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Least Square Simulation and Hierarchical Optimal Control of Distributed Parameter Systems

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Abstract

This paper presents a method for the optimal control of the distributed parameter systems(DPSs) by a hierarchical computational procedure. Approximate lumped parameter systems (LPSs) are derived by using the Galerkin method employing the Legendre polynomials as the basis functions. The DPSs however,are transformed into the large scale LPSs. And thus, the hierarchical control scheme is introduced to determine the optimal control inputs for the obtained LPSs. In addition, an approach to block pulse functions is applied to solve the optimal control problems of the obtained LPSs.
The proposed method is simple and efficient in computation for the optimal control of DPSs.

1. Introduction

One of the main difficulties one meets in the optimal control of DPSs lies obviously in the size of the problem, in particular for numerical computations.
A natural idea is therefore to use, among other things, asymptotic methods, in oder to simplyify the sistuation. This idea has been used extensively for the optimal control of system governed ordinary differential equations(LPSs).
The approximation methods that are founded most often in the literature for the implementation of linear distributed parameter controllers include finite differencing, eigenfunction expansion, orthogonal collcation, and Galerkin's method[1-6].

This paper presents a method for the optimal control of the DPSs by a hierarchical computational procedure. Approximate LPSs are derived by using the Galerkin method employing the shifted Legendre polynomials as the basis functions. Such approximation involves expanding the solution of the partial differential equation as a set of shifted Legendre polynomials. Then it is required to determine the number of terms that must be retained in the series approximation to obtain a satisfactory solution. Increasing the number of terms generally increases solution accuracy and always increases solution computation load and system dimension. Therefore the DPSs are transformed into the large scale LPSs. And thus, the hierarchical control scheme is introduced to determine the optimal control inputs for the obtained LPSs. An approach to block pulse functions is applied to solve the optimal control problems of the obtained LPSs.

The orthogonal functions have been widely applied to control theory. The particular orthogonal functions used up to now are the block pulse functions(BPFs), the Walsh functions, shifted Legendre polynomials and etc[2,7-9].
The main feature of the method of using orthogonal functions is that it reduces the calculus of certain differential equations to a set linear algebraic equations through the use of the well-known operation matrix for integration via orthogonal functions.

2. Least Square Approximate Systems of DPS

Consider a general linear DPSs modelled by

$$\frac{\partial}{\partial t}\mathbf{x}(y,t)=\mathcal{D}\mathbf{x}(y,t)+\mathbf{u}(y,t) \tag{1}$$

with initial conditions,

$$\mathbf{x}(y,0)=\mathbf{x}_0(y) \tag{2}$$

and boundary conditions,

$$\beta\,\mathbf{x}(y,t)=0 \tag{3}$$

where $\mathcal{D}$ is an nxn matrix of linear time-invariant partial differential operators.

The problem is to find the control variables $\mathbf{u}(y,t)$ which minimize the following cost function, J,

$$J=\frac{1}{2}\int_0^{t_f}\int_0^{y_f}\mathbf{x}^T(y,t)\mathbf{Q}\mathbf{x}(y,t)+\mathbf{u}^T(y,t)\mathbf{R}\mathbf{u}(y,t)dydt \quad (4)$$

By using the shifted Legendre polynomial functions, we reduce the optimal control problem of a DPSs to form of the linear regulator problem.

Functions $\mathbf{x}(y,t)$ and $\mathbf{u}(y,t)$ which are absolute integrable in $0\le y\le y_f$ for any time $0\le t\le t_f$, can be represented by a finite series of the shifted Legendre polynomial series, $\psi_i(y)$,

$$\hat{\mathbf{x}}(y,t)=\sum_{i=0}^{N-1}\boldsymbol{\alpha}_i(t)(2i+1)\psi_i(y) \quad (5)$$

$$\hat{\mathbf{u}}(y,t)=\sum_{i=0}^{N-1}\boldsymbol{\gamma}_i(t)(2i+1)\psi_i(y) \quad (6)$$

where $\boldsymbol{\alpha}_i$ and $\boldsymbol{\gamma}_i$ are the expansion coeffitients of the shifted Legendre polynomial functions.

Using in eq.(1) the approximations of eq.(5) and eq.(6) yields the error.

$$\mathbf{e}(y,t)=\frac{\partial}{\partial t}\hat{\mathbf{x}}(y,t)-\mathcal{D}\hat{\mathbf{x}}(y,t)+\hat{\mathbf{u}}(y,t) \quad (7)$$

The Galerkin approach[2,9] is to choose $\boldsymbol{\alpha}_i(t)$ such that to orthogonalize the error with respect to the priori chosen basis function $\psi_i(y)$, i.e, such that

$$\int_0^{y_f}\mathbf{e}(y,t)\psi_i(y)dy=0 \quad (8)$$

Take the differential operator as

$$\mathcal{D}=\mathbf{A}_1\frac{\partial}{\partial y}+\mathbf{A}_0$$

Introducing eq.(5) and (6) in eq.(7), and carring out the integration in eq.(8), yields

$$\frac{d}{dt}\boldsymbol{\alpha}_i(t)=\sum_{j=0}^{N-1}f_{ij}\mathbf{A}_1\boldsymbol{\alpha}_j(t)+\mathbf{A}_0\boldsymbol{\alpha}_i(t)+\boldsymbol{\gamma}_i(t) \quad (9)$$

where $f_{ij}=-\frac{(2j+1)}{y_f}\int_0^{y_f}\psi_j(y)\frac{d}{dy}\psi_i(y)dy$

In matrix form, eq.(9) is written as

$$\frac{d}{dt}\mathbf{x}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{u}(t) \quad (10)$$

where, $\mathbf{A}=-\mathbf{A}_1\otimes\mathbf{F}+\mathbf{A}_0\otimes\mathbf{I}$, $\mathbf{F}=[f_{ij}]$

$$\mathbf{x}(t)=\begin{bmatrix}\boldsymbol{\alpha}_0(t)\\ \boldsymbol{\alpha}_1(t)\\ \vdots\\ \boldsymbol{\alpha}_{N-1}(t)\end{bmatrix},\quad \mathbf{u}(t)=\begin{bmatrix}\boldsymbol{\gamma}_0(t)\\ \boldsymbol{\gamma}_1(t)\\ \vdots\\ \boldsymbol{\gamma}_{N-1}(t)\end{bmatrix}$$

and the symbol $\otimes$ denotes the Kronecker product[10].

Similarly, in the case of the differntial operator as

$$\mathcal{D}=a_2\frac{\partial^2}{\partial y^2}+a_1\frac{\partial}{\partial y}+a_0$$

from eq.(5)-(8) we can obtain the following equation

$$\frac{d}{dt}\mathbf{x}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{u}(t) \quad (11)$$

$$\mathbf{A}=\left[-\frac{(2j+1)}{y_f}a_2g_{ij}+a_1f_{ij}\right]+a_0\mathbf{I}$$

$$g_{ij}=\int_0^{y_f}\frac{d}{dy}\psi_j(y)\cdot\frac{d}{dy}\psi_i(y)dy$$

The cost function, as shown in eq.(4) becomes

$$J=\frac{1}{2}\int_0^{t_f}\mathbf{x}^T(t)\mathbf{Q}'\mathbf{x}(t)+\mathbf{u}^T(t)\mathbf{R}'\mathbf{u}(t)dt \quad (12)$$

where $\mathbf{Q}'=\mathbf{Q}\otimes\mathbf{H}$, $\mathbf{R}'=\mathbf{R}\otimes\mathbf{H}$, $\mathbf{H}=[h_{ij}]$

$$h_{ij}=\begin{cases}0 & i\neq j\\ y_f(2i+1) & i=j\end{cases}$$

A partial differential eqation of DPS is, thus, transformed into a set of ordinary differential eqation of LPS. More leading terms of α_i(large value of N) will lead to the computational results more accurately. However, DPSs are transformed into the large scale LPSs. And thus, the hierarchical control scheme is introduced in optimal control of the obtained LPSs.

3. Hierarchical Optimal Control Scheme of DPS.

Dynamic optimization for an interconnected system can be expressed as

$$\min J=\frac{1}{2}\sum_{i=1}^{n}\int_0^{t_f}\mathbf{x}_i^T(t)\mathbf{Q}_i\mathbf{x}_i(t)+\mathbf{u}_i^T(t)\mathbf{R}_i\mathbf{u}_i(t)dt \quad (13)$$

where $\mathbf{Q}_i$ and $\mathbf{R}_i$ are positive semidefinite and positive definite respectively, and subject to

$$\frac{d}{dt}\mathbf{x}_i(t)=\mathbf{A}_i\mathbf{x}_i(t)+\mathbf{B}_i\mathbf{u}_i(t)+\mathbf{C}_i\mathbf{z}_i(t) \quad (14)$$

$$\mathbf{z}_i(t)=\sum_{i=1}^{n}\mathbf{L}_{ij}\mathbf{x}_j(t) \quad (15)$$

where $\mathbf{z}_i$ denotes the interconnection, the Lagrangian L can be witten as

$$L=\sum_{i=1}^{n}L_i=\sum_{i=1}^{n}\int_0^{t_f}[1/2\,\mathbf{x}_i^T(t)\mathbf{Q}_i\mathbf{x}_i(t)+1/2\mathbf{u}_i^T(t)\mathbf{R}_i\mathbf{u}_i(t)$$
$$+\boldsymbol{\lambda}^T(\mathbf{z}_i(t)-\sum_{j=1}^{n}\mathbf{L}_{ij}\mathbf{x}_j(t))$$
$$+\mathbf{p}_i^T(t)(-\frac{d}{dt}\mathbf{x}_i+\mathbf{A}_i\mathbf{x}_i+\mathbf{B}_i\mathbf{u}_i+\mathbf{c}_i\mathbf{z}_i)]dt \quad (16)$$

where $\boldsymbol{\lambda}_i(t)$ is the Lagrange multiplier and $\mathbf{p}_i(t)$ is an adjoint variable.

The coordination rule for the second level using the interaction-prediction principle[11] is from iteration to K+1

$$\begin{bmatrix} \lambda_i(t) \\ z_i(t) \end{bmatrix}^{K+1} = \begin{bmatrix} -C^T{}_i p_i(t) \\ \sum_{j=1}^{N} L_{ij} x_j(t) \end{bmatrix}^{K} \tag{17}$$

The convergence of this algorithm has been proved by Takahara[12].

Consider the lower-level problems, the Hamiltonian for the i-th subsystem can be written as

$$H_i = 1/2 x_i^T Q_i x_i + 1/2 u_i^T R_i u_i + \lambda_i^T z_i - \sum_{j=1}^{n} \lambda_j^T L_{ji} x_i + p_i^T (A_i x_i + B_i u_i + C_i z_i) \tag{18}$$

and then, from the necessary condition yields

$$u_i(t) = -R_i^{-1} B_i^T p_i(t) \tag{19}$$

$$\frac{d}{dt} x_i(t) = A_i x_i(t) - B_i R_i^{-1} B_i^T p_i(t) + C_i z_i(t) \tag{20}$$

$$-\frac{d}{dt} p_i(t) = -Q_i x_i(t) - A_i^T p_i(t) + \sum_{j=1}^{n} [\lambda_j^T(t) L_{ji}]^T \tag{21}$$

In matrix form, eq.(20) and (21) is written as

$$\begin{bmatrix} \frac{d}{dt} x_i(t) \\ \frac{d}{dt} p_i(t) \end{bmatrix} = M_i \begin{bmatrix} x_i(t) \\ p_i(t) \end{bmatrix} + \begin{bmatrix} N_{1i}(t) \\ N_{2i}(t) \end{bmatrix} \tag{22}$$

where $x_i(0) = x_0$, $p_i(t_f) = 0$

$$M_i = \begin{bmatrix} A_i & -B_i R_i^{-1} B_i \\ -Q_i & -A_i^T \end{bmatrix}$$

$$N_{1i}(t) = C_i z_i(t), \quad N_{2i}(t) = \sum_{j=1}^{n} [\lambda_i^T(t) L_{ij}]^T \tag{23}$$

Let $$\Phi(t_f, t) = \begin{bmatrix} \Phi_{11}(t_f,t) & \Phi_{12}(t_f,t) \\ \Phi_{21}(t_f,t) & \Phi_{22}(t_f,t) \end{bmatrix}$$

be the state transition matrix of eq.(22)

It is well known that the state transition matrix has the following property;

$$\frac{d}{dt}\Phi(t_f,t) = -\Phi(t_f,t) M_i \tag{24}$$
$$\Phi(t_f,t_f) = I$$

Integrating eq.(24) backward from t_f to t, gives

$$I - \Phi(t_f,t) = \int_{t_f}^{t} \Phi(t_f,\tau) M \, d\tau \tag{25}$$

Using eq.(22), from the relation

$$\begin{bmatrix} x(t_f) \\ p(t_f) \end{bmatrix} = \Phi(t_f,t) \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} + \int_t^{t_f} \Phi(t_f,\tau) \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} d\tau = \begin{bmatrix} x(t_f) \\ 0 \end{bmatrix} \tag{26}$$

we have

$$p(t) = -\Phi_{22}^{-1}(t_f,t)\Phi_{21}(t_f,t) - \Phi_{22}^{-1}(t_f,t) \int_t^{t_f} \Omega(t_f,\tau) d\tau \tag{27}$$

where, $\Omega(t_f,\tau) = \Phi_{21}(t_f,\tau) N_1(\tau) + \Phi_{22}(t_f,\tau) N_2(\tau)$

Let $u(t)$ be a modification to give partial feedback control[11], ie,

$$p(t) = K(t) x(t) + s(t) \tag{28}$$

where $K(t)$ is the local feedback gain and $s(t)$ is the open-loop compensation vector

$$K(t) = -\Phi_{22}^{-1}(t_f,t)\Phi_{21}(t_f,t) \tag{29}$$

$$s(t) = -\Phi_{22}^{-1}(t_f,t) \int_t^{t_f} \Omega(t_f,\tau) d\tau \tag{30}$$

From eq.(19) and eq.(27), we obtain local control vector for the ith subsystem.

The task of coordinator level is to improve $N_1(t)$, $N_2(t)$ such that the global optimization is acheived. On the second level, the convergence is measured by the error criterion, where error is defined as

$$\varepsilon = \int_{t_0}^{t_f} \left[\sqrt{\sum_{i=1}^{n} E_1^T(t) E_1(t)} + \sqrt{\sum_{i=1}^{n} E_2^T(t) E_2(t)} \right] dt \tag{31}$$

where $E_1 = N_{1i}^{k+1}(t) - N_{1i}^{k}(t)$

$E_2 = N_{2i}^{k+1}(t) - N_{2i}^{k}(t)$

Once the second level, interaction error is sufficiently small, an optimum solution has been obtained.

4. Hierarchical Optimal Control via BPFs

Block pluse functions(BPFs) $\phi_k(t)$, $k=1,2,3,\cdots,m$ defined in the interval $[0,t_f)$ by

$$\phi_k(t) = \begin{cases} 1 , & \text{for } (k-1)t_f/m \le t \le kt_f/m \\ 0 , & \text{otherwise} \end{cases} \tag{32}$$

The integrals[6] of BPFs can be approximated as

$$\int_0^t \phi_k(\tau) d\tau = \frac{t_f}{m} \sum_{i=k+1}^{m} \phi_i(t) + \frac{t_f}{2m} \phi_k(t) \tag{33}$$

using m BPFs themselves.

Simillary, The backward integrals[6] of BPFs can be approximated as

$$\int_{t_f}^{t} \phi_k(\tau) d\tau = -\frac{t_f}{m} \sum_{i=1}^{k-1} \phi_i(t) - \frac{t_f}{2m} \phi_k(t) \tag{34}$$

Consider the BPF application to hierarchical optimal control of DPS.

The subscript i for the ith subsystem is omitted for the sake of simplicity. In this section, capital letters with the subscript(j) denote the coefficients of Jth BPFs.

Introducing the eq.(34) in eq.(25) , we have

$$\Phi_m = [I - \tfrac{t_f}{2m} M]^{-1}$$
$$\Phi_j = \Phi_{j+1} [I + \tfrac{t_f}{2m} M][I - \tfrac{t_f}{2m} M]^{-1} \tag{35}$$

where $j = m-1, m-2, \cdots, 1$

From eq.(29), we obtain

$$K_j = -(\Phi_{22}^{-1})_j\ (\Phi_{21})_j \tag{36}$$

From eq.(30) and eq.(31), using eq.(34),we obtain

$$S_m = -\frac{t_f}{2m}(\Phi_{22})_m^{-1}\ \Omega_m$$

$$S_j = S_{j+1} + \frac{t_f}{2m}[(\Phi_{22})_j^{-1}\Omega_j + (\Phi_{22})_{j+1}^{-1}\Omega_{j+1}] \tag{37}$$

$$(j = m-1, m-2, .., 1)$$

where, $\Omega_j = [(\Phi_{21})_j \mathbf{N}_{1j} + (\Phi_{22})_j \mathbf{N}_{2j}]$

From eq.(22) and eq.(28), since

$$\frac{d}{dt}x_i(t) = [A_i - B_i R_i^{-1} B_i^T K_i(t)]x_i(t) + N_{1i}(t) - B_i R_i^{-1} B_i^T s(t) \tag{38}$$

integrating eq.(38) from 0 to t_f and then by using eq (33), we obtain

$$\mathbf{X}_1 = \mathbf{M}_{n1}[x(0) + \mathbf{V}_1]$$

$$\mathbf{X}_{j+1} = \mathbf{M}_{nj}[\mathbf{M}_{pj}\mathbf{X}_j + \mathbf{V}_{j+1} + \mathbf{V}_j],\ (j=1,2,...,m) \tag{39}$$

where
$$\mathbf{M}_{nj} = [I - \frac{t_f}{2m}[\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}_j]]^{-1}$$

$$\mathbf{M}_{pj} = I + \frac{t_f}{2m}[\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}_j]$$

$$\mathbf{V}_j = \frac{t_f}{2m}[\mathbf{N}_{1j} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{S}_j]$$

From eq.(19) and eq.(28), we have

$$\mathbf{P}_j = \mathbf{K}_j\mathbf{X}_j + \mathbf{S}_j \tag{40}$$

$$\mathbf{U}_j = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}_j \tag{41}$$

The task of coordinator level is to improve $\mathbf{N}_{1i}(t)$, $\mathbf{N}_{2i}(t)$, such that the global optimum is acheived.

From eq.(17) and eq.(23), we obtain

$$(\mathbf{N}_{1i})_j = (C_i)_j(Z_i)_j \tag{42}$$

$$(\mathbf{N}_{2i})_j = \sum_{r=1}^{N}[(\lambda^T_r)_j(L_{ri})_j]^T \tag{43}$$

Introducing eq.(42) and eq.(43), Equation eq.(31) is written as

$$\varepsilon = \frac{t_f}{m}\left[\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}E_1{}^TE_1} + \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}E_2{}^TE_2}\right] \tag{44}$$

단, $E_1 = [(\mathbf{N}_{1i})_j{}^{K+1} - (\mathbf{N}_{1i})_j{}^K]$

$E_2 = [(\mathbf{N}_{2i})_j{}^{K+1} - (\mathbf{N}_{2i})_j{}^K]$

Once the global system interaction error is sufficiently small, an optimum solution has been obtained. However, the obtained control $\mathbf{u}(t)$ and state $\mathbf{x}(t)$ are the coefficient vector composed of the expansion coefficients of the shifted Legendre polynomial functions. Therefore the optimal control $\mathbf{u}(y,t)$ and state $\mathbf{x}(y,t)$ is determined by inverse Legendre transformation.

5. Numerical Example

Exemple 5.1

For the first example consider the one-dimensional diffusion equation[9,13],

$$\frac{\partial x(y,t)}{\partial t} = \mathcal{D}\,x(y,t) + u(y,t)$$

where $\mathcal{D} = \frac{\partial^2}{\partial y^2}$

and initial condition

$$x(y,0) = 1. + y$$

and boundary condition

$$\frac{\partial x(y,t)}{\partial y} = 0.;\quad \text{at } y=0 \text{ and } y_f=4.$$

The obtained LPS by proposed method, for N=5, is expressed as

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

where
$$\mathbf{A} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -0.75 & 0 & -1.7 & 0 \\ 0 & 0 & -3.75 & 0 & -6.75 \\ 0 & -0.75 & 0 & -10.5 & 0 \\ 0 & 0 & -3.75 & 0 & -22.5 \end{vmatrix}$$

$$\mathbf{B} = \mathbf{I},\ \mathbf{x}_0 = [3\ 2/3\ 0\ 0\ 0]^T$$

The cost function becomes

$$J = \frac{1}{2}\int_0^{t_f}\mathbf{x}^T(t)\mathbf{Q}'\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}'\mathbf{u}(t)\,dt$$

$$\mathbf{Q}' = \mathbf{R}' = y_f\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{vmatrix}$$

Here the system can be decomposed into two subsystem, that is, subsystem 1 and 2 consist of states x_1, x_2, x_3 and x_4, x_5, respectively.

Example 5.2

For the second example, consider the DPS described by the partial differential equation

$$\frac{\partial}{\partial t}\mathbf{x}(y,t) = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix}\frac{\partial}{\partial y}\mathbf{x}(y,t) + \begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix}\mathbf{x}(y,t) + \begin{vmatrix} 0 \\ 2 \end{vmatrix}\mathbf{u}(y,t)$$

$$\mathbf{x}(y,0) = \mathbf{x}_0(y),\ \mathbf{x}(0,t) = \mathbf{0},\ \mathbf{x}(y_f,t) = \mathbf{0}$$

where
$$\mathbf{x}(y,t) = \begin{vmatrix} x_1(y,t) \\ x_2(y,t) \end{vmatrix},\ \mathbf{x}_0(y) = \begin{vmatrix} 1+y \\ 1+y \end{vmatrix}$$

The problem is to find the control vector $\mathbf{u}(y,t)$ which minimizes the following cost function, J,

$$J=\frac{1}{2}\int_0^{t_f}\int_0^{y_f}\mathbf{x}^T(y,t)\mathbf{Q}\mathbf{x}(y,t)+Ru^2(y,t)\,dy\,dt$$

where $\mathbf{Q}=\mathbf{I}, R=1, t_f=1, y_f=6.0$

The obtained LPS for N=5, is written as

$$\frac{d}{dt}\mathbf{x}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)$$

$$\mathbf{x}(0)=\mathbf{x}_0$$

where

$$\mathbf{x}(t)=\begin{bmatrix}\mathbf{x}_0(t)\\ \mathbf{x}_1(t)\\ \mathbf{x}_2(t)\\ \mathbf{x}_3(t)\\ \mathbf{x}_4(t)\end{bmatrix},\quad \mathbf{u}(t)=\begin{bmatrix}\mathbf{u}_0(t)\\ \mathbf{u}_1(t)\\ \mathbf{u}_2(t)\\ \mathbf{u}_3(t)\\ \mathbf{u}_4(t)\end{bmatrix}$$

$$\mathbf{x}_i(t)=\begin{bmatrix}x_{1i}(t)\\ x_{2i}(t)\end{bmatrix},\quad \mathbf{u}_i(t)=\begin{bmatrix}u_{1i}(t)\\ u_{2i}(t)\end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix}-2 & 2 & 1 & 0 & 0 & 0 & 7/3 & 0 & 0 & 0\\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -2 & 2 & 5/3 & 0 & 0 & 0 & 3 & 0\\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -2 & 2 & 7/3 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 3 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2\end{bmatrix}$$

$$\mathbf{B}=\mathbf{I},\quad \mathbf{x}_0=[4\ 4\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0]^T$$

The cost function becomes

$$J=\frac{1}{2}\int_0^{t_f}\mathbf{x}^T(t)\mathbf{Q}'\mathbf{x}(t)+\mathbf{u}^T(t)\mathbf{R}'\mathbf{u}(t)\,dt$$

where $\mathbf{Q}'=\mathbf{Q}\otimes\mathbf{F},\ \mathbf{R}'=R\otimes\mathbf{F}$.

Here the system can be decomposed into five subsystems of equal dimensions.

6. Conclusions

In this paper, the optimal control of DPSs is solved by a hierarchical computational procedure.

The system state variable and the control variable are expressed in terms of the shifted Legendre polymonial function of space coordinate.

The hierarchical control scheme is introduced to determine the optimal control of the large scale LPSs which is derived by using the Galerkin method.

An approach to block pulse functions is applied to solve the optimal control problems of the obtained LPSs.

The obtained LPSs are converted into linear algebraic equations by block pulse transformations. The solution is obtained by solving these equations.

The proposed method is simple and efficient in computation for the optimal control of DPSs.

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