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A Method of Nonlinear Optimal Regulator Using a Liapunov-like Function

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ABSTRACT

In general it is difficult to determine a Liapunov function for a given asymptotically stable, nonlinear differential equations system. But, in the system with control inputs,it is feasible to make a given positive function, except for a small area, globally satisfy the conditions of the Liapunov function for the system. We call such a positive function a Liapunov-like function, and propose a method of nonlinear optimal regulator using this Liapunov-like function. We also use the periodic Liapunov-like function that suits the system whose equilibrium points exist periodically. The relationship between the Liapunov function and cost function which this nonlinear regulator minimizes is considered using inverse optimal method.

1. INTRODUCTION

Most differential equations describing the actual behavior of a system are generally nonlinear. However, the linear optimal regulator is effective when the operation of the system is restricted to a small region around a chosen operating point. In an inherent nonlinear system which shows strong non-linearity the linear optimal regulator often fails, and it is desirable to design the nonlinear optimal regulator for the wide range control. For the nonlinear systems in which the nonlinearity exists on the state vectors only, some methods for a nonlinear regulator are developed[(1)-(3)].

In this paper we propose a method for developing a nonlinear optimal regulator, using a Liapunov-like function in a nonlinear system. In general, it is difficult to determine the Liapunov function for a given asymptotically stable nonlinear differential equations system. But, in a system with control inputs, it is feasible to show that any given positive function can globally satisfy the conditions of the Liapunov function for the system, by adding appropriate control inputs.

For the Liapunov-like function $V(\mathbf{x})$($V(\mathbf{x})$ is chosen arbitrarily), we use the control input $\mathbf{u} = -\mathrm{SB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x})$ where S is a positive definite diagonal matrix and $V_{\mathbf{x}}(\mathbf{x})$ is the gradient of the Liapunov-like function $V(\mathbf{x})$. The time derivative of $V(\mathbf{x})$ becomes $\dot{V}(\mathbf{x}) = V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\dot{\mathbf{x}} = V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathbf{f}(\mathbf{x}) - V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{BSB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x})$ and if we choose the elements of the positive definite diagonal matrix S appropriately large, we can make $\dot{V}(\mathbf{x}) < 0$, notwithstanding the value of inner product $V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathbf{f}(\mathbf{x})$ where $\mathbf{f}(\mathbf{x})$ is a nonlinear vector valued function. Therefore, if we add the control input $\mathbf{u} = -\mathrm{SB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x})$, the Liapunov-like function $V(\mathbf{x})$ satisfies the conditions for a Liapunov function, and the control input $\mathbf{u} = -\mathrm{SB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x})$ makes the system stable.

In general the magnitude of the control input has a constrained limit. The larger the values of S the greater the control input, so it is not desirable to choose large values for S. It is more desirable to choose a value for $V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathbf{f}(\mathbf{x})$ as negative as posible to satisfy the condition of the Liapunov function $\dot{V}(\mathbf{x}) < 0$,because the second term $-V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{BSB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x})$ has a negative value. We consider the region that satisfies the condition $\dot{V}(\mathbf{x}) < 0$, and the relationship between the Liapunov function and cost function. We consider an inverted pendulum with DC motor control as a numerical example.[(4)]

2. OPTIMAL REGULATOR USING LIAPUNOV-LIKE FUNCTION

Consider the system of differential equations in which the nonlinearity exists on the state vectors only.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathrm{B}\mathbf{u} \tag{1}$$

Here, $\mathbf{x}$ is an n-dimensional state vector of the dynamic system and $\mathbf{u}$ is an r-dimensional control vector. $f(\mathbf{x})$ is a nonlinear vector-valued function and B is an $n \times r$ dimensional matrix.

As the optimization criterion, we choose

$$J = [q(\mathbf{x}) + \frac{1}{2}\mathbf{u}^{\mathrm{T}}\mathrm{R}\mathbf{u}]\mathrm{d}t \tag{2}$$

with $q(\mathbf{x}) > 0$. Let R be a positive definite diagonal matrix, then the function which represents the minimum value of the criterion function (2) becomes positive.

$$V(\mathbf{x}, t) = \min_{\mathbf{u}(t)} \int_t^{\infty} [q(\mathbf{x}) + \frac{1}{2}\mathbf{u}^{\mathrm{T}}\mathrm{R}\mathbf{u}]\mathrm{d}t \tag{3}$$

If we assume that the minimum value of (3) is finite, the steady state Hamilton-Jacobi-Bellman equation becomes the following equation.

$$\min_{\mathbf{u}(t)}[q(\mathbf{x}) + \tfrac{1}{2}\mathbf{u}^{\mathrm{T}}\mathrm{R}\mathbf{u} + \mathrm{V}_{\mathbf{x}}(\mathbf{x}, t)^{\mathrm{T}}\{\mathbf{f}(\mathbf{x}) + \mathrm{B}\mathbf{u}\}] = 0 \tag{4}$$

Therefore the optimal control that satisfies (4) is given as follows.

$$\mathbf{u} = -\mathrm{R}^{-1}\mathrm{B}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}, t) \tag{5}$$

where

$$V_{\mathbf{x}}(\mathbf{x}, t) = \frac{\partial V(\mathbf{x}, t)}{\partial \mathbf{x}}$$

When $f(\mathbf{x}) = \mathrm{A}\mathbf{x}$ and $q(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathrm{Q}\mathbf{x}$, we can describe the equation (5) as the linear optimal regulator by choosing the $V(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathrm{K}\mathbf{x}$. But, in general, it is difficult to determine the specific function if $q(\mathbf{x})$ is arbitrarily given. So,we first set the positive function and then we make the control input (6).

$$\mathbf{u} = -\mathrm{SB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \tag{6}$$

where $V_{\mathbf{x}}(\mathbf{x})$ is the gradient of the Liapunov-like function $V(\mathbf{x})$. differentiating $V(\mathbf{x})$ with time t and substituting (1) and (6), we obtain equation (7).

$$\begin{aligned} \dot{V}(\mathbf{x}) &= V_{\mathbf{x}}(\mathbf{x}) \cdot \dot{\mathbf{x}} \\ &= V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}[\mathrm{f}(\mathbf{x}) + \mathrm{Bu}] \\ &= V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{f}(\mathbf{x}) - V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{BSB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \end{aligned} \tag{7}$$

Since the second term of equation (7) has a negative value, if we choose a large value for S, the value of equation (7) becomes negative for any state $\mathbf{x}$ which satisfies $\mathrm{B}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \neq 0$. Then the conditions for the Liapunov function are satisfied in the area $\mathrm{B}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \neq 0$, and the control law (6) makes the system (1) asymptotically stable. But $V(\mathbf{x})$ is not a Liapunov function in a strict sense because the conditions for the Liapunov function are not satisfied in the area $\mathrm{B}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) = 0$. In numerical examples shown later, even if the condition for the Liapunov function is not satisfied, the control input given by equation (6) makes the system asymptotically stable.
On the other hand, the control input (6) minimizes the criterion function (2). In the inverse optimal aspect the function $q(\mathbf{x})$ is given by the following equation.

$$q(\mathbf{x}) = -V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathbf{f}(\mathbf{x}) + \frac{1}{2} V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{BSB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \tag{8}$$

For the positive function $V(\mathbf{x})$, chosen arbitrarily, it does not necessarily follow that the function $q(\mathbf{x})$ becomes positive. To choose a $V(\mathbf{x})$ which the function $q(\mathbf{x})$ becomes positive, trial and error is required. It is desirable to choose the Liapunov-like function that makes a weight coefficient matrix S smaller.

We now compare the condition $\dot{V}(\mathbf{x}) < 0$ and the condition $q(\mathbf{x}) > 0$. Substituting equation (8) into equation (7), we get the following:

$$\dot{V}(\mathbf{x}) = -q(\mathbf{x}) - \frac{1}{2} V_{\mathbf{x}}(\mathbf{x})^{\mathrm{T}}\mathrm{BSB}^{\mathrm{T}} V_{\mathbf{x}}(\mathbf{x}) \tag{9}$$

If the function $q(\mathbf{x})$ is positive, the condition $\dot{V}(\mathbf{x}) < 0$ is always satisfied because the second term of the equation (7) is negative. On the other hand, even if the value of the equation (8) is negative, $q(\mathbf{x})$ is not always positive. Therefore, it is easier to choose a $\dot{V}(\mathbf{x}) < 0$ than to set $q(\mathbf{x}) > 0$. In the stabilization control, the determination of the control input which satisfies $\dot{V}(\mathbf{x}) < 0$ is sufficient for application.

3. SIMULATIONS

3.1 NUMERICAL EXAMPLE

We shall firstly dispense with the trivial form of system (1):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}\mathrm{u}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} \sin \mathrm{x}_2 \\ 0 \end{pmatrix} \qquad \mathrm{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Take the quadratic equation as a Liapunov-like function.

$$V(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathrm{K}\mathbf{x}$$

Then the control input becomes as follows:

$$\begin{aligned} \mathrm{u} &= -\mathrm{s}\mathbf{x}^{\mathrm{T}}\mathrm{K}\mathbf{b} \\ &= -\mathrm{s}(\mathrm{x}_1, \mathrm{x}_2)\begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= -\mathrm{s}(\mathrm{K}_{12}\mathrm{x}_1 + \mathrm{K}_{22}\mathrm{x}_2) \end{aligned}$$

And time derivative of $V(\mathbf{x})$ becomes as follows:

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \mathbf{x}^{\mathrm{T}}\mathrm{K}(\mathbf{f}(\mathbf{x}) - \mathrm{s}\mathbf{x}^{\mathrm{T}}\mathrm{K}\mathbf{b}) \\ &= (\mathrm{K}_{11}\mathrm{x}_1 + \mathrm{K}_{12}\mathrm{x}_2)\sin \mathrm{x}_2 - \mathrm{s}(\mathrm{K}_{12}\mathrm{x}_1 + \mathrm{K}_{22}\mathrm{x}_2)^2 \end{aligned}$$

where K_{11}, K_{12}, K_{22} are the elements of matrix K respectively.

When

$$K_{11} = 1.0,\ K_{12} = 0.8,\ K_{22} = 1.0$$

$V(\mathbf{x})$ becomes the positive function at $\mathbf{x} \neq 0$.
Fig.1 shows the regions of $\dot{V}(\mathbf{x}) > 0$ corresponding to parameter s for the coefficient of the control input. The smaller the values of s the wider the region $\dot{V}(\mathbf{x}) > 0$.

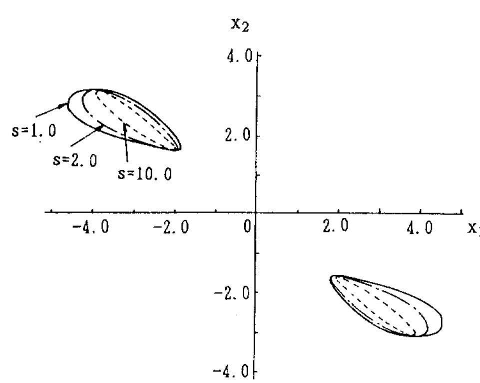

Fig.1 Regions of $\dot{V}(\mathbf{x}) > 0$

Fig.2 shows the phase diagram of x_1 and x_2 of the solutions solved by some initial values. It is observed that the system states with the control input converge on the origin. No control leads the system states to instability as seen from the system equation. It is noted that the solution which departed from the initial value $(\mathrm{x}_1, \mathrm{x}_2) = (2.0, -2.0)$ which is in the area of $\dot{V}(\mathbf{x}) > 0$ in Fig.1, doesn't converge on the origin.

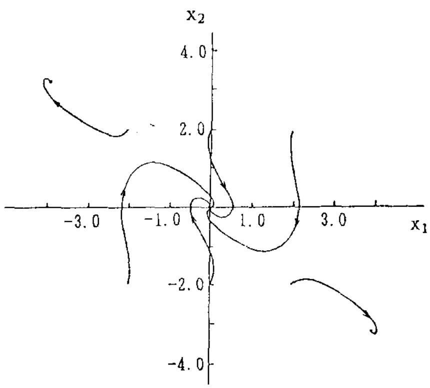

Fig.2 Phase diagram of solutions

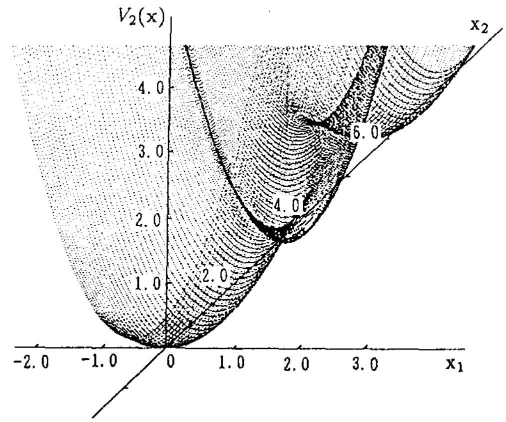

Fig.4 Three dimensional graph of $V_2(\mathbf{x})$

Fig.3 shows the time responses of $V(\mathbf{x})$ and $\dot{V}(\mathbf{x})$ for the initial value (2.0, 2.0).In this example $V(\mathbf{x})$ completely satisfies the condition for a Liapunov function.

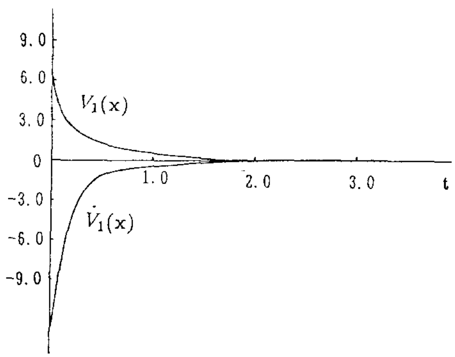

Fig.3 Time response of $V_1(\mathbf{x})$ and $\dot{V}_1(\mathbf{x})$

Next we consider the periodic Liapunov-like function $V_2(\mathbf{x})$.

$$V_2(\mathbf{x}) = \frac{1}{2}(x_1, \sin x_2)\begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ \sin x_2 \end{pmatrix}$$

$$V_{2\mathbf{x}}(\mathbf{x}) = (x_1, \sin x_2)\begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \cos x_2 \end{pmatrix}$$

$$u = -s(x_1, \sin x_2)\begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \cos x_2 \end{pmatrix}$$

$$\dot{V}_2(\mathbf{x}) = V_{2\mathbf{x}}(\mathbf{x})\mathbf{f}(\mathbf{x}) - s(V_{2\mathbf{x}}(\mathbf{x})b)^2$$

$$= (K_{11}x_1 + K_{12}\sin x_2)\sin x_2 - s(K_{12}x_1\cos x_2 + K_{22}\sin x_2\cos x_2)^2$$

Fig.4 shows the three dimensional graph of $V_2(\mathbf{x})$, where $V_2(\mathbf{x})$ becomes 0 at $(0, n\pi)$ ($n = 0, 1, 2, \ldots$).

The comparison of the phase curves of the solutions using $V_1(\mathbf{x})$ and $V_2(\mathbf{x})$ is represented in Fig.5 where the initial state is (2.0, 3.0). It can be seen that the state converges on the new equilibrium state $(0, \pi)$ in the case using $V_2(\mathbf{x})$. And the comparison of the control inputs and the comparison of the values of the cost function are represented in Fig.6 and Fig.7 respectively. It is seen that the state in the case using $V_2(\mathbf{x})$ moves to the equilibrium point by smaller control input than the case using $V_1(\mathbf{x})$.

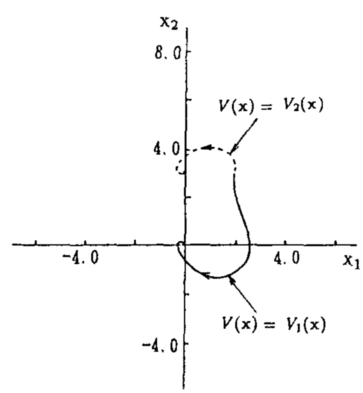

Fig.5 Phase diagram of solutions

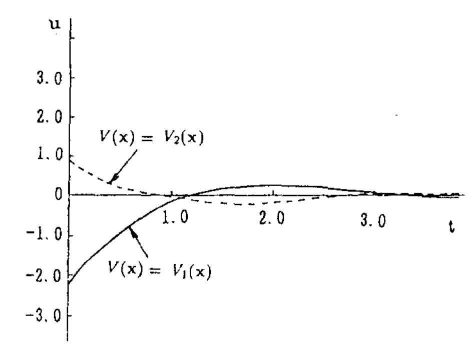

Fig.6 Comparison of control inputs

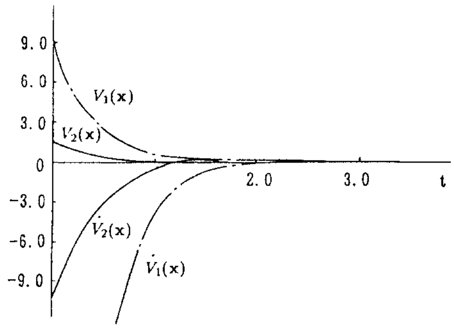

Fig.7 Comparison of the time response of Liapunov-like function

Fig.8 shows the region $\dot{V}(\mathbf{x}) > 0$. Note that the area of $\dot{V}(\mathbf{x}) > 0$ does not always mean the area of instability.

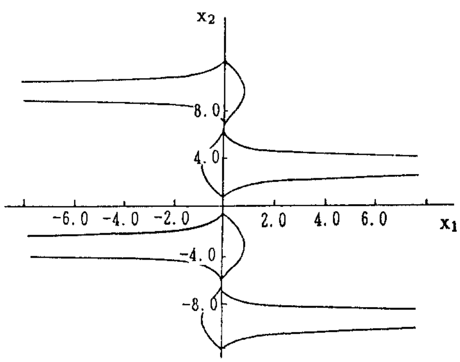

Fig.8 Region of $\dot{V}_2(\mathbf{x}) > 0$

3.2 INVERTED PENDULUM WITH DC MOTOR CONTROL

Next we consider an inverted pendulum with DC motor control as illustrated in Fig.9. The pendulum kinematics can be described by the following:

$$ml^2\ddot{\theta} = lmg\sin\theta - T_p$$

$$T_p = 10K_mI$$

where m is the mass of the pendulum, l is the length of the pendulum, θ is the angle of the pendulum measured from the inverted state, and g is the gravity constant. The torque T_p of the pendulum is proportional to the current of DC motor, where K_m is a torque constant. Let V be the input voltage; the circuit equation is represented by the following equation:

$$L\dot{I} + RI + V_b = V$$

$$V_b = 10K_b\dot{\theta}$$

where L and R are the inductance and resistance of the armature winding of the DC motor. The induced voltage V_b is proportional to the angular velocity, where K_b is the induced voltage constant. Regarding θ, $\dot{\theta}$ and I as state variables, and V as control variable, we let $x_1 = \theta, x_2 = \dot{\theta}, x_3 = I$ and $u = V$. Then we obtain the following simultaneous differential equations system.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g}{l}\sin x_1 + 10\frac{K_m}{l^2 m}x_3$$

$$\dot{x}_3 = -10\frac{K_b}{L}x_2 - \frac{R}{L}x_3 + \frac{1}{L}u$$

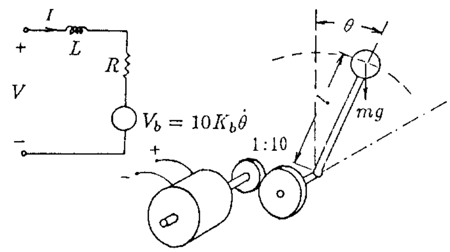

Fig.9 Inverted pendulum with a DC motor

We shall carry out numerical calculations for the case $g = 9.8\ m/s^2$, $m = 0.5\ Kg$, $l = 0.3\ m$, $K_m = 0.01\ Nm/A$, $K_b = 0.02\ Vs/rad$, $L = 20\ mH$, $R = 0.1\Omega$. In the simulations, we set the desired state to an unstable equilibrium point to test the usefulness of the method.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 32.67\sin x_1 + 2.22x_3$$
$$\dot{x}_3 = -x_2 - 5.0x_3 + 50.0u$$

Linearizing the system around the equilibrium point, we get a linearized matrix A. As the cost function we choose the quadratic function where Q is a unit matrix and R=1. Then,

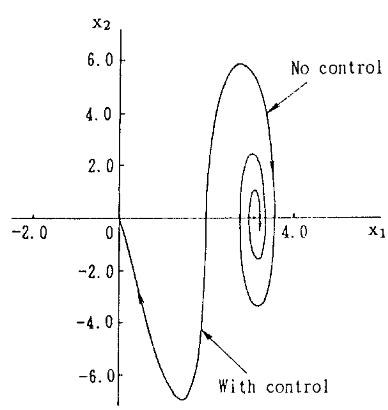

Fig.10 Phase diagram of solutions

the Riccati equation is calculated as follows:

$$K = \begin{pmatrix} 96.73 & 16.53 & 0.66 \\ 16.53 & 2.93 & 0.11 \\ 0.66 & 0.11 & 0.02 \end{pmatrix}$$

We set the Liapunov-like function $V_1(\mathbf{x})$ using the K matrix. Fig.10 shows the phase diagram of x_1 and x_2 of the solutions solved by initial state $(2.0, 0.0, 0.0)$.
It is observed that the system states with the control input converge on the origin, on the other hand no control input leads the system state to an equilibrium stable point $(\pi, 0, 0)$ with oscillation. Fig.11 shows the time responses of $V_1(\mathbf{x})$ and $\dot{V}_1(\mathbf{x})$. $V_1(\mathbf{x})$ satisfies the condition of a Liapunov function.

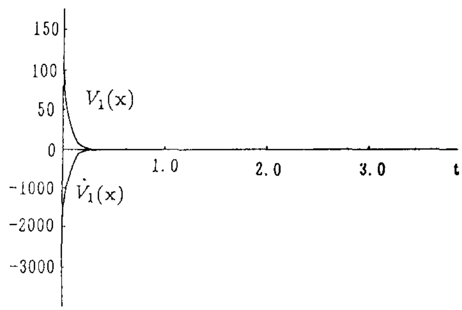

Fig.11 Time response of $V_1(\mathbf{x})$ and $\dot{V}_1(\mathbf{x})$

Next we consider the periodic Liapunov-like function $V_2(\mathbf{x})$.

$$V_2(\mathbf{x}) = \frac{1}{2}(\sin x_1, x_2, x_3) \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{pmatrix} \begin{pmatrix} \sin x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$V_{2\mathbf{x}}(\mathbf{x}) = (\sin x_1, x_2, x_3) \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{pmatrix} \begin{pmatrix} \cos x_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \dot{V}_2(\mathbf{x}) &= V_{2\mathbf{x}}(\mathbf{x})\mathrm{f}(\mathbf{x}) - \mathrm{s}(V_{2\mathbf{x}}(\mathbf{x})\mathrm{b})^2 \\ &= (K_{11}x_1 + K_{12}\sin x_2)\sin x_2 - \mathrm{s}(K_{12}x_1\cos x_2 \\ &\quad + K_{22}\sin x_2 \cos x_2)^2 \end{aligned}$$

The comparison of the phase curves of the solutions using $V_1(\mathbf{x})$ and $V_2(\mathbf{x})$ is represented in Fig.12 where the initial state is $(5.0, 0.0)$.

It is observed that the state converges on the new equilibrium state $(2\pi, 0)$ in the case using $V_2(\mathbf{x})$. The comparison of the control inputs and the comparison of the values of the cost function are represented in Fig.13 and Fig.14 respectively. It is also seen that the state in the case using $V_2(\mathbf{x})$ moves to the equilibrium point by smaller control input than the case using $V_1(\mathbf{x})$.

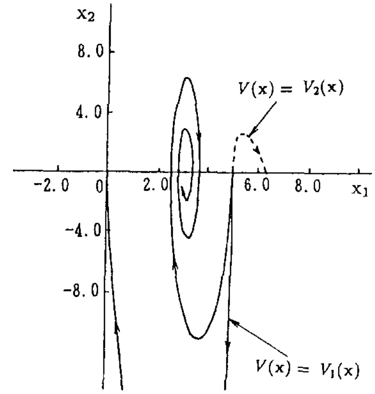

Fig.12 Phase diagram of solutions

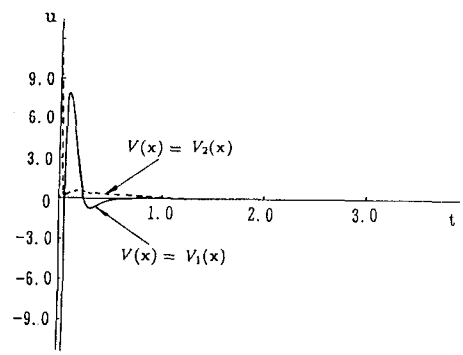

Fig.13 Comparison of control inputs

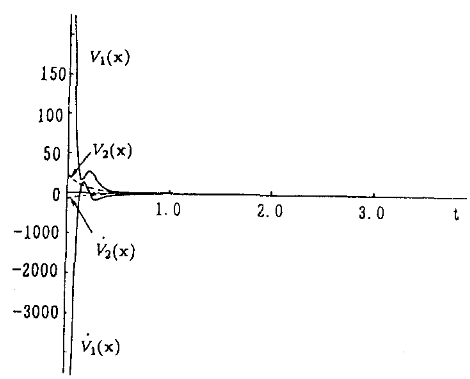

Fig.14 Comparison of Liapunov-like function

4. SUMMARY AND DISCUSSION

In this paper we have assumed a positive function as a candidate for the Liapunov-like function, and, using the control input $\mathbf{u} = -\mathrm{R}^{-1}\mathrm{B}^T V_{\mathbf{x}}(\mathbf{x})$, we showed that a useful regulator for a nonlinear system is obtained. For an arbitrarily chosen $V(\mathbf{x})$, it is theoretically feasible that this positive function satisfies the condition for a Liapunov function by choosing a reasonable large coefficient s. But, if s becomes large, the control input also has a large value coresponding to it, so the control input within the limit is not always obtained. It is necessary to choose the positive function such that the condition of a Liapunov function is satisfied as broadly as possible. It is difficult to choose a Liapunov-like function that satisfies the condition of the Liapunov function in a wide range area. In general, if the area that satisfies $B^T V_{\mathbf{x}}(\mathbf{x}) = 0$ is a plane or a line in a n-dimensional space, the condition $\dot{V}(\mathbf{x}) < 0$ is not satisfied. But, as seen in numerical examples,the solution pathed through the state which is not satisfied by $\dot{V}(\mathbf{x}) < 0$ does not necessarily become unstable. Since a Liapunov function merely gives a sufficient condition for the system to be asymptotically stable, the solution in the area $V(\mathbf{x}) > 0$ is not always unstable. In Fig.8 and Fig.14 showing the time response $V(\mathbf{x})$ and $\dot{V}(\mathbf{x})$, the times of $\dot{V}(\mathbf{x}) > 0$ exist, but over all, it is seen that the condition of Liapunov function is satisfied. The solution's curve in a phase diagram transverses the area $\dot{V}(\mathbf{x}) > 0$ and finally converges on the equilibrium point. For a general n-dimensional system, describing the phase diagram of two priority variables for stability, we can see the outline of the graph of $\dot{V}(\mathbf{x}) = 0$. Fig.15 represents a graph of $\dot{V}_2(\mathbf{x}) = 0$ in x_1x_2 plane when $x_3 = 0$ and s=1.4. When s=1.0 the graph of $\dot{V}_2(\mathbf{x}) = 0$ dissapears.

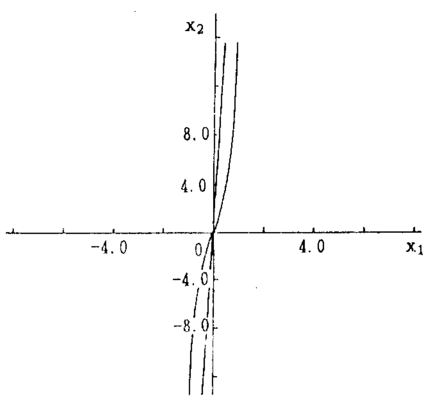

Fig.15 Region of $\dot{V}_1(\mathbf{x}) > 0$

Generally, a cost function is given first in a control problem which is mathematically defined. The problem is to determine an optimal control which minimizes the cost function. Since the cost function is calculated by the inverse optimal method in this paper, it can not be given first arbitrarily. It is not very important to minimize the cost function itself. The purpose is to get the control input that leads the system to a stable state with a fast and non-oscillatory manner. Though a little experience and trial and error are still necessary to determine the positive Liapunov-like function where the area $\dot{V}(\mathbf{x}) < 0$ is wide and $q(\mathbf{x})$ becomes a positive function, the method proposed in this paper provides a strategy for designing a nonlinear control system.

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