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An Iterative Learning and Adaptive Control Scheme for a Class of Uncertain Systems

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Abstract

An iterative learning control scheme for tracking control of a class of uncertain nonlinear systems is presented. By introducing a model reference adaptive controller in the learning control structure, it is possible to achieve zero tracking of unknown system even when the upperbound of uncertainty in system dynamics is not known apriori. The adaptive controller pull the state of the system to the state of reference model via control gain adaptation at each iteration, while the learning controller attracts the model state to the desired one by synthesizing a suitable control input along with iteration numbers. In the controller role transition from the adaptive to the learning controller takes place in gradually as learning proceeds. Another feature of this control scheme is that robustness to bounded input disturbances is guaranteed by the linear controller in the feedback loop of the learning control scheme. In addition, since the proposed controller does not require any knowledge of the dynamic parameters of the system, it is flexible under uncertain environments. With these facts, computational easiness makes the learning scheme more feasible. Computer simulation results for the dynamic control of a two-axis robot manipulator shows a good performance of the scheme in relatively high speed operation of trajectory tracking.

1 Introduction

The interest in application of learning control [2, 5, 7, 8, 11, 12, 13] to the repeatible dynamic systems, such as robot manipulators, has been growing because of its simplicity and straightforwardness. Since the iterative learning control does not require of knowledge of exact dynamic model of the system, it provides a flexible and adaptively operating characteristic under uncertain environments. One of significant advantages of the iterative learning control over the usual parametric adaptive control lies in its computational easiness due to the fact that it makes a direct correction to actuating signal for the exact response of the system, instead of input correction via parameter adaptation.

In this paper we present an iterative learning control method. Firstly, for a class of uncertain multi-input systems zero tracking is achieved via the learning control procedure with iteration number. Then it will be shown that the learning control can be applied for tracking control of unknown nonlinear dynamical systems by combining a model reference adaptive controller. A suitable reference input along the desired trajectory is systhesized by the learning controller, while the response of the model is followed by that of the plant through control gain adaptation in the adaptive controller. After perfect learning, the feedforward control input signal for the desired system response will be automatically generated from the learning controller, while the adaptive controller plays a main role at initial stage of learning.

The presentation of the paper is as follows. In section 2, an iterative learning control scheme for the target class of uncertain systems is introduced and zero tracking is achieved with iteration numbers. As an extension of the control scheme, in order to apply the learning control scheme to an unknown system in which the upperbound of the uncertain system dynamics are not available apriori, a model reference adaptive control technique is utilized. Finally, section 3 contains discussion and concluding remarks.

2 An Iterative Learning Control

2.1 Uncertain Multi-input Systems with Known Bound

Consider a class of multi-input linear systems described by

$$\dot{x}_L(t) = A_L x_L(t) + B_L U_L(t) \tag{1}$$

where $x_L \in R^{2n}$, $U_L \in R^n$, and A_L, B_L are given by

$$x_L(t) = \begin{bmatrix} x_{L1} \\ x_{L2} \end{bmatrix}$$

$$A_L = \begin{bmatrix} O_{n\times n} & I_{n\times n} \\ -\Lambda_{1_{n\times n}} & -\Lambda_{2_{n\times n}} \end{bmatrix}$$

$$B_L = \begin{bmatrix} O_{n\times n} \\ B_{L1} \end{bmatrix}.$$

We assume that $\Lambda_1, \Lambda_2, B_{L1}$ are unknown matrices, but known to be bounded such that

$A1)\quad ||\Lambda_j|| \le \lambda_j$

$A2)\quad 0 < \delta_1 I \le B_{L1} \le \delta_2 I$

$$A3) \quad ||\dot{B}_{L1}|| \le \sigma_1$$

for $j = 1,2$, where an induced matrix norm $||A||$ for $A \in R^{n\times n}$ is defined as

$$||A|| = sup\{M : M = \frac{|Av|}{|v|} \; for \; |v| \neq 0\}$$

with uclidean norm $|v|$ for $v \in R^n$.

Our control objective is as follows: for an addmissible trajectory $x_d(t)$ given in a compact subset of R^{2n}, it is desired for the uncertain system (1) to track x_d for all $t \in [0, t_f]$ with t_f being final time.

We will demonstrate in the sequal that, in spite of uncertainties in system dynamics, the control objective may be achieved by an iterative learning control method to be presented. The control input in the iterative learning control scheme consists of a feedback error signal from a linear controller and a feedforward control input to be adjusted through repetitive trial for each step. That is, as for the reference control input and the updatation law at the $i-th$ trial, we choose U_L^i and H^{i+1} such that

$$U_L^i(t) = H^i(t) + E_L^i(t) \tag{2}$$

$$H^{i+1}(t) = H^i(t) + \beta E_L^i(t) \tag{3}$$

$$E_L^i(t) = Lz^i(t) \tag{4}$$

where $L \in R^{n\times n}$ represents a positive definite feedback gain matrix to be chosen below and β a positive constant ($0 < \beta < 2$), and z^i stands for

$$z^i(t) = e_{L2}^i(t) + ae_{L1}^i(t)$$

for a positive constant a and $e_L^i(t) = x_d(t) - x_L^i(t)$ denotes the system state deviation from the desired command trajectory $x_d(t) \in R^{2n}$. Further, we set the initial conditions $e_L^i(0) = 0$ for all $i = 1,2,...$ and $H^1(t) = 0$ for all $t \in [0, t_f]$. The error equation is obtained as

$$\dot{e}_L^i = A_L e_L^i + B_L(U_d - U_L^i), \tag{5}$$

where U_d denotes the desired unknown input for the uncertain system (1) to track the desired state trajectory x_d.

Now, the system reponse of the reference model is shown to be convergent.

Theorem 1: Suppose that the feedback gain a and L are chosen such that

$$C1)B_0 \equiv (B_{L1} - \frac{1}{a}\dot{B}_{L_1}I)L > 0$$

$$C2)D_0 \equiv B_{L1}L > 0$$

$$C3)d_1 \equiv \lambda_{min}((2-\beta)D_0) - 2(a + \lambda_2) > 0$$

$$C4)d_2 \equiv \lambda_{min}((2-\beta)B_0 + aI) - \frac{1}{a}\lambda_1 > 0$$

$$C5)d_3 \equiv d_1 - \frac{\lambda_1^2}{a^2 d_2} > 0,$$

where D_0 denotes a symmetric positive definite matrix and $\lambda_{min}(\cdot)$ the minimum eigenvalue of $(\cdot)$.

Then the iterative learning control scheme (2),(3),(4) applied to the uncertain system (1) is convergent in the sense that $lim_{i\to\infty} e_L^i(t) = 0$ for all $t \in [0, t_f]$.

Proof: At $i = 1$, it is not difficult to show the exisitence of feddback gains a, L such that for a given constant e_0

$$|e_L^1| \le \frac{\delta_2 |U_d|}{\alpha} < e_0$$

where a positive constant α depends on feedback gains a, L and system constants λ_j. Hence, V^1 is bounded for all $t \in [0, t_f]$. At $i \ge 2$, define an index functional

$$V^i(t) = \int_0^t (z^i(\tau))^T \Gamma(z^i(\tau)) d\tau \qquad \text{for } t \in [0, t_f] \tag{6}$$

where

$$\Gamma = \beta D_0$$

Letting

$$z(t) = z^{i+1}(t) - z^i(t)$$

$$w(t) = e_{L1}^{i+1}(t) - e_{L1}^i(t),$$

we get

$$z(t) = \dot{w}(t) + aw.$$

Substituting (2) for $U_L^i(t)$ in (5) yields

$$\begin{bmatrix} \dot{e}_{L1}^i \\ \dot{e}_{L2}^i \end{bmatrix} = \begin{bmatrix} O & I \\ -\Lambda_1 - aD_0 & -\Lambda_2 - D_0 \end{bmatrix} \begin{bmatrix} e_{L1}^i \\ e_{L2}^i \end{bmatrix} + \begin{bmatrix} O \\ B_{L1} \end{bmatrix} (U_d - H^i) \tag{7}$$

By subtracting this from the equation at $(i+1)-th$ iteration and using (3), we obtain

$$\dot{z} = -(D_0 - aI + \Lambda_2)z - a(aI - (\Lambda_2 - \frac{1}{a}\Lambda_1))w - \Gamma z^i. \tag{8}$$

Define

$$\Delta V(t) = V^{i+1}(t) - V^i(t).$$

Then, after some calculation with z^i, z, we obtain

$$\begin{aligned} \Delta V &= \int_0^t (z^T \Gamma z + 2z^T \Gamma z^i) d\tau \\ &= -z^T z - \int_0^t (z^T D_1 z \\ &+ 2az^T(aI - (\Lambda_2 - \frac{1}{a}\Lambda_1))w) d\tau \\ &= -z^T z - a(2-\beta) w^T D_0 w \end{aligned}$$

$$
\begin{aligned}
&- \int_0^t (\dot{w}^T D_1 \dot{w} + \dot{w}^T \Lambda_1 w + w^T \Lambda_1 \dot{w} + a^2 w^T D_2 w) d\tau \\
&\leq -z^T z - a(2-\beta) w^T D_0 w \\
&- \frac{1}{2} \int_0^t (d_3(a^2 w^T w + \dot{w}^T \dot{w})) d\tau \\
&\leq 0, \qquad (9)
\end{aligned}
$$

where

$$
\begin{aligned}
D_1 &\equiv (2-\beta) D_0 - 2aI + 2\Lambda_2 \\
D_2 &\equiv (2-\beta) B_0 + aI + \frac{1}{a}\Lambda_1.
\end{aligned}
$$

Therefore, $V^i(t)$ converges to a constant value forcing $\Delta V(t)$ to be driven to zero. In addition, since

$$
\Delta V \leq -z^T z \leq 0, \qquad (10)
$$

it is clear that $z(\dot{w}$ and $w) \to 0$ for all $t \in [0, t_f]$ and hence $\dot{z} \to 0$ as i goes to infnity for all $t \in [0, t_f]$. With this fact the equation (8) implies that $z^i(e^i_{L1}$ and $e^i_{L2}) \to 0$ as $i \to \infty$. $\Delta\Delta$

From this result and the learning rule (3), one can see that the feedforward control input H^i converges to a constant value for each $t \in [0, t_f]$. That is, $|H^{i+1} - H^i| \to 0$ as $i \to \infty$. Actually, the eror equation (7) implies that the fixed value is the desired control input U_d for each $t \in [0, t_f]$, since B_{L1} is positive definite.

Figure 1 shows the schematic diagram of learning controller with a feedback and fedforward controller. After perfect learning, the feedforward controller is equiped with an inverse dynamic model of the uncertain plant. Note also that for the updation and generation of the sampled feedforward control input sequence H^i in the learning controller, a mapping rule similar to the one introduced in [8] may be utilized to save the memory storage.

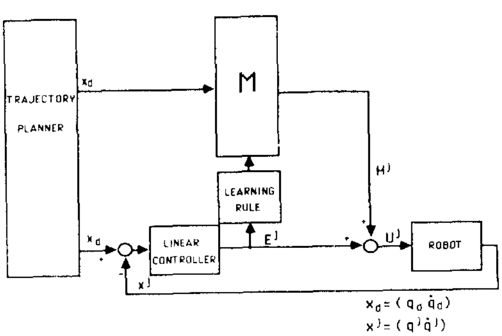

Figure 1: Schematic Diagram of the Learning Controller

2.2 Extension to a Class of Nonlinear Systems with Unknown Bound

In this subsection we will apply the iterative learning control scheme to a class of unknownn nonlinear systems by providing a suitable reference input for a model reference adaptive controller. Let the target class of nonlinear systems be described by the following set of equations.

$$
\dot{x}(t) = A(x(t))x(t) + B(x(t))U \qquad (11)
$$

where

$$
\begin{aligned}
A(x(t)) &= \begin{bmatrix} O_{n\times n} & I_{n\times n} \\ A_1(x(t)) & A_2(x(t)) \end{bmatrix} \\
B(x(t)) &= \begin{bmatrix} O_{n\times n} \\ B_1(x(t)) \end{bmatrix}
\end{aligned}
$$

with $x \in R^{2n}$ and the control input vector $U \in R^n$, and bounded unknown matrices $A_1, A_2, B_1 \in R^{n\times n}$ and B_1 positive definite.

Since the upperbound of system submatrices are assumed not to be known apriori, the iterative learning control in previous subsection can not be applied directly. Hence, a model reference adaptive controller is incorporated so as to achieve zero tracking of the unknown system (11). Assume that a reference model is given by (1) in which the submatrices Λ_1, Λ_2 are chosen to be matrices of strictly Herwitz and B_{L1} positive definite constant matrix. Let the control input for the plant (11) at $i-th$ iteration be

$$
U^i(t) = K^i_x(t) x^i(t) + K^i_u(t) U^i_L(t), \qquad (12)
$$

where K^i_x, K^i_u are given by

$$
\begin{aligned}
K^i_x &= \left[\, K^i_{x_1} | K^i_{x_2} \, \right]_{n\times 2n} \\
K^i_{x_j} &= (k^i_{x_j})_{n\times n} \\
K^i_u &= (k^i_u)_{n\times n}
\end{aligned}
$$

for $j = 1, 2$.

Further, we assume that the rate of control gain adaptation in (12) is much faster than that of the system matrices change at each sampling interval. Then, by utilizing the MRAC theory, we obtain an adaptation law for the control input gains K_x and K_u such that

$$
\begin{aligned}
\dot{K}^i_x &= \gamma_1 E^i_p x^{i^T} \\
\dot{K}^i_u &= \gamma_2 E^i_p U^{i^T}_L, \qquad (13)
\end{aligned}
$$

where E^i_p denotes

$$E_p^i = P_2 e_1^i + P_3 e_2^i$$

with $e^i = x_L^i - x^i$ and P_2, P_3 submatrices of a solution matrix P for the Lyapunov equation(Appendix).

In view of the result on learning control in the previous subsection along with the adaptive scheme, the operating characteristic of the closed-loop system is described as follows:

Theorem 2: Consider the class of unknown nonlinear systems (11), and the stable and minimal reference model (1) with the adaptive control action and the control gain adaptation law given by (12),(13). Let the reference control input $U_L^i(t)$ be synthesized iteratively via the learning rule (2),(3),(4) with the initial conditions $e_L^i(0) = 0$ for all $i = 1, 2, \ldots$. Then, the state $x^i(t)$ of the system (11) is convergent in the sense that $lim_{i \to \infty} e^i(t) = 0$ for all $t \in [0, t_f]$.

We give a brief sketch of a proof of the theorem.

Proof: At each trial, the model reference adaptive control mechanism will force the system state $x^i(t)$ to follow the response of the reference model $x_L^i(t)$, while the learning controller will drive $x_L^i(t)$ to the desired command state $x_d(t)$. This implies that the tri-states linkage in the controller is convergent such that $x^i(t) \to x_L^i(t) \to x_d(t)$ as $i \to \infty$. That is, $e^i(t)$ and $e_L^i(t)$ vanish asymptotically for all $t \in [0, t_f]$. $\Delta\Delta$

Figure 2 shows the schematic diagram of the controller. It consists of two feeedback loops for an adaptive and a learning controller:
the inner loop drives the system states to that of the reference model by tuning the control parameters with an adaptation algorithm and the outer loop synthesizes an appropriate control input for the state of reference model to follow the desired trajectory. At the initial stage of learning, where there may exist large tracking error, the adaptive controller will play a dominant role, while the learning controller will generate a desired input signal after perfect learning.

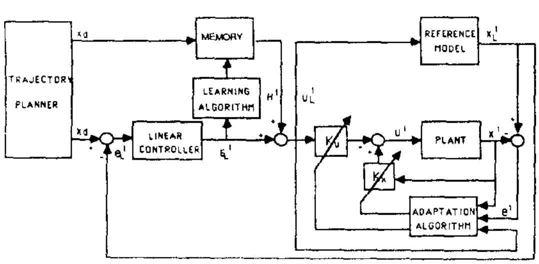

Figure 2: Schematic Diagram of the Adaptive and Learning Controller

An attractive feature of the controller described in this section is that it does not require the knowledge of any system parameters and is entirely based on the desired trajectory and the actual state trajectories of the system and the reference model which are directly available.

3 Discussion and Conclusions

An iterative learning control scheme for a class of uncertain dynamical systems is described in this paper. In case where the upperbound of the system submatrices are not known apriori, a model reference adaptive controller is introduced in the learning controller, so that they cooperate in the closed loop operation by exchanging their role in generating a suitable control input action and control parameters. An attractive feature of the control scheme is that it does not requires any knowledge of the dynamic parameters of the system. The simplicity of arithmetic operation invoved in calculation of the control action makes the proposed controller suitable for implementation in on-line control of the target class of dynamic systems with high sampling rates.

Appendix : Derivation of a MRAC

On applying the adaptive control input (12) to the nonlinear system (11), we obtain

$$\dot{x} = \hat{A}x + \hat{B}U_L, \tag{14}$$

where $\hat{A}, \hat{B}$ denote

$$\hat{A} = \begin{bmatrix} O & I \\ A_1 + B_1 K_{x1} & A_2 + B_1 K_{x2} \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} O \\ B_1 K_u \end{bmatrix}$$

and we suppressed the superscript i. The error equation for the reference model (1) and the controlled system (11) becomes

$$\dot{e} = A_L e + \tilde{A}x + \tilde{B}U_L \tag{15}$$

where

$$e = x_L - x$$

$$\tilde{A} = A_L - \hat{A} = \begin{bmatrix} O & O \\ \tilde{A}_1 & \tilde{A}_2 \end{bmatrix}$$

$$\tilde{B} = B_L - \hat{B} = \begin{bmatrix} O \\ \tilde{B}_1 \end{bmatrix}.$$

In deriving an adaptation law for the control gains, we take a Lyapunov function candidate

$$V(t) = e^T P e + tr(\tilde{A}^T R_1 \tilde{A}) + tr(\tilde{B}^T R_2 \tilde{B}) \tag{16}$$

where tr denotes trace with symmetric positive definite matrices P and R_1, R_2. Differentiating V along the system trajectory yields

$$\begin{aligned}\dot{V}(t) &= e^T(A_L^T P + PA_L)e \\ &+ 2tr(\tilde{A}^T(Pex^T + R_1\dot{\tilde{A}})) \\ &+ 2tr(\tilde{B}^T(PeU_L^T + R_2\dot{\tilde{B}})). \end{aligned} \quad (17)$$

Letting P be the solution of Lyapunov equation for the reference model

$$A_L^T P + PA_L = -Q, \quad (18)$$

and letting $\dot{\tilde{A}}, \dot{\tilde{B}}$ be

$$\begin{aligned}\dot{\tilde{A}}_1 &= -R_{13}^{-1}E_p x_1^T \\ \dot{\tilde{A}}_2 &= -R_{13}^{-1}E_p x_2^T \\ \dot{\tilde{B}}_1 &= -R_{23}^{-1}E_p U_L^T, \end{aligned} \quad (19)$$

where E_p denotes

$$E_p = P_2 e_1 + P_3 e_2,$$

and $P_2, P_3, R_{i3}(i = 1,2)$ represent submatrices of P and R_i such that

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} R_{11} & O \\ O & R_{13} \end{bmatrix}, R_2 = \begin{bmatrix} R_{21} & O \\ O & R_{23} \end{bmatrix},$$

we obtain

$$\dot{V}(t) = -e^T(t)Qe(t), \quad (20)$$

which is a negative definite function of ϵ, in view of a positive definite symmetric matrix Q. Applying the control input and adaptation law (12),(19) to the system (11), we have from (20) that the state error e asymptotically converges to zero in time.

However, in order for the adaptive law (19) to be feasible with unknown system matrices A, B, the adaptation law should be modified in light of detailed consideration of system dynamics. This is done via a reasonable assumption in the operating condition of closed-loop system. That is, since we have assumed that the controller gains K_x and K_u change relatively much faster than the unknown system matrices A, B in each sampling interval, the system matrices can be treated as slowly time-varying compared with the controller gains. This assumption implies that the adaptation law (19) is simplified as

$$\begin{aligned}\dot{\tilde{K}}_{x1} &= \gamma_1 E_p x_1^T \\ \dot{\tilde{K}}_{x2} &= \gamma_1 E_p x_2^T \\ \dot{\tilde{K}}_u &= \gamma_2 E_p U_m^T, \end{aligned} \quad (21)$$

where we have chosen $R_{i3}(i = 1,2) \in R^{n\times n}$ in (22) such that for positive constants γ_1, γ_2

$$R_{i3} = \frac{1}{\gamma_i} B_1^{-1}.$$

With the control input and gain adaptation law (12),(22), the negative semi-definite $\dot{V}(t)$ guarantees that the state error $\epsilon(t)$ is driven zero asymptotically as $t \to \infty$. However, since $\dot{V}(t)$ is not a function of system parameter error, it is not obvious whether the system parameter errors $\tilde{A}$ and $\tilde{B}$ will converge to zero. Nevertheless, note that the adaptation laws (22) and control input (12) do not depend on the system dynamic model (11), but depend only on the states and its errors between the reference model and the controlled system.

References

[1] James S. Albus,"Brains, Behavior, and Robotics," N.H. Byte Books, 1981.

[2] S. Arimoto, S.Kawamura, and F. Miyasaki, "Bettering operation of robots by learning," J. Robotic Syst., pp 123-140. 1984.

[3] A.Balestrino, G.De Maria, and L. Sciavicco, "An Adaptive Model Following Control for Robot Manipulators", ASME J. Dynam. Syst., Meas. Contr., Vol.105, pp143-151, Sept. 1983.

[4] A. K. Bejezy, " Robot Arm Dynamics and Control," Technical Memo 33-669, Jet Propulsion Laboratory, Pasadena, CA, 1974.

[5] Poala Bondi, Giuseppe Casaline, and Lucia Gambardella, " On the iterative learning control theory for robotic manipulators , " J.Robotics and Automation, Vol.4, No.1, Feb.1988.

[6] J.J. Craig, P.Hsu and S.S. Sastry, "Adaptive Control of Mechanical Manipulators", The Int. J. Robotics Res., Vol.6,No.2, Sum. 1987.

[7] Sadao Kawamura, Fumio Miyazaki, and Suguru Arimoto, " Realization of robot motion based on a learning method ," Trans. Sys. Man.,Cyber., Vol.18, No.1, Jan./Feb.1988.

[8] Tae-yong Kuc and Kwanghee Nam, "CMAC Based Iterative Learning Control of Robot Manipulators", IEEE Conf. Dec. Contr., Tampa Fl. Dec. 1989.

[9] C.S.G. Lee and M.J. Chung, "An adaptive control strategy for mechanical manipulators," IEEE Trans. Automatic Contr., AC-29, pp 837-840, 1984.

[10] Kye Y. Lim and M. Eslami, "Robust Adaptive Controller Designs for Robot Manipulator Systems", IEEE J. Robotics and Automation, Vol.RA-3, No.1,Feb.1987.

[11] W. Thomas Miller III, Filson H. Glanz, L. Gordon Kraft III, "Application of a general learning algorithm to the control of robotic manipulators, " Int.,J.Robotics R. Vol.6,No.2, Sum.1987.

[12] H. Miyamoto, M. Kawato, T. Setoyama, and R. Suzuki, " Feedback-Error-Learning Neural Network for Trajectory Control of a Robot Manipulator," N. Networks, Vol. 1, pp251-265, 1988.

[13] S. Oh, Z. Bien, and I.H. Suh, "An Iterative Learning Control Method with Application for the Robot Manipulator," IEEE J. Robotics and Aotomation, Vol.4,No.5, October 1988.

[14] F. Pourboghrat, "Adaptive Learning Control for Robots", IEEE Conf. Dec.Contr.,1988.

[15] H. Seraji,"A New Approach to Adaptive Control of Manipulators", J. Dynam. Syst., Meas., and Contr. Vol.109, Sep. 1987.

[16] J-J.E. Slotine and W. Lee, "Composite Adaptive Control of Robot Manipulators", Automatica, Vol.25, No.4, pp.509-519, 1989.

[17] M. Tsai and M. Tomizuka, "Model Reference Adaptive Control and Repetitive Control for Robot Manipulators", IEEE Conf. Robotics and Automation, 1989.