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A NEW OBJECT RECOGNITION ALGORITHM USING GENERALIZED INCREMENTAL CIRCLE TRANSFORM

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ABSTRACT

A method of recognizing 2-dimensional polygonal objects is proposed by using a concept of generalized incremental circle transform. The generalized incremental circle transform, which maps boundaries of an object into a circular disc, represents efficiently the shape of the boundaries that are obtained from digitized binary images of the objects. It is proved that the generalized incremental circle transform of an object is invariant to object translation, rotation, and size, and can be used as feature information for recognizing two dimensional polygonal object efficiently.

1. INTRODUCTION

Adoption of sensor-based automation in manufacturing is often motivated by increasing need of the flexibility of the system and widening the scope of applications of robotic systems. In particular, the machine vision is incorporated into the process of classifying objects, and many robotic assembly tasks. In these cases, the machine vision must perform reliable high speed object recognition for two dimensional robotic assembly[1-7] for enhanced productivity.

There are proposed numerous pattern recognition methods. One of the earliest of these techniques is the method of moments[1]. The method can be shown that a simple linear combination of moments have values which are invariant under a number of similarly transforms. Such *"moment invariants"* have been used for pattern recognition. The major disadvantage of this methodology is that although the first few moments convey significant information for simple objects, they fail to do so for more complicated ones[2].

Another possibility is to take the two-dimensional Fourier Transform[3-6]. The transform can be expressed in terms of tangent angle versus arc length, or as complex functions of characteristic functions of an object. The coefficients of the transform convey shape information, but their computation is quite extensive.

The medial axis transform(MAT), proposed by H. Blum[7], is one of the earliest but most widely studied techniques. In that approach, a "full" figure is transformed into a set of line drawing points that are in the boundary. This skelton is then used to derive information about the shape of the original figure, but the computation of the skeleton can be quite time consuming and very sensitive to noise.

Summing up, one may note that existing methods suffer from common problems of expensive computational inefficiency and sensitivity to noise. In this paper, we define a new concept of generalized incremental circle transform and propose a fast and efficient object recognition algorithm that can be applicable for two dimensional polygonal objects based on the generalized incremental circle transform.

2. THEORY OF GENERALIZED INCREMENTAL CIRCLE TRANSFORM

Let us consider a simple(non-self-intersecting) closed curve, **C**. This curve may represent the boundary contour of an occlusion-free object lying on the x-y plane and can be expressed by a parametric vector function

$$\boldsymbol{\alpha}(l) = (x(l), y(l)), \quad 0 \le l \le L$$

here L denotes the total length of the curve **C**, such that for each l,

$$\boldsymbol{\alpha}(l + L) = \boldsymbol{\alpha}(l)$$

It is assumed that an increment of l in $\boldsymbol{\alpha}(l)$ results in counterclockwise(CCW) directional displacement in the curve $\mathbf{C}$.

As a means of effective shape description of a curve $\boldsymbol{\alpha}(l)$, the notion *incremental circle transform* was first introduced in [8]. The incremental circle transform, which maps boundaries of an object onto a circle can be used in finding the orientation of an object in a very fast manner independent of position and starting point of boundary coding.

The incremental circle transform(ICT) can further be utilized for simple object recognition as well. But since many points of boundary of an object may be mapped to a point on a circle, the ICT may not distinguish complicated objects. Another limitation is that the incremental circle transforms of objects of different shapes can be the same. In fact, we find that an indented polygon and protruded polygon have the same ICT. To overcome the difficulties of ICT, we propose a modified concept called *generalized incremental circle transform*.

Definition) Generalized Incremental Circle Transform

Let there be given a simple closed curve, $\boldsymbol{\alpha}(l)$, $0 \le l \le L$ and a feasible radius r. Let $\Delta_S\boldsymbol{\alpha}(l)$ denote the vector

$$\Delta_S\boldsymbol{\alpha}(l) = (\ \Delta_S x(l)\ ,\ \Delta_S y(l)\ )$$

where

$$\boldsymbol{\alpha}(l + \Delta l) = \boldsymbol{\alpha}(l) + \Delta_S\boldsymbol{\alpha}(l)$$

and

$$\Delta_S x^2(l) + \Delta_S y^2(l) = s^2(l) \tag{1}$$

where

$$s(l) = r\ (1 - \frac{\gamma l}{L})$$

for some Δl, $0 \le \Delta l \le L$, and for some γ, $0 \le \gamma \le 1$. We call $\Delta_S\boldsymbol{\alpha}(l)$, $0 \le l \le L$ as the generalized incremental circle transform of $\boldsymbol{\alpha}(l)$, $0 \le l \le L$.

This new transform shows characteristics of translation invariance, rotation invariance, and scaling invariance as discussed in the following. To be specific, refer to Fig. 1, where $\mathbf{C}_p$ is translated contour of the contour $\mathbf{C}$ with total length L. $\mathbf{C}_r$ is a rotated contour with respect to the center of mass of $\mathbf{C}_p$ in CCW-direction. $\mathbf{C}_s$ is a m times scaled contour of $\mathbf{C}$, where $m = \int_{\mathbf{C}_s} dt \,/\, \int_{\mathbf{C}} dt$. As shown in Fig. 1, let each contours $\mathbf{C}$, $\mathbf{C}_p$, $\mathbf{C}_r$ and $\mathbf{C}_s$ be expressed by $\alpha(l)$, $\alpha_p(l)$, $\alpha_r(l)$ and $\alpha_s(ml)$, $0 \le l \le L$,

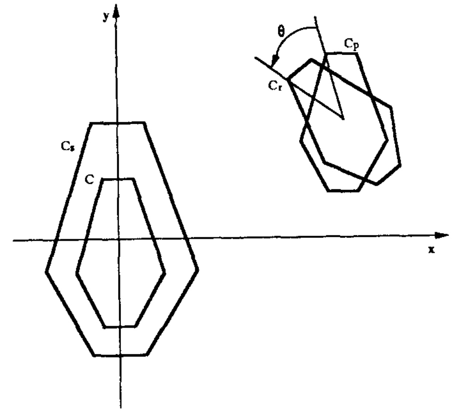

Fig. 1. Contours in X-Y image plane

respectively.

(Property 1) Translation Invariance

Let us consider two simple closed contour $\mathbf{C}$ and $\mathbf{C}_p$, as shown in Fig. 1. Let the X-axis and Y-axis differences of the centers of $\mathbf{C}$ and $\mathbf{C}_p$ be expressed by x_p and y_p, respectively. And if the starting points of the contours $\mathbf{C}$ and $\mathbf{C}_p$ are chosen to coincide with one another, then

$$\boldsymbol{\alpha}_p(l) = \boldsymbol{\alpha}(l) + (\ x_p\,,\ y_p) \tag{2}$$

$$\boldsymbol{\alpha}_p(l+\Delta l) = \boldsymbol{\alpha}(l+\Delta l) + (\ x_p\,,\ y_p) \tag{3}$$

then, subtracting (3) from (2)

$$\begin{aligned} \Delta_S\boldsymbol{\alpha}_p(l) &= \boldsymbol{\alpha}_p(l+\Delta l) - \boldsymbol{\alpha}_p(l) \\ &= \boldsymbol{\alpha}(l+\Delta l) - \boldsymbol{\alpha}(l) \\ &= \Delta_S\boldsymbol{\alpha}(l) \end{aligned}$$

Therefore, $\Delta_S\boldsymbol{\alpha}(l)$ is invariant to object translation.

(Property 2) Rotation Invariance

Let us consider two simple closed contour $\mathbf{C}_p$ and $\mathbf{C}_r$, as shown in Fig. 1. If the starting points of the contours $\mathbf{C}_p$ and $\mathbf{C}_r$ are chosen to coincide with one another, and the rotation angle in CCW-direction with respect to the z-axis at the center of $\mathbf{C}_p$ is expressed by θ, then the following equation is satisfied.

$$\boldsymbol{\alpha}_r(l) = \mathbf{R} \cdot \boldsymbol{\alpha}_p(l) \tag{4}$$

where

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Also, for the other corresponding points,

$$\boldsymbol{\alpha}_p(l+\Delta l) = \boldsymbol{\alpha}_p(l) + \Delta_S\boldsymbol{\alpha}_p(l)\ ,$$

$$\boldsymbol{\alpha}_r(l+\Delta l) = \boldsymbol{\alpha}_r(l) + \Delta_S\boldsymbol{\alpha}_r(l)$$

the following equation is satisfied

$$\boldsymbol{\alpha}_r(l+\Delta l) = \mathbf{R} \cdot \boldsymbol{\alpha}_p(l+\Delta l) \tag{5}$$

then, subtracting (5) from (4)

$$\begin{aligned}\Delta_S \boldsymbol{\alpha}_r(l) &= \boldsymbol{\alpha}_r(l+\Delta l) - \boldsymbol{\alpha}_r(l) \\ &= \mathbf{R} \cdot [\ \boldsymbol{\alpha}_p(l+\Delta l) - \boldsymbol{\alpha}_p(l)\ ] \\ &= \mathbf{R} \cdot \Delta_S \boldsymbol{\alpha}_p(l)\end{aligned}$$

Let $\Delta_S \boldsymbol{\alpha}_p(0)$ be $(\ \Delta_S x_{p0}, \Delta_S y_{p0}\ )$, and let us map this point to $(\ 0, r\ )$. Then,

$$\begin{bmatrix} 0 \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta_{pn} & -\sin\theta_{pn} \\ \sin\theta_{pn} & \cos\theta_{pn} \end{bmatrix} \begin{bmatrix} \Delta_S x_{p0} \\ \Delta_S y_{p0} \end{bmatrix} \tag{6}$$

is satisfied and from (6),

$$\cos\theta_{pn} = \frac{\Delta_S y_0}{r}, \quad \sin\theta_{pn} = \frac{\Delta_S x_0}{r}$$

Let $\Delta_S \boldsymbol{\alpha}_r(0)$ be $(\ \Delta_S x_{r0}, \Delta_S y_{r0}\ )$, and let us map this point to $(\ 0, r\ )$. Then,

$$\begin{bmatrix} 0 \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta_{rn} & -\sin\theta_{rn} \\ \sin\theta_{rn} & \cos\theta_{rn} \end{bmatrix} \begin{bmatrix} \Delta_S x_{r0} \\ \Delta_S y_{r0} \end{bmatrix} \tag{7}$$

is satisfied and from (7),

$$\cos\theta_{rn} = \frac{\Delta_S y_{r0}}{r}, \quad \sin\theta_{rn} = \frac{\Delta_S x_{r0}}{r}$$

If we let

$$\mathbf{R}_{pn} = \begin{bmatrix} \dfrac{\Delta_S y_{p0}}{r} & -\dfrac{\Delta_S x_{p0}}{r} \\ \dfrac{\Delta_S x_{p0}}{r} & \dfrac{\Delta_S y_{p0}}{r} \end{bmatrix} \tag{8}$$

and

$$\mathbf{R}_{rn} = \begin{bmatrix} \dfrac{\Delta_S y_{r0}}{r} & -\dfrac{\Delta_S x_{r0}}{r} \\ \dfrac{\Delta_S x_{r0}}{r} & \dfrac{\Delta_S y_{r0}}{r} \end{bmatrix}$$

and rotate all vector $\Delta_S \boldsymbol{\alpha}_p(l)$ with degree θ_{pn} and rotate all vector $\Delta_S \boldsymbol{\alpha}_r(l)$ with degree θ_{rn}, we obtain

$$\mathbf{R}_{rn}\ \Delta_S \boldsymbol{\alpha}_r(l) = \mathbf{R}_{pn}\ \Delta_S \boldsymbol{\alpha}_p(l)$$

Therefore, $\mathbf{R}_{pn}\ \Delta_S \boldsymbol{\alpha}_p(l)$ is invariant to rotation.

(Property 3) Scaling Invariance

Let us consider two closed contour $\mathbf{C}$ and $\mathbf{C}_s$, as shown in Fig. 1. $\mathbf{C}_s$ is m times scaled contour of $\mathbf{C}$ such that $\alpha_s(ml) = m \cdot \alpha(l)$. If we let circle radius r be proportional to perimeter of object, then

$$\begin{aligned}\Delta_{mS} \boldsymbol{\alpha}_s(l) &= \boldsymbol{\alpha}_s(m(l+\Delta l)) - \boldsymbol{\alpha}_s(ml) \\ &= m \cdot (\boldsymbol{\alpha}(l+\Delta l) - \boldsymbol{\alpha}(l)) \\ &= m \cdot \Delta_S \boldsymbol{\alpha}(l)\end{aligned}$$

Therefore, $\Delta_S \boldsymbol{\alpha}(l)$ is proportional to scaling variation. If we let circle radius r be proportional to the object perimeter and normalize it, then $\Delta_S \boldsymbol{\alpha}(l)$ is invariant to object scaling.

So far, we investigate three properties of translation invariance, rotation invariance, and scaling invariance. We can summarize the properties of *generalized incremental circle transform* as follows

At first, calculate the perimeter, L, of contour $\mathbf{C}$, and decide circle radius r

$$r = \frac{L}{k}, \quad L = perimeter, \quad k = const$$

r stands for object scaling, and we obtain the data $\Delta\boldsymbol{\alpha}(l)$ which is proportional to object scaling.

$$\Delta x^2(l) + \Delta y^2(l) = r^2$$

Let the starting point of $(\ \Delta x(l), \Delta y(l)\ )$ be $(\ \Delta x_0, \Delta y_0\ )$, and the rotation angle θ_n which map the starting point $(\ \Delta x_0, \Delta y_0\ )$ to $(\ 0, r\ )$ is

$$\cos\theta_n = \frac{\Delta y_0}{r}, \quad \sin\theta_n = \frac{\Delta x_0}{r}$$

Rotate all vector $(\ \Delta x(l), \Delta y(l)\ ), 0 \le l \le L$ with degree θ_n, we obtain

$$\Delta\boldsymbol{\alpha}_r(l) = \begin{bmatrix} \dfrac{\Delta y_0}{r} & -\dfrac{\Delta x_0}{r} \\ \dfrac{\Delta x_0}{r} & \dfrac{\Delta y_0}{r} \end{bmatrix} \Delta\boldsymbol{\alpha}(l)$$

then, $(\ \Delta x_r(l), \Delta y_r(l)\ )$ is invariant to object rotation, and following equation is satisfied.

$$\Delta x_r^2(l) + \Delta y_r^2(l) = r^2 \tag{9}$$

Multiplying both side of (9) by $\dfrac{s^2(l)}{r^2}$, we obtain

$$\frac{\Delta x_r^2(l)\ s^2(l)}{r^2} + \frac{\Delta y_r^2(l)\ s^2(l)}{r^2} = s^2(l),$$

where

$$s(l) = (\ 1 - \frac{\gamma l}{L}\ )$$

Then, *generalized incremental circle transform*, $\Delta_S \boldsymbol{\alpha}(l) = (\ \dfrac{\Delta x_r(l) s(l)}{r}, \dfrac{\Delta y_r(l) s(l)}{r}\ )$ maps boundary points of an object into an unit circle and is invariant to translation, rotation, and scaling.

3. OBJECT RECOGNITION ALGORITHM

For recognition of objects, the generalized incremental circle transform of objects can be used as feature information. Before we present an pattern recognition algorithm based on generalized incremental circle transform, we discuss the initial point problem.

In Section 2, we assumed that the initial point of GICT is the same. But, the assumption is very hard to satisfy in practice. Therefore, we first specify the corner point of a shape and choose one of the corner point as be the initial point of GICT.

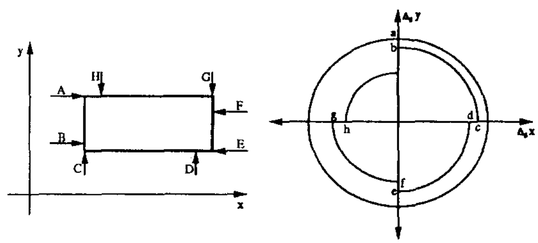

Fig. 2. Rectangular and it's result of Generalized Incremental Circle Transform

As shown in Fig. 2 the straight line on the object boundary curve, from A to B is mapped to the line of GICT from a to b. The straight line from B to C is mapped to the curve of GICT from b to c, and here exists a variation of $\Delta\alpha(l)$, for $\gamma = 0$ case. The interval at which there is no angle change satisfy

$$\frac{d\Delta_S\alpha(l)}{dl} = 0$$

and the interval at which there is angle change satisfy

$$\frac{d\Delta_S\alpha(l)}{dl} \neq 0$$

The corner point is the point where the angle change disappear and defined as follows

$$\begin{cases} \lim\limits_{h \to -0} \dfrac{\Delta_S\alpha(l+h) - \Delta_S\alpha(l)}{h} \neq 0 \\ \lim\limits_{h \to +0} \dfrac{\Delta_S\alpha(l+h) - \Delta_S\alpha(l)}{h} = 0 \end{cases} \tag{10}$$

We can find corner points of object by calculating which satisfying the equation (10).

The GICT of an object is invariant to object translation, rotation, and scaling. We can recognize the object by comparing the GICT's of model objects with the GICT's of real object. The object recognition algorithm using GICT is depicted in Fig. 3, and the part denoted by * is depicted in Fig. 4 in detail.

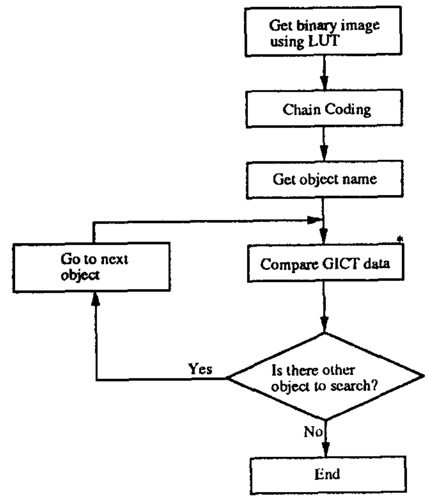

Fig. 3. Object Recognition Algorithm

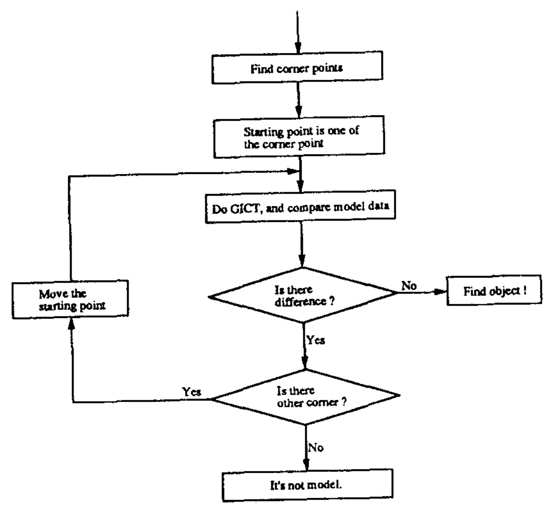

Fig. 4. Object Recognition Algorithm in detail

The difference between GICT's of two objects can be calculated by equation (11)

$$\int_{l=0}^{L} |\ \Delta_S\alpha(l) - \Delta_S\bar{\alpha}(l)\ |\ dl \tag{11}$$

where $\Delta_S\alpha(l)$ and $\Delta_S\bar{\alpha}(l)$ is the GICT's of the objects.

4. EXPERIMENTAL RESULTS

We have implemented the object recognition algorithm on a VME-based vision system. The visual processing unit, which is composed of image digitizer, real-time frame grabber, LUT/ALU for gray level image binarization, and MC68000 based system controller equipped with a coprocessor MC68881 for floating point arithmetic, recognize many rotated, scaled polygon effectively.

Fig. 5 shows the boundary curves of three squares, which has been rotated by 45^o and scaled by an area factor of 2, and it's results of GICT. For three cases, the results of GICTs are the same. Fig. 6 shows the boundary curves of three trapezoids, and it's results of GICT. The results of GICT are same, and compare to square case, the GICT of objects have different shape. From this difference we can classify trapezoid with easy. Fig. 7 shows the boundary curves of three regular triangles and it's results of GICT.

So far, numerous pattern recognition methods have been proposed for two dimensional object recognition problems. Fourier Descriptor method is one of the popular pattern recognition method. we compare our method to Fourier Descriptor method for recognition of two dimensional polygonal objects. The process and processing time

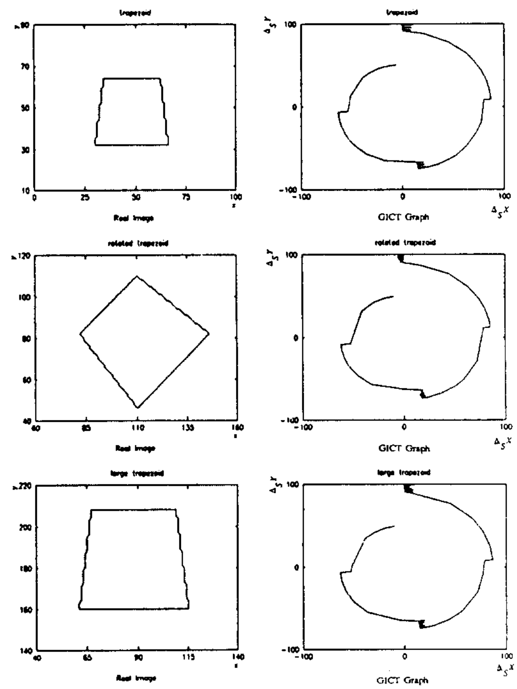

Fig. 6. Trapezoid and it's result of Generalized Incremental Circle Transform

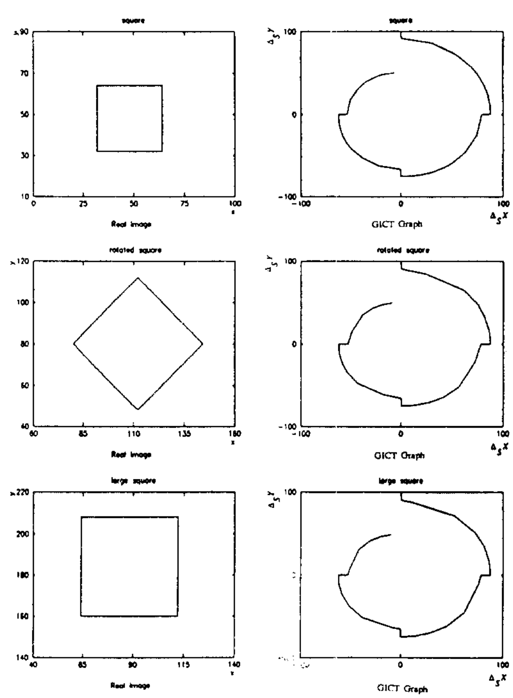

Fig. 5. Square and it's result of Generalized Incremental Circle Transform

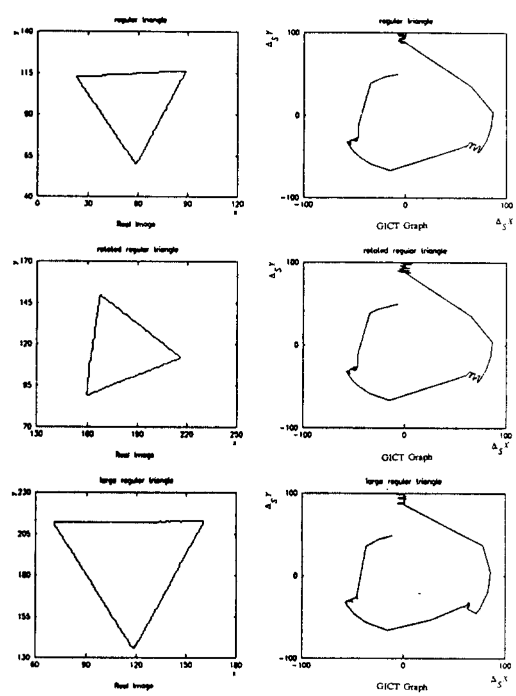

Fig. 7. Regular Triangle and it's result of Generalized Incremental Circle Transform

of each method which apply for recognition of 50 × 50(pixel × pixel) square, is shown in Table 1.

Fourier Descriptor		MICT	
process	time(ms)	process	time(ms)
o Extract FD o Normalization	3,381 37	o Corner Finding o MICT	352 210

Table 1. Comparison of processing time between Fourier Descriptor method and MICT method

5. CONCLUSION

This paper has proposed a concept of generalized incremental circle transform, which maps boundaries of object into an unit circle. Using this method, we successfully recognize two dimensional polygonal objects very fast and efficient manner and it is applicable to classifying the object on a vision-based robotic assembly systems.

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