

# A Millimeter-Sized Master-Slave Robot Driven by Conduit-Guided Wires (Part 1. Force and position control of a joint)

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## Abstract

This paper presents a fundamental study of a millimeter-sized master-slave robot driven by conduit-guided wires, which is expected to be applied to the delicate surgical operations, the assembling precise and small parts and so on.

This system consists of a millimeter-sized slave robot and a master manipulator of which the size is adapted to a human finger. Displacement and torque of the master side can be reduced and transferred to the slave robot by controlling the motor torque against the master torque by feeding back tension signals. The master can feel the tensions by the motor torque.

In this paper, the design method and making process of the master-slave system and the dynamical characteristic of displacement and torque control are proposed.

## 1. Introduction

A millimeter-sized robot is necessary for delicate surgical operations under microscope, manipulations of very small sized materials in area of biotechnology, and assemblies of small machines[1]. So far, some papers in the field of a millimeter-sized robot are reported[2],[3],[4]. These papers([2],[3]) proposed a millimeter-sized robot using shape memory alloy(=SMA). However, this robot can not control the displacement and torque of robot precisely because any displacement and torque sensor is not used, which is too small to be attached to the robot. Moreover, SMA actuator used in this robot is not powerful enough to drive the robot. These defects should be solved for the above

millimeter-sized robot.

In this paper, in order to improve the above defects, conduit-guided wires are used for designing a millimeter-sized robot, focusing on its merits of 1) very big transmitting force, whose maximum can be attainable at the yield stress of the wire, 2) precise displacement and torque control can be achieved by using sensors attached to the outside of a millimeter-sized robot. These wires are controlled by master-slave type operating system shown in Fig.1. This system have next advantages for the millimeter-sized robot, that is, a) the linear motion of the wire at the master side can be reduced and transferred to the millimeter-sized robot by the pulley(A), b) the torque of the master side can be reduced and transferred to the millimeter-sized robot by controlling the motor torque against the master torque by feeding back tension signals from straingages, c) the master can feel the tensions by the above motor torque. On the other hand, there is a following disadvantage. The tension sensors which are far from the millimeter-sized robot will bring torque control error due to the friction inside the conduit. This defect will be compensated by using the friction force model of the conduit-guided wires system.

The design method and making process of the master-slave system for a millimeter-sized joint shown in Fig.1 will be presented. Then the dynamical characteristics of the displacement and torque control of this system will be reported.

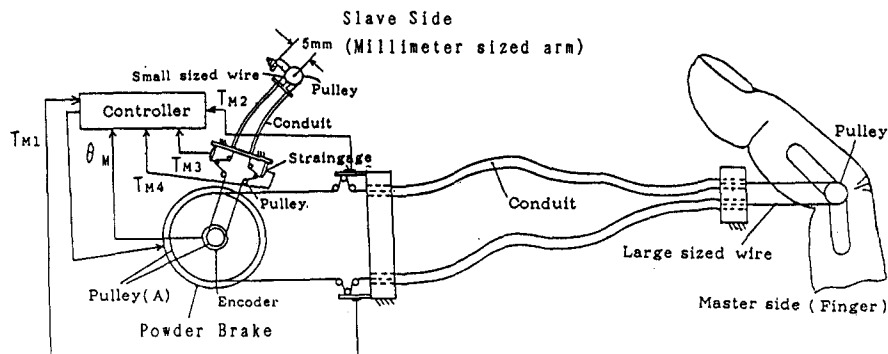


Fig.1 displacement-torque control a master-slave system

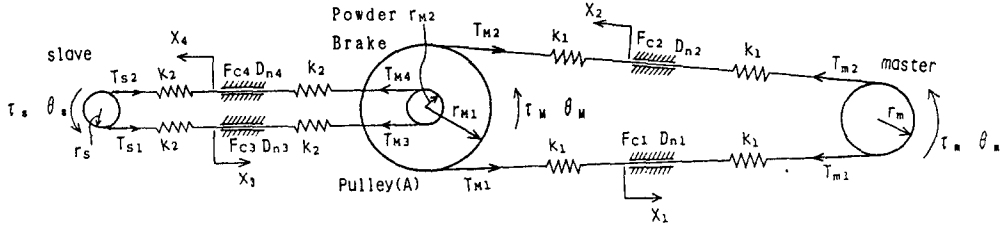


Fig.2 Model of displacement-torque control system

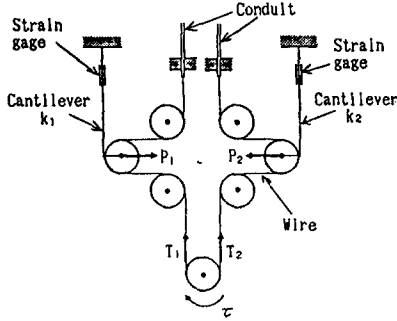


Fig.3 Detectors of wire-tension

## 2. Mechanical System Design

This system consists of a millimeter-sized slave arm and a master whose size is adapted to a human finger. A conduit is used for guiding a wire. The power is transmitted from the master to the slave by plastic coated multistrand steel wires which passes through conduits. For no slippage between the wire and pulley, the wire is fixed at a point of the pulley. Displacement and torque of the master is reduced and transferred to the slave robot arm, and the master perceives the force acting on the slave side, which is magnified and fedback to the master side. A schematic representation of the system is shown in Fig.2. Also, for detecting the tensions of wires, straingages is used as shown in Fig.3.

## 3. System Model

By using wires and conduits, the rubbing of the wire against the inner walls of the conduit results. Coulomb friction and linear and nonlinear damping forces contribute to the energy dissipative forces which are dependent on wire tension, speed and conduit geometry. The springs shown in Fig.2 represent the combined compliance of conduits and wires. Therefore, as taking friction and compliance into consideration, the following equations describing the dynamics of the system can be obtained[5].

$$\begin{aligned}\tau_m &= (T_{m1} - T_{m2}) \cdot r_m & (1) \\ T_{m1} &= k_1(r_m \cdot \theta_m - X_1) + T_{m10} & (2) \\ T_{m1} &= T_{m1} - \text{sgn}(\dot{X}_1) \cdot F_{c1} - D_{n1} \cdot \dot{X}_1 & (3) \\ T_{m1} &= k_1(X_1 - r_{m1} \cdot \theta_m) + T_{m10} & (4) \\ T_{m2} &= k_2(X_2 - r_m \cdot \theta_m) + T_{m20} & (5)\end{aligned}$$

$$T_{m2} = T_{m2} + \text{sgn}(\dot{X}_2) \cdot F_{c2} + D_{n2} \cdot \dot{X}_2 \quad (6)$$

$$T_{m2} = k_1(r_{m1} \cdot \theta_m - X_2) + T_{m20} \quad (7)$$

$$J \ddot{\theta}_m + b \dot{\theta}_m = \tau_m + (T_{m1} - T_{m2}) \cdot r_{m1} + (T_{m4} - T_{m3}) \cdot r_{m2} \quad (8)$$

$$T_{m3} = k_2(r_{m2} \cdot \theta_m - X_2) + T_{m30} \quad (9)$$

$$T_{s1} = T_{m3} - \text{sgn}(\dot{X}_3) \cdot F_{c3} - D_{n3} \cdot \dot{X}_3 \quad (10)$$

$$T_{s1} = k_2(X_3 - r_s \cdot \theta_s) + T_{s10} \quad (11)$$

$$T_{m4} = k_2(X_4 - r_{m2} \cdot \theta_m) + T_{m40} \quad (12)$$

$$T_{s2} = T_{m4} + \text{sgn}(\dot{X}_4) \cdot F_{c4} + D_{n4} \cdot \dot{X}_4 \quad (13)$$

$$T_{s2} = k_2(r_s \cdot \theta_s - X_4) + T_{s20} \quad (14)$$

$$J_s \ddot{\theta}_s + b_s \dot{\theta}_s = \tau_s - \tau_{ext} - \text{sgn}(\dot{\theta}_s) \cdot \tau_f \quad (15)$$

$$\tau_s = r_s(T_{s1} - T_{s2}) \quad (16)$$

where :

$b, b_s$  : linear damping coefficient about  $\theta_m, \theta_s$  axis respectively

$D_{n1}$  : linear damping in the conduits

$F_{c1}$  : Coulomb friction force between conduits and wires

$J, J_s$  : moment of inertia about  $\theta_m, \theta_s$  axis respectively.

$k_1$  : equivalent spring constant

$r_m$  : radius of a pulley of the master joint

$r_s$  : radius of a pulley of the slave joint

$\text{sgn}(\cdot)$  : sign which is identified that of velocity( $\cdot$ )

$T_{m1}$  : wire tension in around master joint

$T_{m1}$  : wire tension in around motor shaft

$T_{s1}$  : wire tension in around slave joint

$T_{m10}, T_{m20}, T_{s10}$  : wire pretensions

$X_1$  : wire displacement at the Coulomb friction element

$\theta_m, \theta_m, \theta_s$  : angular displacement

$\tau_m$  : torque of a pulley of the master joint

$\tau_s$  : torque of a pulley of the slave joint

$\tau_m$  : motor torque

$\tau_{ext}$  : external torque about  $\theta_s$  axis

$\tau_f$  : Coulomb friction torque at the manipulator joint

Outside the nonlinear part of backlash, the equation of motion can be linearized as follows.

From equations (2), (5), and (4), (7) :

$$T_{m1} - T_{m2} = T_{m1} - T_{m2} = k_1(r_m \theta_m - r_{m1} \theta_m) \quad (17)$$

From equations (9), (12), and (11), (14) :

$$T_{m3} - T_{m4} = T_{s1} - T_{s2} = k_2(r_{m2} \theta_m - r_s \theta_s) \quad (18)$$

Substituting equations (17), (18) into (8) :

$$J \ddot{\theta}_m + b \dot{\theta}_m = \tau_m + r_{m1} k_1(r_m \theta_m - r_{m1} \theta_m) - r_{m2} k_2(r_{m2} \theta_m - r_s \theta_s) \quad (19)$$

From equations (15),(16) and (18) :

$$J_s \ddot{\theta}_s + b_s \dot{\theta}_s = K_s r_s (r_{M2} \theta_M - r_s \theta_s) - \tau_{ext} \quad (20)$$

The above equations (19),(20) are the motion equations concerning pulley(A) and slave arm shaft respectively.

#### 4. Controller Design

The following linear control law is used to control the slave side torque and to feel it at the master side.

$$\tau_M = K_{M2}(T_{M4}-T_{M3}) + K_{M1}(T_{M1}-T_{M2}) \quad (21)$$

where :

$K_{M1}$  ,  $K_{M2}$  : Gain of feedback

##### 4.1 Torque Control

While putting  $J=b=0$  for simplicity , by substituting equation (21) into (8) :

$$(T_{M3}-T_{M4}) = \frac{r_{M1}+K_{M1}}{r_{M2}+K_{M2}} (T_{M1}-T_{M2}) = K(T_{M1}-T_{M2}) \quad (22)$$

Because  $K_{M1}$  is the feedback gain of brake torque of the powder brake,  $K_{M1}$  gets the region as follows :

$$K_{M1} < 0$$

Meanwhile, because the reduced torque from master side must be transmitted to the slave side, the coefficient of the right term of equation(22) must be below 1.  
Therefore,

$$\frac{r_{M1}+(K_{M1})_{min}}{r_{M2}+(K_{M2})_{max}} \leq K = \frac{r_{M1}+K_{M1}}{r_{M2}+K_{M2}} \leq 1 \quad (23)$$

Inside the above region, feedback gains  $K_{M1}$ ,  $K_{M2}$  should be selected. Then the tension ratio of equation (22) can be designed within the range of equation (23) arbitrary. By assuming that the system is linear and in case of  $J_s=b_s=D_{M1}=0$ , we will get the following equations, which demonstrate relationship between  $(T_{M1}-T_{M2})$  and  $(T_{S1}-T_{S2})$ ,  $\tau_m$  and  $\tau_s$  respectively.  
From equation (17),(18) and (22) :

$$T_{S1}-T_{S2} = \frac{K_{M1}+r_{M1}}{K_{M2}+r_{M2}} (T_{M1}-T_{M2}) \quad (24)$$

From equations(1),(16), and (22) :

$$\tau_s = \frac{(K_{M1}+r_{M1})r_s}{(K_{M2}+r_{M2})r_m} \tau_m \quad (25)$$

In case of linear system, the relations of  $(T_{M3}-T_{M4})$  to  $(T_{M1}-T_{M2})$ ,  $(T_{S1}-T_{S2})$  to  $(T_{M1}-T_{M2})$ , and  $\tau_s$  to  $\tau_m$  are plotted in Fig.4, Fig.5 and Fig.6 respectively. From Fig.5 and Fig.6, we know that there are linear relationships with slopes of  $K=(K_{M1}+r_{M1})/(K_{M2}+r_{M2})$  and  $K\alpha = \{(K_{M1}+r_{M1})r_s\} / \{(K_{M2}+r_{M2})r_m\}$  respectively.

Next, by considering nonlinear elements of the system and  $D_{M1}=0$ , the relationship between  $(T_{M1}-T_{M2})$  and  $(T_{S1}-T_{S2})$ ,  $\tau_m$  and  $\tau_s$  can be obtained as follows :

$$T_{S1}-T_{S2} = K(T_{M1}-T_{M2}) - K\{\text{sgn}(\dot{X}_1)F_{c1} + \text{sgn}(\dot{X}_2)F_{c2} - \{\text{sgn}(\dot{X}_3)F_{c3} + \text{sgn}(\dot{X}_4)F_{c4}\} \quad (26)$$

$$\tau_s = K\alpha \tau_m - K r_s \{\text{sgn}(\dot{X}_1)F_{c1} + \text{sgn}(\dot{X}_2)F_{c2} - r_s \{\text{sgn}(\dot{X}_3)F_{c3} + \text{sgn}(\dot{X}_4)F_{c4}\} \quad (27)$$

In case of considering nonlinear element in the system, the relations of  $(T_{S1}-T_{S2})$  to  $(T_{M1}-T_{M2})$ , and  $\tau_s$  to  $\tau_m$  are shown in dotted line in Fig.5 and Fig.6. The system exhibits a hysteresis effect due to backlash. From equations (26) and (27), width of backlash region are obtained respectively as follows :

$$\delta (T_{M1}-T_{M2}) = (F_{c1} + F_{c2}) + (F_{c3} + F_{c4})/K \quad (27a)$$

$$\delta \tau_m = r_m \cdot \delta (T_{M1}-T_{M2}) \quad (27b)$$

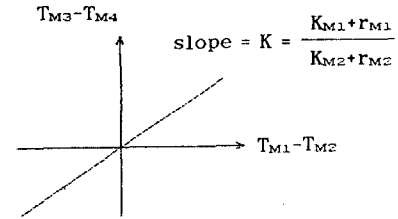


Fig.4 Relation between  $(T_{M1}-T_{M2})$  and  $(T_{M3}-T_{M4})$

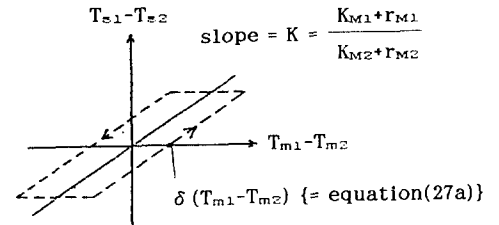


Fig.5 Relation between  $(T_{M1}-T_{M2})$  and  $(T_{S1}-T_{S2})$  and hysteresis effect of them

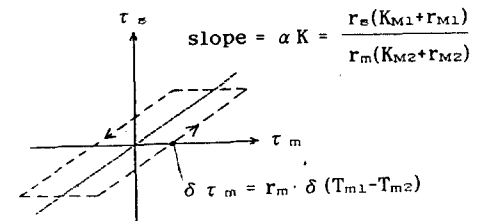


Fig.6 Relation between  $\tau_m$  and  $\tau_s$  and hysteresis effect of them

## 4.2 Angular Displacement Control

Assuming the case of  $J=b=J_s=b_s=D_{r1}=0$ , we will investigate the static relationship between  $\theta_m$  and  $\theta_M$ ,  $\theta_M$  and  $\theta_s$  respectively. From equations (2),(5), and (4),(7) :

$$\theta_M = \frac{r_m}{r_{M1}} \theta_m - \frac{\tau_m}{k_1 r_{M1} r_m} \quad (28)$$

From equations (9),(12), and (11),(14) :

$$\theta_s = \frac{r_{M2} \theta_M}{r_s} - \frac{\tau_s}{k_2 r_s^2} \quad (29)$$

Therefore, the static relationship between  $\theta_m$  and  $\theta_s$  can be obtained as follows :

$$\theta_s = \frac{r_{M2}}{\alpha r_{M1}} \theta_m - \frac{r_{M2} \tau_m}{k_1 r_s r_{M1} r_m} - \frac{\tau_s}{k_2 r_s^2} \quad (30)$$

Assuming that the system is nonlinear, then we can write the following equations.

$$\theta_M = \frac{r_m}{r_{M1}} \theta_m - \frac{\tau_m}{k_1 r_{M1} r_m} + \{ \text{sgn}(\dot{X}_1) F_{c1} + \text{sgn}(\dot{X}_2) F_{c2} \} \frac{1}{2k_1 r_m} \quad (31)$$

Similarly,

$$\theta_s = \frac{r_{M2}}{r_s} \theta_M - \frac{\tau_s}{k_2 r_s^2} - \{ \text{sgn}(\dot{X}_3) F_{c3} + \text{sgn}(\dot{X}_4) F_{c4} \} \frac{1}{2k_2 r_s} \quad (32)$$

Therefore, the static relationship between  $\theta_m$  and  $\theta_s$  can be obtained as follows :

$$\theta_s = \frac{r_{M2}}{\alpha r_{M1}} \theta_m - \frac{\tau_m r_{M2}}{k_1 r_m r_s r_{M1}} - \frac{\tau_s}{k_2 r_s^2} + \frac{\{ \text{sgn}(\dot{X}_1) F_{c1} + \text{sgn}(\dot{X}_2) F_{c2} \} r_{M2}}{2k_1 r_s r_{M1}} - \frac{\{ \text{sgn}(\dot{X}_3) F_{c3} + \text{sgn}(\dot{X}_4) F_{c4} \}}{2k_2 r_s} \quad (33)$$

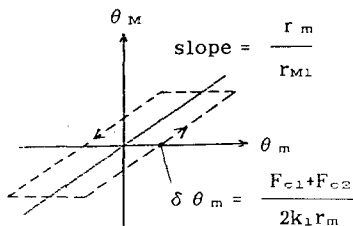


Fig.7 Relation between  $\theta_m$  and  $\theta_M$  and hysteresis effect of them

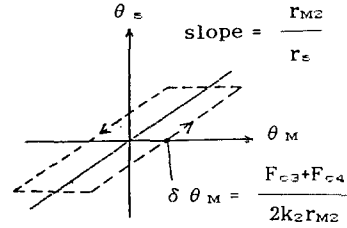


Fig.8 Relation between  $\theta_M$  and  $\theta_s$  and hysteresis effect of them

the static relationships between  $\theta_m$  and  $\theta_M$ ,  $\theta_M$  and  $\theta_s$  are shown in Fig.7 and Fig.8. In case of considering nonlinear elements, the relationships between  $\theta_m$  and  $\theta_M$ ,  $\theta_M$  and  $\theta_s$  are the part shown in dotted line in Fig.7 and Fig.8. From Fig.7 and 8, we obtain the linear relationship with slopes of  $(r_m/r_{M1})$  and  $(r_{M2}/r_s)$  respectively. From equations (31) and (32), width of backlash region are obtained as follows :

$$\delta \theta_m = \{F_{c1} + F_{c2}\} / 2k_1 r_m \quad (33a)$$

$$\delta \theta_M = \{F_{c3} + F_{c4}\} / 2k_2 r_{M2} \quad (33b)$$

## 4.3 Input-Output Relationship

For investigating the static relationship of inputs and outputs, by assuming that pretensions of the wires are equal and all of nonlinear terms are neglected, we will get block diagrams such as Fig.9 and 10. Assuming that  $\tau_m, \theta_s$  are input and  $\tau_{ext}, \theta_m$  are output, a block diagram representation of the mechanical system is shown in Fig.9. It is corresponding to control  $\tau_{ext}$  by handling  $\tau_m$ , in case that the slave arm is set to be the angular displacement  $\theta_s$  desired. Also, by assuming that  $\tau_{ext}, \theta_m$  are input and  $\tau_m, \theta_s$  are output, we will get a block diagram as shown in Fig.10. It is corresponding to control  $\theta_s$  to be angular displacement desired by handling  $\theta_m$ , in case that external torque  $\tau_{ext}$  apply to the slave arm.

From these block diagrams, the transfer functions of inputs and outputs are obtained as follows :

$$\theta_m = G_1(s) \tau_m + G_2(s) \theta_s \quad (34)$$

$$\tau_{ext} = G_3(s) \tau_m + G_4(s) \theta_s \quad (35)$$

$$\theta_s = G_5(s) \tau_{ext} + G_6(s) \theta_m \quad (36)$$

$$\tau_m = G_7(s) \tau_{ext} + G_8(s) \theta_m \quad (37)$$

where :

$$G_1(s) = B/A, G_2(s) = C/A, C = k_2 r_s (r_{M2} + K_{M2})$$

$$A = (Js^2 + bs + r_{M2} k_2 (r_{M2} + K_{M2})) \cdot r_m / r_{M1}$$

$$B = (Js^2 + bs + r_{M1}^2 k_1 + r_{M2}^2 k_2 + K_{M1} k_1 r_{M1} - K_{M2} k_2 r_{M2}) / r_m r_{M1} k_1$$

$$G_3(s) = (B k_2 r_s r_{M2} r_m / A r_{M1} - r_s r_{M2} k_2 / r_m r_{M1} k_1)$$

$$G_4(s) = (C k_2 r_s r_{M2} r_m / A r_{M1} - Js^2 - bs - r_s^2 k_2)$$

$$G_5(s) = B / (B G_4 - C G_3), G_6(s) = A G_3 / (B G_4 - C G_3)$$

$$G_7(s) = -C / (C G_3 - B G_4), G_8(s) = -A G_4 / (C G_3 - B G_4)$$

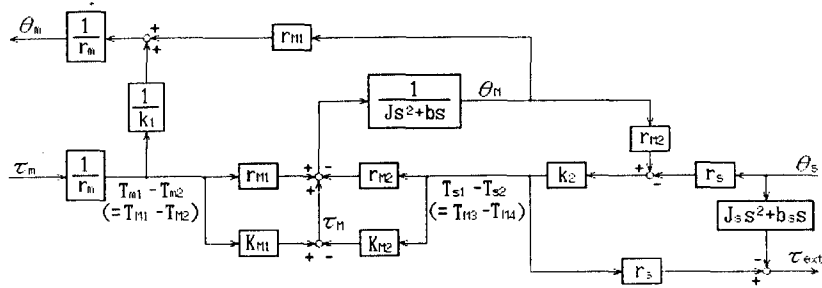


Fig.9 Block diagram of position-force control system. ( $\tau_m$ ,  $\theta_s$  : input,  $\tau_{ext}$ ,  $\theta_m$  : output)

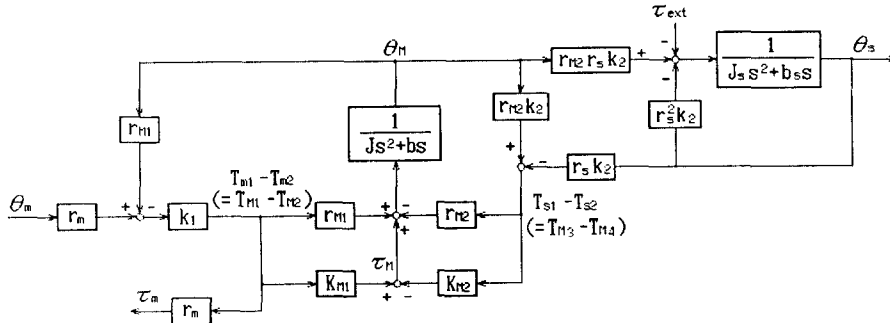


Fig.10 Block diagram of position-force control system. ( $\tau_{ext}$ ,  $\theta_m$  : input,  $\tau_m$ ,  $\theta_s$  : output)

From these transfer functions, by putting  $s=0$ , the static relationship of inputs and outputs are obtained as follows :

$$\theta_m = g_1 \tau_m + g_2 \theta_s \quad (38)$$

$$\tau_{ext} = g_3 \tau_m + g_4 \theta_s \quad (39)$$

$$\theta_s = g_5 \tau_{ext} + g_6 \theta_m \quad (40)$$

$$\tau_m = g_7 \tau_{ext} + g_8 \theta_m \quad (41)$$

where :

$$g_1 = 1/(k_1 r_m) + K/(\beta k_2 r_m), \quad g_2 = \alpha / \beta, \\ g_3 = -1/(k_2 r_s) - \beta / (K k_1 r_s), \quad g_4 = -\beta / \alpha, \\ g_5 = \alpha K, \quad g_6 = g_3 = 0, \quad g_7 = -1/(\alpha K), \quad (\beta = r_{m2}/r_{m1})$$

## 5. Conclusion

The design method and making process of a millimeter-sized master-slave robot driven by conduit-guided wires are proposed. Also, the dynamical characteristic of the displacement and torque control are presented.

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