Receding Horizon Tracking Control as A Predictive Control for the Continuous-time Systems

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ABSTRACT

This paper proposed a predictive tracking controller for the continuous-time systems by using the receding horizon concept in the optimal tracking control. This controller is the continuous-time version of the previous RHTC (Receding Horizon Tracking Control) for the discrete-time state space models. The problems in implementing the feedforward part of this controller is discussed and a approximate method of implementing this controller is presented.

This approximate method utilizes the information of the command signals on the receding horizon and has simple constant feedback and feedforward gain. To perform the offset free control, the integral action is included in the continuous time RHTC. By simulation it is shown that the proposed method gives better performance than the conventional steady state tracking control.

1. INTRODUCTION

For regulator problems, the infinite time optimal control has been widely used because the control law is obtained from ARE (Algebraic Riccati Equation). But, for the tracking problems, the infinite time optimal control is difficult to implement unless the command signals are known for the infinite horizon. Even if the command signals are known priori for the infinite horizon, the feedforward part of the optimal controller can not be computed easily.

Recently the concept of receding horizon control in the regulator problem is developed in papers [1],[2] for the continuous-time systems and extended to the case of the discrete-time systems [3].

The receding horizon concept is also applied to the tracking problems for the discrete-time systems [4]. In [7] this receding horizon tracking control law is shown to be equivalent to the well known predictive controller 'GPC' (Generalized Predictive Control [7]).

In this paper the receding horizon concept will be applied to the tracking problem for the continuous-time systems. The obtained control law will be called the receding horizon tracking control [RHTC] for the continuous time systems. Unlike the infinite-time optimal tracking problem, the RHTC requires a known command signal over the finite horizon. Even though the finite horizon command signal is employed, the computation of the feedfoward part of the RHTC requires some efforts.

So this paper suggest approximate receding horizon tracking controller which can be implemented easily. (we call it ARHTC from now). This controller assumes that the command signals can be approximated by the stepwise functions. It is reasonable in most real plant process because the only smooth component of command signals are important.

To guarantee zero offset in tracking of constant command signal, integrator must be included in this controller. It is accomplished by introducing a modified cost function and solving the same problem for the augmented object systems.

This work is organized as follows. In Section 2, the problems are formulated in the receding horizon concepts. In Section 3, the control law of the continuous-time RHTC is derived. In Section 4, approximate receding horizon tracking control (ARHTC) is derived and its implementation and the properties are discussed. The method of including the integrator into the RHTC controller is discussed in Section 5. Simulation studies are provided in Section 6. It is demonstrated that the presented controller gives superior performance to the conventional steady state optimal tracking control. Finally conclusions are given in section 7.

2. PROBLEM FORMULATION

We consider a linear-time invariant continuous-time system described by

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}u(t)\\ y(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned} \tag{1}$$

Where $\mathbf{x}(t)\epsilon R^n$,$u(t)\epsilon R^m$ and $y(t)\epsilon R^p$ are the state vector, the control vector and the output vector respectively.

At the present time t, it is assumed that the command signal $z[\cdot]$ over the future horizon $[t,t+T]$ are available.

First, the problem is obtaining the control input $u[\sigma]$, $\sigma \,\epsilon\, [t,t+T]$ which minimize the quadratic cost:

$$\mathbf{J}(t,T) = \frac{1}{2}e^T(t+T)\mathbf{F}e(t+T) + \frac{1}{2}\int_t^{t+T}[e^T(\sigma)\mathbf{Q}e(\sigma) + u^T(\sigma)\mathbf{R}u(\sigma)]d\sigma \tag{2}$$

Where $e(\sigma) = z(\sigma) - y(\sigma)$

With the assumption that T is specified, that R is positive definite and Q,F are positive semi-definite.

In conventional optimal tracking control, $u[\sigma]$, $\sigma \,\epsilon\, [t,t+T]$ is applied to the controlled systems and at time t+T the above procedure is repeated over the horizon [t+T,t+T+T']. But the receding horizon concept utilize the only u(t) at time t and the control horizon [t,t+T] is moving continuously. At arbitrary time t, u(t) is the control which minimizes the cost over the horizon [t,t+T] and after Δt (where Δt is arbitrary small time interval), $u(t+\Delta t)$ minimize the cost over the horizon $[t+\Delta t, t+T+\Delta t]$.

3. CONTINUOUS-TIME RHTC

If t is fixed (for example t_0) in equation (2), the solution is obtained by the conventional optimal problem as follows [5].

$$u(\sigma) = \mathbf{R}^{-1}\mathbf{B}^T[g(\sigma) - \mathbf{K}(\sigma)\mathbf{x}(\sigma)] \qquad \sigma \in [t, t+T] \tag{3}$$

Therefore the solution u(t) which we want is given as follows.

$$u(t) = \mathbf{R}^{-1}\mathbf{B}^T[g(t) - \mathbf{K}(t)\mathbf{x}(t)] \tag{4}$$

The n × n real symmetric and positive definite matrix K(t) is obtained from the solution $\mathbf{K}(\sigma)$ of the Riccati differential equation.

$$\begin{aligned}\dot{\mathbf{K}}(\sigma) = &-\mathbf{K}(\sigma)\mathbf{A} - \mathbf{A}^T\mathbf{K}(\sigma)\\ &+ \mathbf{K}(\sigma)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}(\sigma) - \mathbf{C}^T\mathbf{Q}\mathbf{C}\\ &\sigma \in [t, t+T]\end{aligned} \tag{5}$$

Where $\dot{\mathbf{K}}(\sigma) = \partial\mathbf{K}(\sigma)/\partial\sigma$.
With the boundary condition

$$\mathbf{K}(t+T) = \mathbf{C}^T\mathbf{F}\mathbf{C} \tag{6}$$

The vector g(t) (with n-components) is obtained from $g(\sigma)$, $\sigma \in [t, t+T]$, solution of the following linear vector differential equation.

$$\dot{\mathbf{g}}(\sigma) = -[\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}(\sigma)]^T\mathbf{g}(\sigma) - \mathbf{C}^T\mathbf{Q}z(\sigma) \qquad \sigma \in [t, t+T] \tag{7}$$

With the boundary condition

$$g(t+T) = \mathbf{C}^T\mathbf{F}z(t+T) \tag{8}$$

We note that the Riccati differential equation (5) and boundary condition (6) are independent of the desired ouput $z(\sigma), \sigma \in [t, t+T]$. This means that the gain $\mathbf{K}(\sigma)$ is completely specified, once the system, the cost (F,Q,R) and the termianl time interval T are specified.

Especially K(t) is constant gain in the linear time invariant system, that is, the receding horizon controller have a constant feedback gain K = K(t).

Also the gain K is identical to that of the receding horizon regulation problem from the identifiation of the Riccati differential equation [1][2]. This means that the feedback structure of the receding horizon tracking control is the same as that of the receding horizon ouptput regulation control case, that is, the properties of stability and robustness are identical to the previous results of regulation problems.

Since in the receding horizon control the finite horizon is assumed, the weigthing matrix F,which is essentially a design parameter,plays a crucial role in determining the properties of the stability in this presented control.

case 1 : $\mathbf{F} = 0$
case 2 : $\mathbf{F} = \infty$
case 3 : $\{\ \mathbf{F} : \mathbf{F} > 0\ ,\ \ \mathbf{A}^T\mathbf{F} + \mathbf{F}\mathbf{A} + \mathbf{F}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{F} + \mathbf{Q} \leq 0\ \}$

For the above three different case of **F**,the following several results is proved in Kwon and Pearson's[2] and these facts are aslo applied to continuous-time RHTC.

Fact 1 : If {**A**,**B**} is controllable and $\mathbf{F} = 0, \mathbf{Q} > 0, \mathbf{R} > 0$, then there exists a finite horizon T, such that the above receding horizon tracking control law stabilizes the system.

Fact 2 : If {**A**,**B**} is controllable and $\mathbf{F} = \infty, \mathbf{Q} \geq 0, \mathbf{R} > 0$, then the above RHTC law stabilizes the system.

Fact 3 : If the **F** is belong to the case 3,then **K**,the solution of the Riccati equation (5), satisfies the following inequalities

$$\mathbf{K}(\tau)|_{T=\sigma_1} \geq \mathbf{K}(\tau)|_{T=\sigma_2} \quad \text{for } \tau \leq \sigma_1 \leq \sigma_2 \tag{9}$$

If **K** solution of (5) has lower and upper bound ,inequalities (9) is satisfied and {**A**,**B**} is controllable, then the above continuous time RHTC stabilizes the system.

The receding horizon tracking control is summarized in the following Fig. 3.1 and equations (10),(11),(12),(13).

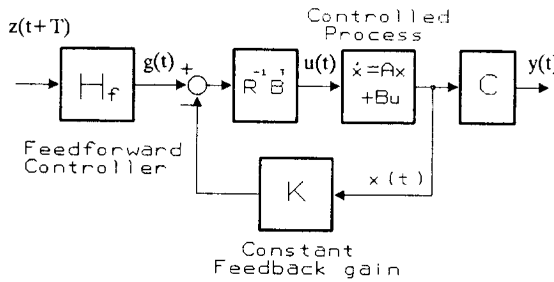


Fig. 3.1 Continuous time RHTC

$$u(t) = \mathbf{R}^{-1}\mathbf{B}^T[\mathbf{g}(t) - \mathbf{K}\mathbf{x}(t)] \tag{10}$$

Where K is obtained from the Riccati-differential equation (10) off line ($\mathbf{K} = \mathbf{K}(\tau)|_{\tau=0}$).

$$\begin{aligned}\dot{\mathbf{K}}(\tau) = &-\mathbf{K}(\tau)\mathbf{A} - \mathbf{A}^T\mathbf{K}(\tau)\\ &+ \mathbf{K}(\tau)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}(\tau) - \mathbf{C}^T\mathbf{Q}\mathbf{C}\\ &\tau \in [0, T]\end{aligned} \tag{11}$$

With $\mathbf{K}(T) = \mathbf{C}^T\mathbf{F}\mathbf{C}$

Feedforward controller H_f is

H_f :

$$\mathbf{g}(t) = g(\tau)|_{\tau=0} \tag{12}$$

$$\dot{g}(\tau) = -[\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}(\tau)]^T g(\tau) - \mathbf{C}^T\mathbf{Q}z(t+\tau) \qquad \tau \in [0, T] \tag{13}$$

With $g(T) = \mathbf{C}^T\mathbf{F}z(t+T)$
Where $K(\tau)$ has been obtained from equation (11) off line in advance.

We note that feedback part of this continuous RHTC is easily implementable in continuous-time state space. But feedforward controller H_f is not, because it is not simple that g(t) is obtained by solving the differential equation (13) continuously on line at arbitrary time t and because to solve the eq.(13), $\mathbf{K}(\tau)\ \tau \in [0, T]$ should be always saved somewhere.

So although the controlled process is linear time-invariant, feedforward part of the controller is not the form of constant gain controller

4. APPROXIMATE RHTC (ARHTC)

In Section 3, the defects of continuous-time RHTC is discussed. It is clear that if the feedforward controller is easily implementable, continuous-time RHTC is very useful to real plant because of its good performance. If the following form of feedforward controller is developed, the above defects can be solved easily.

$$\dot{\mathbf{g}}(t) = \Phi_g \mathbf{g}(t) + \Gamma z(t+T) \tag{14}$$

But in this paper, approximation method of feedforward controller is presented. This controller is approximate continuous RHTC (we call it ARHTC from here).

Consider arbitrary time interval T_i , $1 \le i \le N$ like the following

$$0 = T_1 < T_2 < \cdots < T_i < \cdots < T_N < T_{N+1} = T \tag{15}$$

and define the function $\Pi(\tau, l)$ as follows.

$$\Pi(\tau,l) = \begin{cases} 1 & \text{if } 0 \le \tau \le l \\ 0 & otherwise \end{cases} \tag{16}$$

Then we can obtain a set of functions

$$\Pi_i(\sigma) = \Pi(\sigma - t - T_i, T_{i+1} - T_i) \quad 1 \le i \le N \quad \sigma \in [t, t+T] \tag{17}$$

Given $z(\sigma)$, $\sigma \in [t,t+T]$, if only the information of $z(\sigma)$ at $\sigma = t + T_i$, $i = 1, \cdots, N$ are utilized, $z(\sigma)$ can be approximated as follows.

$$z(\sigma) \approx \sum_{i=1}^{N} \hat{z}_i(t)\, \Pi_i(\sigma) \qquad \sigma \in [t, t+T] \tag{18}$$

Where $\hat{z}_i(t) = z(t + T_i)$, i.e. the sampling value of command signal in future at the instant of $T_i + t$.

As ,in most real cases, the only low frequency part of the command signal is important, the appoximation of (18) is valid only if $T_{i+1} - T_i$ is appropriately small.

Let $\Psi(t, t_0)$ be the state transition matrix of the differential equation (7).
then

$$\mathbf{g}(t) = \Psi^{-1}(t+T,t)\mathbf{C}^T\mathbf{F}z(t+T) + \int_t^{t+T} \Psi^{-1}(\tau,t)\mathbf{C}^T\mathbf{Q}z(\tau)d\tau \tag{19}$$

From the above eq. (19) and approximation of eq. (18), the output of feedforward controller in continuous-time RHTC is given approximately.

$$\mathbf{g}(t) = \sum_{i=1}^{N} \hat{z}_i(t)\phi_i(t) \tag{20}$$

Where

$$\phi_i(t) = \Psi^{-1}(t+T,t)\mathbf{C}^T\mathbf{F}L_i + \int_{t+T_i}^{t+T_{i+1}} \Psi^{-1}(\tau,t)\mathbf{C}^T\mathbf{Q}d\tau \tag{21}$$

where

$$L_i = \begin{cases} 1 & \text{if } i = N \\ 0 & otherwise \end{cases}$$

Since the functions $\Pi_i(\sigma)$ in eq.(17) is not a function of t, but τ ($= \sigma - t$) and the state transition matrix $\Psi(\, , \,)$ of eq.(7) or eq.(13) are independent of present time t, t in (21) can be fixed to 0. So ϕ_i is given off-line and constant parameters in ARHTC, once T_i $i \in [1,N]$ is specified.

Therefore ϕ_i is given as follows.

$$\phi_1 = \int_0^{T_2} \Psi^{-1}(\tau,0)\mathbf{C}^T\mathbf{Q}d\tau \tag{22}$$

$$\phi_2 = \int_{T_2}^{T_3} \Psi^{-1}(\tau,0)\mathbf{C}^T\mathbf{Q}d\tau$$

$$\cdots \qquad \cdots$$

$$\phi_N = \int_{T_N}^{T} \Psi^{-1}(\tau,0)\mathbf{C}^T\mathbf{Q}d\tau + \Psi^{-1}(T,0)\mathbf{C}^T\mathbf{F}$$

Fig. 4.1 shows the structure of feedforward controller H_f of ARHTC. Also feedback structure is the same as that of continuous-time RHTC in Section 3 or receding horizon regulator.

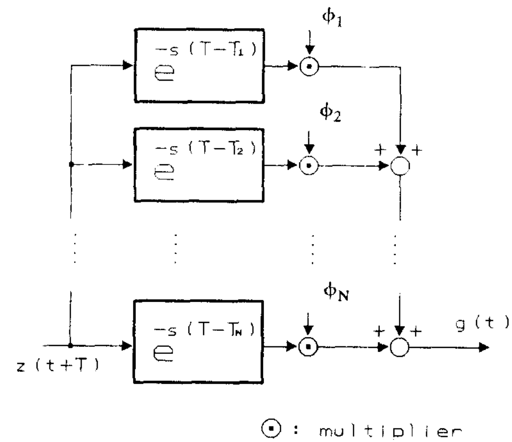


Fig 4.1 Feedforward Controller H_f of ARHTC

The design Parameters in ARHTC are as follows.

(a) Weight matix Q,R,F in cost (2)
(b) Time interval T in cost (2)
(c) Number of partitions N
(d) Partitions of interval [0,T] : $T_1, T_2, \cdots, T_N, T$

In designing the controller parameters (a), Q,R,F must be selected to satisfy the desired degree of stabilty and robustness primarily by classic LQ optimal control theory. It is suggested that T is sufficiently large for feedback gain K, solution of eq.(5) to be steady state solution(i.e. $\dot{\mathbf{K}} \approx 0$) and that T is not necessarily longer than the period of the command signals.

N is concerned with closeness to the continuous time RHTC. Larger N results in better performance in tracking the command signal. Since Ψ in (13) is usually ustable, Ψ^{-1} is stable. So it can be shown that if the $T_{i+1} - T_i$ is same for all i, influence of $z(t+T_i)$ is larger than that of $z(t+T_j)$ for i<j.

Therfore by choosing so that $T_{i+1}-T_i < T_{j+1}-T_j$ for i<j, more nice performance of tracking control is expected.

Tracking control in conventional optimal problem assumes that T is infinite time and that z(t+T) in future is constant, so the constant gain feedback controller and feedforward controller are obtained [5]. The latter assumption is identical to that N is fixed to T in proposed controller ARHTC.

The conventional optimal tracking controller is given as follows [5:pp801 -pp803].

$$u(t) = \mathbf{R}^{-1}\mathbf{B}_T [\, \hat{\mathbf{g}} - \hat{\mathbf{K}}\mathbf{x}(t) \,] \tag{23}$$

Where $\hat{\mathbf{K}}$ and $\hat{\mathbf{g}}$ are the steady state values of $\mathbf{K}$ and $\mathbf{g}(t)$ with the assumptions that T is infinite and z(t+T) = z(t) for all T. $\hat{\mathbf{K}}$ and $\hat{\mathbf{g}}$ is obtained from the following equations.

$$-\hat{\mathbf{K}}\mathbf{A} - \mathbf{A}^T\hat{\mathbf{K}} + \hat{\mathbf{K}}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\hat{\mathbf{K}} - \mathbf{C}^T\mathbf{Q}\mathbf{C} = 0 \tag{24}$$

$$\hat{\mathbf{g}} = -(\mathbf{G}^T)^{-1}\mathbf{C}\mathbf{Q}z(t) \tag{25}$$

Where $\mathbf{G} = \mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\hat{\mathbf{K}}$

5. CONTINUOUS-TIME RHTC WITH INTEGRATOR

It has been known that the derivative of control $\dot{u}(t)$ replacing control u(t) in the quadratic cost (2) gives integral action to the controller. In this way Continuous-time RHTC or ARHTC can possess integral action as well and it can be shown that the zero offset is guaranteed for a constant command signal. To do this we transform the model system (1) into the following augmented system:

$$\begin{aligned} \dot{\mathbf{x}}_a(t) &= \mathbf{A}_a\mathbf{x}_a(t) + \mathbf{B}_a u_a(t) \\ y(t) &= \mathbf{C}_a\mathbf{x}_a(t) \end{aligned} \tag{26}$$

where $\mathbf{x}_a(t)$, $u_a(t)$, $\mathbf{A}_a$, $\mathbf{B}_a$, and $\mathbf{C}_a$ are defined as

$$\mathbf{x}_a(t) = [\, \mathbf{x}(t)^T \;\; u(t) \,]^T, \qquad u_a(t) = \dot{u}(t)$$

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}, \quad \mathbf{B}_a = [\, \mathbf{O} \;\; \mathbf{B}^T \,]^T$$

$$\mathbf{C}_a = [\, \mathbf{C} \;\; \mathbf{O} \,]$$

where $\mathbf{O}$ is zero vector or matrix.

Then the control problem is minimizing the following cost function with satisfying the receding concept.

$$\mathbf{J}_a(t,T) = \frac{1}{2}e^T(t+T)\mathbf{F}e(t+T) + \frac{1}{2}\int_t^{t+T} [e^T(\sigma)\mathbf{Q}e(\sigma) + u_a^T(\sigma)\mathbf{R}u_a(\sigma)]d\sigma \tag{27}$$

By the same way as the above method, the optimizing $u_a(t) = \dot{u}(t)$ is obtained and the control input which is applied to plant is given through the integrator from $\dot{u}(t)$.

$$\begin{aligned} u(t) &= \int^t u_a(\tau)d\tau \\ &= \int^t \mathbf{R}^{-1}\mathbf{B}_a^T[\mathbf{g}_a(\tau) - \mathbf{K}_a(\tau)\mathbf{x}_a(\tau)d\tau \end{aligned} \tag{28}$$

Where $\mathbf{K}_a(\tau)$ and $\mathbf{g}_a(\tau)$ are obtained from (11),(12) and (13) with $\mathbf{A}$,$\mathbf{B}$ and $\mathbf{C}$ replaced by $\mathbf{A}_a$,$\mathbf{B}_a$ and $\mathbf{C}_a$

6. SIMULATION

The objected plant is the following 2-order single-input single-output linear time-invariant system.

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}u \\ y &= [\, 1 \;\; 0 \,]\mathbf{x} \end{aligned} \tag{29}$$

The parameters in cost (2) are

$$\mathbf{Q} = 4, \mathbf{R} = 1, \mathbf{F} = 5, T = 5.0 \tag{30}$$

The feedforward controller is designed by

$$\begin{aligned} N &= 10 \\ T_i &= \{\, 0, 0.05, 0.15, 0.3, 0.5, 0.8, 1.3, 2.0, 3.2, 4.8 \} \end{aligned} \tag{31}$$

The gain of feedback and feedforward ARHTC controller is obtained through numerical method with $\Delta t = 0.002$ by computer.

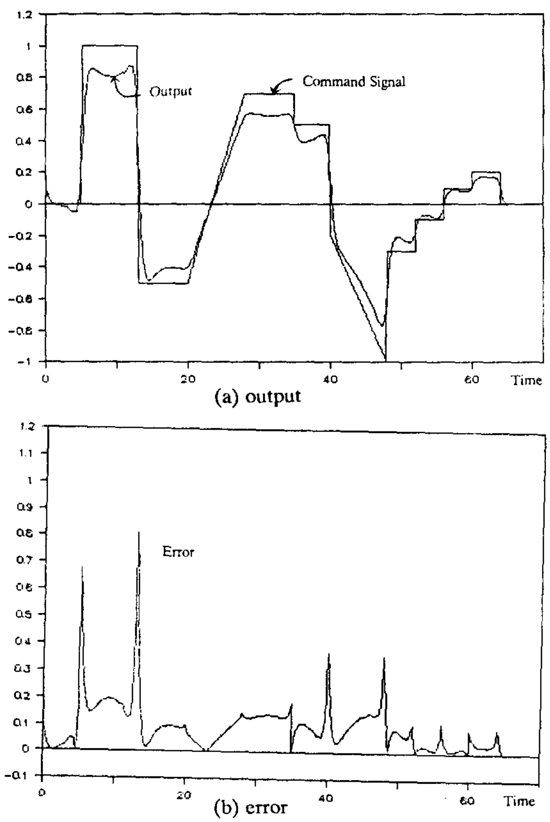
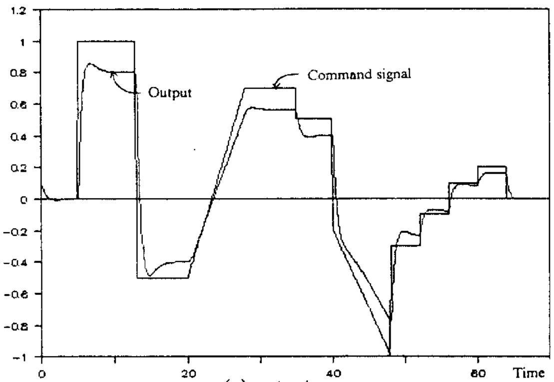
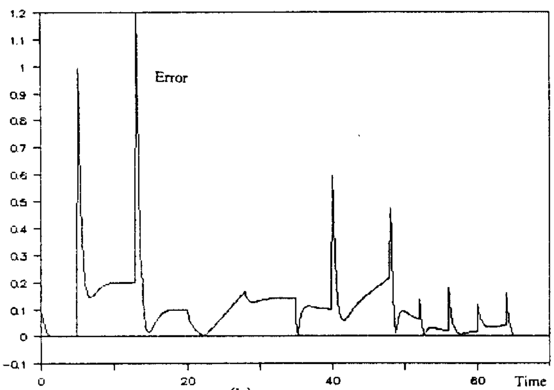


Fig. 1 (a) output and (b) error of ARHTC with Q=4

Fig.1 (a) shows that when ARHTC is used,the output of system is tracking the command signals from t=0 to 70 and Fig.1 (b) displays the error e(t)=z(t)−y(t) in the same case. Fig.2 (a),(b) are the results of simulation when conventional optimal tracking controller of the equation (23),(24),(25).



(a) output



(b) error

Fig. 2 (a) output and (b) error of conventional controller with Q=4

To compare the performance, let's introducce the following performance criteria.

$$\hat{J}(t,T) = \int_0^{70} [e^T(t)\mathbf{Q}e(t) + u^T(t)\mathbf{R}u(t)]dt \qquad (32)$$

The above criteria (32) include the accumulated error between desired command signals and the output of system and control costs over the simulation interval. Also it reflects Q and R of the design cost function (2).

The comparizion of performance for the both controllers in Fig.1 and Fig.2 is as follows

ARHTC	$\hat{J}$= 15.6748
Standrad Controller	$\hat{J}$ = 18.4011

From these data, it is clear that approximate RHTC controller gives better performance than the conventional optimal tracking controller of eq.(23), (24),(25) in the viewpoint of minimizing the criteria of eq. (32). Also from the Fig.1 (b) and Fig.2 (b),it can be known that when ARHTC is used, the closed loop system has small error at the abrupt transition of command signal,

7. CONCLUSION

Recently RHTC for the discrete-time state space is introduced and is known to be useful in real plant. In this paper a predictive controller,continuous-time RHTC, is proposed which is the continuous-time version of the previous RHTC [4] by introducing the receding horizon concept into the optimal tracking control problem. The properties of stability of this controller is discussed by the results of the receding horizon regulation problem.

Since the continuous time RHTC controller has a little complex feedforward part,a approximate continuous-time controller is proposed in this paper. Unlike the original continuous time RHTC,its feedforward control signal can be computed with constant gains, so its implementation is simple The method of obtaining the zero offset in tracking constant command.

By computer simulation,it is shown that the proposed controller gives better performance, especially at the time of abrupt transition of the command signal. It is probably due to the properties of the predictive control laws.

In continuous time state space, many results of stabilty and robustness of feedback control is available, especially in the area of optimal LQ control. The RHTC will share the good properties of the optimal LQ regulator problem since they are similar in nature.

More studies are demanded in developing more nice feedforward controller, or RHTC controller for the output feedback form. Also results of the application of continuous time RHTC to real process will be necessary.

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