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OPTIMAL TRAJECTORY TRACKING CONTROL OF A ROBOT MANIPULATOR

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ABSTRACT

In order to find the optimal control law for the precise trajectory tracking of a robot manipulator, a perturbational control method is proposed based on a linearized manipulator dynamic model which can be obtained in a very compact and computationally efficient manner using the dual number algebra. Manipulator control can be decomposed into two parts: the nominal control and the corrective perturbational control. The nominal control is precomputed from the inverse dynamic model using the quantities of a desired trajectory. The perturbational control is obtained by applying the second-variational method on the linearized dynamic model. Simulation results for a PUMA-560 robot show that, by using this controller, the desired trajectory tracking performance of the robot can be achieved, even in the presence of large initial positional disturbances.

1. INTRODUCTION

As the role of industrial robots is growing rapidly in the various areas of manufacturing processes, the necessity of precise path tracking control is also increasing not only to improve the productivity and quality of a job but also to broaden the range of potential applications of robots. In this context, there have been proposed numerous methods for robot manipulator control: the PD or PID control [1,2]; the local linearization method [3,4]; the computed torque technique [5,6]; the feedforward compensation method [7,8]; the nonlinear feedback control method [9,10,11]; the variable structure systems method [12,13,14]; the adaptive and learning control method [15,16,17,18], etc. However, except the PD (or PID) control method, most of those methods are kept out of practical implementation mainly because of its structural and computational complexity and/or inaccurate dynamic models. Although the PD (or PID) method is easy to implement, it generally lacks in the ability to achieve precise control since its controller gains are ususally determined without considering the dynamics of a controlled system.

In this paper, we use the perturbational control approach, for which the linearized dynamic model and required second-order partial derivative functions can be efficiently computed by using the dual number dynamic formulations [20]. Manipulator control may be decomposed into two control problems: the nominal control and the corrective perturbational control. The nominal control deals with how to determine the control torque vector which produces the predetermined nominal trajectory of a robot manipulator without concerning any kind of disturbances. The nominal control can be precomputed from the inverse dynamic model using the quantities of a desired trajectory.

The perturbational control deals with how to obtain the variational control torques which deviate from the nominal values in accordance with the variational trajectory generated due to various kinds of disturbances such as model uncertainties, parameter variations, contacts with environments, etc. To find the optimum perturbational control, the second-variational method is used on the linearized dynamic model. The two controls obtained as above are then summed to provide the input of the robot manipulator with an optimal torque vector to track the desired trajectory under disturbances.

The structure of this paper is as follows: In Section 2, the nominal feedforward control is reviewed using the inverse system concept. In Section 3, the linearized dynamic equations are stated, which can be conveniently obtained using the dual number algebra. In Section 4, an optimal tracking control problem is formulated for a robot manipulator. In Section 5, the optimal perturbational controller is derived using the sweep method based on the linearized dynamic model. In Section 6, simulation is performed using the actual parameter values of a PUMA-560 manipulator, and its results are shown. Finally, Section 7 discusses the results of the paper and draws some conclusions.

2. NOMINAL FEEDFORWARD CONTROL

By applying the inverse system theory [21], we can compute the input torque to the robot dynamic system as an output of its right inverse system:

$$\tau = \hat{H}(q)u + \hat{b}(q,\dot{q}) + \hat{g}(q) \quad (1)$$

where u is the input to the right inverse system and the hat symbols denote the quantities of the inverse system corresponding to those of the original system. Thus, from the desired output trajectory $\{q^0, \dot{q}^0, \ddot{q}^0\}$, the nominal input torque vector can be computed as follows.

$$\tau^0 = \hat{H}(q^0)\ddot{q}^0 + \hat{b}(q^0,\dot{q}^0) + \hat{g}(q^0) \quad (2)$$

Given the trajectory of a robot manipulator, the problem to find the required control torques is called the inverse dynamics of the robot. In an ideal environment where the perfect inverse dynamic model is available, by simply applying these torques to the input of a robot, we may control the manipulator to track the desired trajectory. In practical cases, however, there exist various kinds of disturbances due to parameter variations, model uncertainties, contact with environments, etc. Thus, in order to track the output trajectory as closely as possible, it is extremely important to find the perturbational control which minimizes the effects of all the disturbances.

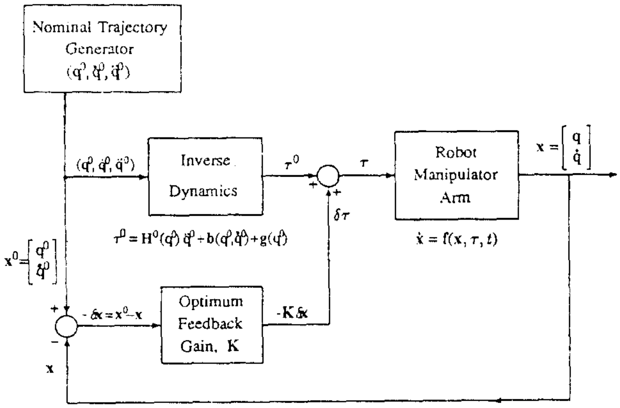

Figure 1. Block diagram for perturbational feedback control of a robot manipulator

Besides there may exist control variations due to the change of certain conditions such as the terminal constraints. In the following sections, we will formulate a perturbational control problem and solve for the perturbational control law which tracks the output trajectory of a manipulator as accurately as possible.

3. PERTURBATIONAL DYNAMIC EQUATIONS

Consider the dynamic equation for a manipulator of the following form:

$$\tau = \tau(q, \dot{q}, \ddot{q}) \tag{3}$$

where $q = [q_1, q_2, \cdots, q_N]^T$, $\dot{q} = dq/dt$ and $\ddot{q} = d\dot{q}/dt$, and variations in the trajectory of the manipulator about a nominal trajectory $\{q^0, \dot{q}^0, \ddot{q}^0\}$

$$q = q^0 + \delta q, \quad \dot{q} = \dot{q}^0 + \delta\dot{q}, \quad \ddot{q} = \ddot{q}^0 + \delta\ddot{q}. \tag{4}$$

Eq. (3) can be expanded about the nominal trajectory as follows:

$$\tau = \tau^0 + \delta\tau = \tau(q^0, \dot{q}^0, \ddot{q}^0) + A\delta\ddot{q} + B\delta\dot{q} + C\delta q + H.O.T. \tag{5}$$

where

$$A = \frac{\partial\tau(q, \dot{q}, \ddot{q})}{\partial\ddot{q}}, \quad B = \frac{\partial\tau(q, \dot{q}, \ddot{q})}{\partial\dot{q}}, \quad C = \frac{\partial\tau(q, \dot{q}, \ddot{q})}{\partial q}, \tag{6}$$

and H.O.T. stands for higher-order terms. Since $\tau^0 = \tau(q^0, \dot{q}^0, \ddot{q}^0)$ is a nominal dynamic equation, the perturbational dynamic equation is given by the following form:

$$\delta\tau(t) \approx A(t)\delta\ddot{q}(t) + B(t)\delta\dot{q}(t) + C(t)\delta q(t). \tag{7}$$

The coefficient matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$ and $\mathbf{C}(t)$ can be derived in both a recursive form and a nonrecursive form. In both cases, those matrices can be obtained in a very compact and compuatationally efficient manner using the dual number algebra [20].

The dynamic equation, Eq. (1), can be rewritten in the following state-space form:

$$\dot{x} = \begin{bmatrix} x_2 \\ -H^{-1}(x_1)\{b(x_1, x_2) + g(x_1)\} \end{bmatrix} + \begin{bmatrix} 0 \\ H^{-1}(x_1) \end{bmatrix}\tau \tag{8}$$

where $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T = (\mathbf{q}, \dot{\mathbf{q}})^T$. By denoting the right-hand side of this equation by $f(\mathbf{x}, \tau, t)$, we have

$$\dot{x}(t) = f(x, \tau, t). \tag{9}$$

Linearizing this equation along the nominal trajectory, we obtain the following perturbational dynamic equation in a state-space form:

$$\delta\dot{x}(t) = F(t)\delta x(t) + G(t)\delta\tau(t), \quad \text{with } \delta x(t_0) \text{ specified} \tag{10}$$

where

$$\delta x = \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} = \begin{bmatrix} \delta q \\ \delta\dot{q} \end{bmatrix} = \begin{bmatrix} q - q^0 \\ \dot{q} - \dot{q}^0 \end{bmatrix}, \tag{11}$$

$$F(t) = \frac{\partial f(x, \tau, t)}{\partial x} = \begin{bmatrix} 0 & I \\ -A^{-1}(t)C(t) & -A^{-1}(t)B(t) \end{bmatrix}, \tag{12}$$

$$G(t) = \frac{\partial f(x, \tau, t)}{\partial\tau} = \begin{bmatrix} 0 \\ A^{-1}(t) \end{bmatrix} \tag{13}$$

where the matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$ and $\mathbf{C}(t)$ are defined the same as Eq. (7).

4. FORMULATION OF A TRACKING CONTROL PROBLEM

For the problem formulation, we will use the short-written dynamic equation Eq. (9). We carry out the analysis with a fixed terminal time t_f to simplify the discussion. Thus, consider a general performance index of the following form:

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x, \tau, t)dt. \tag{14}$$

The problem is to minimize the performance index J subject to the system equations given by Eq. (9) with $\mathbf{x}(t_f)$ specified, and the following terminal constraints:

$$\psi(x(t_f), t_f) = 0. \tag{15}$$

The augmented performance index $\bar{J}$ is obtained by adjoining the constraints due to the originanal performance index J using Lagrange multipliers $\lambda(t)$ and ν.

$$\begin{aligned} \bar{J} &= J + \nu^T\psi(x(t_f), t_f) + \int_{t_0}^{t_f} \lambda^T(t)(f(x, \tau, t) - \dot{x})\,dt \\ &= \phi(x(t_f), t_f) + \nu^T\psi(x(t_f), t_f) + \int_{t_0}^{t_f} \left\{L(x, \tau, t) + \lambda^T(t)(f(x, \tau, t) - \dot{x})\right\}dt. \end{aligned} \tag{16}$$

We may now define the Hamiltonian as

$$H(x, \tau, \lambda, t) \overset{def}{=} \lambda^T(t)f(x, \tau, t) + L(x, \tau, t). \tag{17}$$

Then, the augmented performance index can be rewritten as:

$$\bar{J} = \phi(x(t_f), t_f) + \nu^T\psi(x(t_f), t_f) + \int_{t_0}^{t_f}\left(H(x, \tau, \lambda, t) - \lambda(t)^T\dot{x}\right)dt. \tag{18}$$

We now consider a perturbation in $\bar{J}$ due to the perturbations $\delta\tau$ and $\delta\mathbf{x}$ in the control $\tau(t)$ and the state $x(t)$, respectively. The perturbations are assumed to be weak ones such that $\delta\mathbf{x}(t)$ and $\delta\tau(t)$ are both small and bounded. From the necessary conditions for an extremum, we have:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = -F^T(t)\lambda(t) - \frac{\partial L(x, \tau, t)}{\partial x}, \quad \lambda(t_f) = \left[\frac{\partial\phi}{\partial x} + \left\{\frac{\partial\psi}{\partial x}\right\}^T\nu\right]_{t=t_f}, \tag{19}$$

$$\frac{\partial H}{\partial\tau} = 0. \tag{20}$$

By applying the necessary conditions, the nonlinear problem becomes the linear quadratic type problem with the cross-product terms of the variations $\delta \mathbf{x}$ and $\delta\tau$ in the integrand of the performance index: find $\delta\tau$ which minimizes

$$\delta \bar{J} = \frac{1}{2}\left[\delta \mathbf{x}^T\left\{\phi_{\mathbf{xx}} + (\psi_{\mathbf{x}}^T \nu)_{\mathbf{x}}\right\}\delta \mathbf{x}\right]_{t=t_f} + \int_{t_0}^{t_f}\left\{\frac{1}{2}\left[\delta \mathbf{x}^T\ \delta\tau^T\right]\begin{bmatrix} H_{\mathbf{xx}} & H_{\mathbf{x}\tau} \\ H_{\tau\mathbf{x}} & H_{\tau\tau}\end{bmatrix}\begin{bmatrix}\delta \mathbf{x} \\ \delta\tau\end{bmatrix}\right\}dt \quad (21)$$

subject to

$$-\delta\dot{\mathbf{x}}(t) + \mathbf{F}(t)\delta\mathbf{x}(t) + \mathbf{G}(t)\delta\tau(t) = 0, \quad \delta\mathbf{x}(t_0)\ \text{specified}, \quad (22)$$

$$\delta\psi = (\psi_{\mathbf{x}}\delta\mathbf{x})_{t=t_f}, \qquad \delta\psi \text{ is specified}, \quad (23)$$

where

$$H_{\mathbf{xx}} = \frac{\partial^2 H}{\partial \mathbf{x}^2},\ H_{\tau\tau} = \frac{\partial^2 H}{\partial \tau^2},\ H_{\mathbf{x}\tau} = \frac{\partial^2 H}{\partial \mathbf{x}\partial\tau},\ \psi_{\mathbf{x}} = \frac{\partial\psi}{\partial \mathbf{x}},\ \phi_{\mathbf{xx}} = \frac{\partial^2\phi}{\partial \mathbf{x}^2} \quad (24)$$

To determine variations in the control torques, $\delta\tau(t)$, which minimize $\delta\bar{J}$ subject to Eqs. (22) and (23), we define a new Hamiltonian using multipliers $\delta\lambda$ and $d\nu$.

$$\delta H \overset{def}{=} \delta\lambda^T\{\mathbf{F}\delta\mathbf{x} + \mathbf{G}\delta\tau\} + \frac{1}{2}\left\{\delta\mathbf{x}^T H_{\mathbf{xx}}\delta\mathbf{x} + \delta\tau^T H_{\tau\tau}\delta\tau + 2\delta\mathbf{x}^T H_{\mathbf{x}\tau}\delta\tau\right\} \quad (25)$$

By applying necessary conditions, we obtain $\delta\tau$:

$$\delta\tau(t) = -H_{\tau\tau}^{-1}(t)\left\{H_{\mathbf{x}\tau}^T(t)\delta\mathbf{x}(t) + \mathbf{G}^T(t)\delta\lambda(t)\right\}. \quad (26)$$

provided that $\mathbf{H}_{\tau\tau}$ is nonsingular. $\delta\mathbf{x}$ and $\delta\lambda$ are solutions of the following two-point boundary value problem.

$$\delta\dot{\mathbf{x}}(t) = \mathbf{M}(t)\,\delta\mathbf{x}(t) - \mathbf{D}(t)\,\delta\lambda(t), \quad \delta\mathbf{x}(t_0) = 0, \quad (27)$$

$$\delta\dot{\lambda}(t) = -\mathbf{E}(t)\,\delta\mathbf{x}(t) - \mathbf{M}^T(t)\,\delta\lambda(t),$$
$$\delta\lambda(t_f) = \left[\left\{\phi_{\mathbf{xx}} + (\psi_{\mathbf{x}}^T\nu)_{\mathbf{x}}\right\}\delta\mathbf{x} + \psi_{\mathbf{x}}^T d\nu\right]_{t=t_f} \quad (28)$$

where

$$\mathbf{M}(t) = \mathbf{F}(t) - \mathbf{G}(t)H_{\tau\tau}^{-1}(t)H_{\mathbf{x}\tau}^T(t), \quad (29)$$

$$\mathbf{D}(t) = \mathbf{G}(t)H_{\tau\tau}^{-1}(t)\mathbf{G}^T(t), \quad (30)$$

$$\mathbf{E}(t) = H_{\mathbf{xx}}(t) - H_{\mathbf{x}\tau}(t)H_{\tau\tau}^{-1}(t)H_{\mathbf{x}\tau}^T(t). \quad (31)$$

5. PERTURBATIONAL SOLUTION

The two-point boundary-value problem formulated in the previous section can be solved using the sweep method [19]. To do this, the following linear transformation equations are introduced:

$$\delta\lambda(t) = \mathbf{S}(t)\delta\mathbf{x}(t) + \mathbf{R}(t)d\nu \quad (32)$$

$$\delta\psi = \mathbf{R}^T(t)\delta\mathbf{x}(t) + \mathbf{Q}(t)d\nu \quad (33)$$

where $d\nu$ and $\delta\psi$ are constant infinitesimal vectors. $\mathbf{S}(t)$, $\mathbf{R}(t)$ and $\mathbf{Q}(t)$ are time-varying matrices. These transformations lead to the following matrix equations:

$$\dot{\mathbf{S}}(t) = -\mathbf{M}^T(t)\mathbf{S}(t) - \mathbf{S}(t)\mathbf{M}(t) + \mathbf{S}(t)\mathbf{D}(t)\mathbf{S}(t) - \mathbf{E}(t), \quad (34)$$
$$\mathbf{S}(t_f) = \left[\phi_{\mathbf{xx}} + (\psi_{\mathbf{x}}^T\nu)_{\mathbf{x}}\right]_{t=t_f}$$

$$\dot{\mathbf{R}}(t) = \left(\mathbf{S}(t)\mathbf{D}(t) - \mathbf{M}^T(t)\right)\mathbf{R}(t), \quad \mathbf{R}(t_f) = \left[\psi_{\mathbf{x}}^T\right]_{t=t_f} \quad (35)$$

$$\dot{\mathbf{Q}}(t) = \mathbf{R}^T(t)\mathbf{D}(t)\mathbf{R}(t), \quad \mathbf{Q}(t_f) = 0. \quad (36)$$

The perturbational torque vector $\delta\tau$ is then obtained as follows:

$$\delta\tau(t) = -H_{\tau\tau}^{-1}(t)\left[\left\{H_{\tau\mathbf{x}}(t) + \mathbf{G}^T(t)\left(\mathbf{S}(t) - \mathbf{R}(t)\mathbf{Q}^{-1}(t)\mathbf{R}^T(t)\right)\right\}\delta\mathbf{x}(t) + \mathbf{G}^T(t)\mathbf{R}(t)\mathbf{Q}^{-1}(t)\,\delta\psi\right] \quad (37)$$

This is a linear feedback control law which minimizes the performance index J, and can be rewritten in the following form:

$$\delta\tau(t) = -\mathbf{K}_1(t)\delta\mathbf{x}(t) - \mathbf{K}_2(t)\delta\psi, \quad (38)$$

where

$$\mathbf{K}_1(t) = H_{\tau\tau}^{-1}(t)\left\{H_{\tau\mathbf{x}}(t) + \mathbf{G}^T(t)\left(\mathbf{S}(t) - \mathbf{R}(t)\mathbf{Q}^{-1}(t)\mathbf{R}^T(t)\right)\right\} \quad (39)$$

$$\mathbf{K}_2(t) = H_{\tau\tau}^{-1}(t)\,\mathbf{G}^T(t)\mathbf{R}(t)\mathbf{Q}^{-1}(t). \quad (40)$$

Note that the optimal feedback gain matrices, $\mathbf{K}_1(t)$ and $\mathbf{K}_2(t)$, change with time but can be precomputed along the predetermined, desired nominal trajectory and stored for later use. No knowledge of the initial state, $\mathbf{x}_0$, is required for the computation of the gain matrices.

The second-order partial derivatives of the Hamiltonian H, such as $H_{\tau\tau}$, $H_{\tau x}$ and H_{xx}, constitute the above feedback gain matrices as well as the matrix differential equations, Eqs. (34) through (36). Hence, in order to evaluate the gain matrices, it is necessary to compute the second-order partial derivatives of the dynamic equations, and these partial derivatives are computed using the higher-order dual number algebra [20].

Finally, the overall control law for real-time optimal trajectory tracking is found by adding the above perturbational feedback control to the nominal control τ^0 which can be found using the inverse dynamics as given by Eq. (2).

$$\tau(t) = \tau^0(t) + \delta\tau(t) \quad (41)$$

6. SIMULATION AND RESULTS

To illustrate the use of the techniques developed in the previous sections, a case study for a PUMA-560 manipulator is made in this section. All the required computer programs are written in Ada (VADS version 5.5).

To find the optimal control law which tracks the desired trajectory, we used a performance index with the following quadratic forms:

$$\phi(\mathbf{x}(t_f), t_f) = \frac{1}{2}\left(\mathbf{x}(t_f) - \mathbf{x}_f^d\right)^T \mathbf{H}\left(\mathbf{x}(t_f) - \mathbf{x}_f^d\right) \quad (42)$$

$$L(\mathbf{x}, \tau, t) = \frac{1}{2}\left(\mathbf{x}(t) - \mathbf{x}^d(t)\right)^T\mathbf{Q}\left(\mathbf{x}(t) - \mathbf{x}^d(t)\right) + \frac{1}{2}\left(\tau(t) - \tau^d(t)\right)^T\mathbf{R}\left(\tau(t) - \tau^d(t)\right) \quad (43)$$

and the following constraint equations on final states:

$$\psi\left(\mathbf{x}(t_f), t_f\right) = \mathbf{x}(t_f) - \mathbf{x}_f^d = 0 \quad (44)$$

where $\mathbf{x}^d$ is a desired trajectory, and $\mathbf{Q}$ and $\mathbf{R}$ are positive semidefinite and definite matrices, respectively. Specifically, both $\mathbf{Q}$ and $\mathbf{R}$ are defined as diagonal matrices such that the diagonal elements of $\mathbf{Q}$ are set as 580.0, 100.0, 35.0, 250.0, 220.0, 40.0 30.0 3.5, 1.5, 1.0, 0.8, 1.0, and the values of the diagonal elements of $\mathbf{R}$ are set to be as 0.1, 0.04, 0.1, 0.37, 0.2, 0.35. We also assume that the terminal

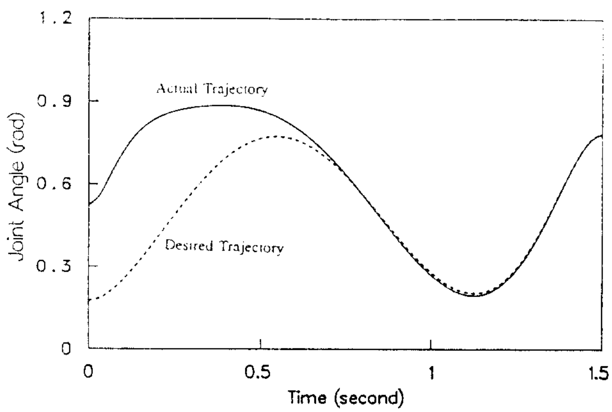

Fig.2. Desired and Actual Position Functions of Joint 3

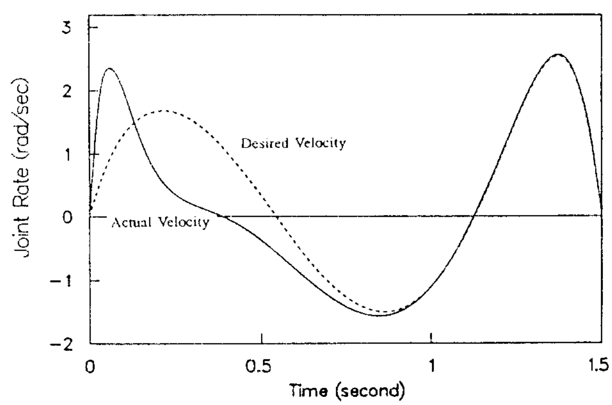

Fig.3. Desired and Actual Velocity Functions of Joint 3

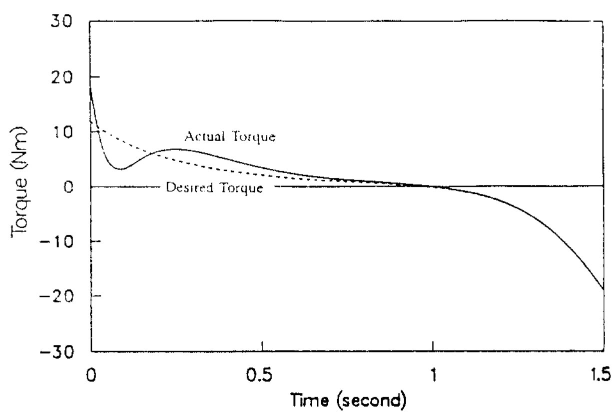

Fig.4. Desired and Actual Torque Functions of Joint 3

conditions are fixed so that $\delta\psi(t_f)=0$.Before simulation or actual real-time control, we precompute the nominal control torques, τ^d, and the optimum feedback gain matrix, **K**, for a perterbational control using the method described in the previous section.

Part of the simulation results are shown in Figs. (2) through (4). Fig.2 (joint position) and Fig.3 (joint rate) show the desired trajectory and actual trajectory of joint 3. Fig.4 shows the desired torque function and actual torque function. The graphs for the other joints are omitted due to space limitation. To test the robustness of the tracking controller, we introduced initial positional disturbances of various magnitude. In the presence of the large initial disturbances, the actual trajectories converge to the desired trajectories in a very satisfactory manner as shown in the figures, and at the final time, each actual trajectory becomes almost the same as the desired final state of each joint.

7. CONCLUSIONS

We have presented a method for precise trajectory tracking control of a robot manipulator. The gain matrix of the perturbational controller, as well as the nominal control, can be computed off-line, thereby eliminating real-time computation. The simulation results for a PUMA-560 robot show that, by using this controller, the trajectory of each manipulator joint converges in an excellent manner to the desired trajectory even in the presence of large initial positional disturbances.

The method presented in this paper facilitates the practical implementation of an advanced controller for a robot manipulator without the burden of real-time computational difficulty. Notice, however, that this method is not suitable for systems with constrained control torques. Extension to such a case is a subject for further research.

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REFERENCES

[1] J.Y.S. Luh, "Conventional Controller Design for Industrial Robots - A Tutorial," IEEE trans. Systems, Man, and Cybern., vol. SMC-13, no.3, pp. 298-316, 1983.

[2] S. Arimoto and F. Miyazaki, "Stability and Robustness of PID Feedback Control for Robot Manipulators of Sensory Capability," The First Int. Symp. Robotics Res., pp. 783-799, MIT Press, Cambridge, MA, 1983.

[3] D.L. Pieper, The Kinematics of Manipulators Under Computer Control, Ph.D. Dissertation, Stanford A.I. Lab Memo 72, Stanford University, CA, 1968.

[4] D.E. Whitney, "Resolved Motion Rate Control of Manipulators and Human Prothesis," IEEE Trans. on Man, Machine, System, pp. 47-53, Vol. MMS-10, 1969.

[5] R. P. Paul, "Modeling, Trajectory Calculation and Servoing of a Computer-Controlled Arm," Stanford Artificial Intelligence Laboratory, A.I. Memo 177, 1972.

[6] B.R. Markiewicz, "Analysis of the Computed Torque Drive Method and Comparison with Conventional Position Servo for a Computer Controlled Manipul-

ator," Tech. Memo 33-601, JPL, Mar. 1973.

[7] A. K. Bejczy, "Robot Arm Dynamics and Control," Technical Memo 33-669, Jet Propulsion Laboratory, Feb. 1974.

[8] A. Liegeois, A Fournier, & M. Aldon, "Model Referenced Control of High-Velocity Industrial Robots," Proc. Joint Automatic Control Conf., San Francisco, CA, pp. , Aug. 1980.

[9] E. Freund, "Fast Nonlinear Control with Arbitrary Pole-Placement for Industrial Robots and Manipulators," Int. Jour. Robotics Research, Vol. 1, No. 1, 1982.

[10] E.G. Gilbert and I.J. Ha, "An Approach to Nonlinear Feedback Control with Applications to Robotics," IEEE Conf. Decision and Control, San Antonio, 1983.

[11] T.J. Tarn et al., "Nonlinear Feedback in Robot Arm Control," Proc. 23rd IEEE Conf. Decision and Control, Las Vegas, Nevada, Dec. 1984.

[12] V.I. Utkin, "Variable Structure Systems with Sliding Modes," IEEE Trans. Auto. Control, Vol. AC 22, No. 2, pp. 212-222, Apr. 1977.

[13] J.J. Slotine, "The Robust Control of Robot Manipulators," Int. Jour. Robotics Research, Vol. 4, No. 2, 1985.

[14] K.-K. D. Young, "Controller Design for a Manipulator Using Theory of Variable Structure Systems," IEEE Trans. Systems, Man, Cybernetics, Vol. 8, No. 2, 1978.

[15] S. Dubowsky and D.T. DesForges, "The Application of Model-Referenced Adaptive Control to Robotic Manipulators," ASME Jour. Dyn. Syst. Meas. and Control, Vol. 101, pp. 193-200, 1979.

[16] A.J. Koivo and T.H. Guo, "Adaptive Linear Controller for Robotic Manipulators," IEEE Trans. Auto. Contr., Vol. AC 28, No. 2, pp. 162-170, Feb. 1983.

[17] H. Seraji, "Adaptive Independent Joint Control of Manipulators: Theory and Experiment," Proc. IEEE Int. Conf. Robotics and Automation, pp. 854-861, Philadelphia, 1988.

[18] S. Kawamura, F. Miyazaki, and S. Arimoto, "Applications of Learning Method for Dynamic Control of Robot Manipulators," Proc. 24th IEEE Conf. Decision and Control, pp. , 1985.

[19] A.E. Bryson and Y.C. Ho, Applied Optimal Control, Hemisphere, New York, 1975.

[20] G.-H. Lee and M.W. Walker, "Linearization of the Dynamics of a Robot Manipulator Using Dual Number Algebra," Proc. 18th Annual Conf. on Modeling and Simulation, pp. 1327-1334, Pittsburgh, Apr. 1987.

[21] R.H. Hirschorn, "Invertability of Multivariable Nonlinear Control Systems," IEEE Trans. on Automatic Control, Vol. AC-24, No. 6, pp. 855-865, Dec. 1979.