\mathbb{F}(s_2)=-\begin{bmatrix}\mathbb{C}_1\mathbb{A}^{d_1}(s_2)\mathbb{B}(s_2)\\ \vdots\\ \mathbb{C}_p\mathbb{A}^{d_p}(s_2)\mathbb{B}(s_2)\end{bmatrix}^{-1}\mathbb{C}\mathbb{A}(s_2)\mathbb{T}(s_2) \tag{49}$$

$$=\{f_{n_1}(s_2)/\hat{f}(s_2)\}\hat{\mathbb{F}}(s_2)\mathbb{T}(s_2)\ . \tag{50}$$

($\hat{\mathbb{F}}(s_2)$ is at most $n_2(2p-1)$ degree polynomial matrix)

$\hat{\mathbb{F}}(s_2)$ is supposed to be decomposed into

$$\hat{\mathbb{F}}(s_2)=\hat{\mathbb{F}}_1(s_2)\cdot\hat{\mathbb{F}}_2(s_2) \tag{51}$$

($\hat{\mathbb{F}}_1(s_2)$ is $p\times n_1p$ at most $(2n_2p-n_3)$ degree polynomial matrix and $\hat{\mathbb{F}}_2(s_2)$ is $n_1p\times n_1p$ at most (n_3-n_2) degree polynomial)

and we let $\mathbb{T}(s_2)$ in (50) be

$$\mathbb{T}(s_2)=\{\delta(s_2)/f_{n_1}(s_2)\}\hat{\mathbb{F}}_2^{-1}(s_2)\ , \tag{52}$$

then, $\mathbb{F}(s_2)$ is

$$\mathbb{F}(s_2)=\{1/\hat{\delta}(s_2)\}\hat{\mathbb{F}}_1(s_2)\ . \tag{53}$$

And, $\hat{\mathbb{F}}(s_2)$ in (50) is supposed to be decomposed into

$$\hat{\mathbb{F}}(s_2)=\bar{\mathbb{F}}_1(s_2)\cdot\bar{\mathbb{F}}_2(s_2) \tag{54}$$

($\bar{\mathbb{F}}_1(s_2)$ is $p\times n_1p$ at most $(2n_2p-n_2-n_3)$ degree polynomial matrix and $\bar{\mathbb{F}}_2(s_2)$ is $n_1p\times n_1p$ n_3 degree polynomial matrix)

and we let $\mathbb{T}(s_2)$ in (50) be

$$\mathbb{T}(s_2)=\delta(s_2)\bar{\mathbb{F}}_2^{-1}(s_2)\ , \tag{55}$$

then,

$$\mathbb{F}(s_2)=\{f_{n_1}(s_2)/\hat{\delta}(s_2)\}\bar{\mathbb{F}}_1(s_2) \tag{56}$$

and $\mathbb{F}(s_2)$ is proper.

In this case, (23) is proper and its McMillan degree is at most $(2n_2p-n_3)p$. It is reduced by n_3p.

As $\mathbb{P}(s_2)$ in (18) is the product of $\mathbb{D}(s_2)$ and $\mathbb{F}(s_2)$, and $\mathbb{H}(s_2)$ in (18) is the product of $\mathbb{D}(s_2)$ and $\mathbb{G}(s_2)$, both of $\mathbb{P}(s_2)$ and $\mathbb{H}(s_2)$ are proper.

Consequently, (26) is proper and its McMillan degree is at most $(2n_2p-n_3)p$. It is reduced by n_3p.

But, for any n_3, it is not necessary that decomposition of $\hat{\mathbb{F}}(s_2)$ in (46), that of $\hat{\mathbb{F}}(s_2)$ in (51),(54) exist.

4. POLE ASSIGNMENT PROBLEM OF THE DECOUPLED SYSTEM

Here, we consider the system whose decoupling indices of the first realization are all 0, too.

We let the transfer function matrix of the desired decoupled two-dimensional system be

$$\tilde{\mathbb{F}}(s_1,s_2)=\begin{bmatrix}1/(\alpha_1+s_1) & & \mathbb{O}\\ & \ddots & \\ \mathbb{O} & & 1/(\alpha_p+s_1)\end{bmatrix}\ . \tag{57}$$

In this case, $\mathbb{G}(s_2)$ in (13) is obtained by (15).

We designate $\mathbb{F}(s_2)$ as in the next equation.

$$\mathbb{F}(s_2)=-\mathbb{G}(s_2)\begin{bmatrix}\mathbb{C}_1(s_2)\{\alpha_1\mathbb{I}_{n_1p}+\mathbb{A}(s_2)\}\\ \vdots\\ \mathbb{C}_p(s_2)\{\alpha_p\mathbb{I}_{n_1p}+\mathbb{A}(s_2)\}\end{bmatrix} \tag{58}$$

$\mathbb{F}(s_2)$ in (58) is expressed by

$$\mathbb{F}(s_2)=-\mathbb{G}(s_2)\left\{\begin{bmatrix}\alpha_1 & & \mathbb{O}\\ & \ddots & \\ \mathbb{O} & & \alpha_p\end{bmatrix}\begin{bmatrix}\mathbb{C}_1(s_2)\\ \vdots\\ \mathbb{C}_p(s_2)\end{bmatrix}+\begin{bmatrix}\mathbb{C}_1(s_2)\mathbb{A}(s_2)\\ \vdots\\ \mathbb{C}_p(s_2)\mathbb{A}(s_2)\end{bmatrix}\right\} \tag{59}$$

$$=-\mathbb{G}(s_2)[\mathrm{diag}\{\alpha_1\cdots\alpha_p\}\mathbb{C}(s_2)+\mathbb{C}(s_2)\mathbb{A}(s_2)]\ . \tag{60}$$

Consequently, in this case, (23),(25) are proper and their McMillan degree are same with those in 3.

Therefore, in case that the denominator of each entry of the transfer function matrix of the decoupled system are designated as polynomials of one variable s_1, dimension needed to decouple is same with that in 3, 4.

If the denominator of each entry of the transfer function matrix of the decoupled system is the two-dimensional polynomial like

$$\alpha_i(s_2)+s_1\quad(i=1,2,\cdots,p)\ , \tag{61}$$

in case that s_2 is the dynamical variable, (23), (25) can not be realized. In case that s_2 is the statical variable, we give the variable transformation in the next equation

$$s_2=\hat{s}_2^{-1} \tag{62}$$

to (23),(25) and realize them with respect to $\hat{s}_2$.

5. CONCLUSION

In this paper, we proposed a method for decoupling the multi-input multi-output two-dimensional system and analyzed the realization dimension of the dynamical feedback, feedforward given. Moreover, we considered the possibility of the reduction of the dynamical dimension needed to decouple. Besides, in order to stabilize the decoupled two-dimensional system, we suggested a method for assigning the poles of each entry of its transfer function matrix to the desired positions.

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