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A New Pattern Classification Algorithm for Two-dimensional Objects

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ABSTRACT

Pattern classification is an essential step in automatic robotic assembly which joins together finite number of seperated industrial parts.

In this paper, a fast and systematic algorithm for classifying occlusion-free objects is proposed, using the notion of *incremental circle transform* which describes the boundary contour of an object as a parametric vector function of incremental elements. With similarity transform and line integral, *normalized determinant curve* of an object classifies each object, independent of position, orientation, scaling of an object and cyclic shift of the starting point for the boundary description.

1. INTRODUCTION

In robotic assembly which joins together finite number of industrial parts, the machine vision is used effectively for classifying the industrial parts and determining the position and orientation of each industrial part being out of order. In case of parts classification, each industrial part is often required to be classified regardless of its variations in position, orientation and scaling. In addition, the pattern recognition must be performed in a reliable and fast manner for the visual processing capability often determines the efficiency of automated robotic assembly.

So far, there have been proposed a number of methodologies [1-6] to improve the efficiency such as processing time or recognition rate. Specifically, Hu[1] utilized moments invariants of an object, and Perkins[2] represented boundary contours of industrial parts as a set of straight lines and circular arcs, and classified the parts by cross-correlating curvatures of the concurves with those of the models. In [3], normalized Fourier descriptors of a cumulative angular function, representing the boundary contour of an object, are adopted to classify the object. The cumulative angular function denotes the net amount of angular bend between tangential lines at the starting point and at each point of the boundary contour. Ballard[4] proposed a generalized Hough transform which transforms boundary points of an object into points in Hough transform space by using directional information from each boundary point to a referecne point of the object. The most clustered point in the Hough transform space corresponds to the shape of the object.

Ayache and Faugeras[5] described the boundary contour of an object as an approximate polygonal and then generate a number of hypotheses to locate the object by comparing local intrinsic features of the object with those of the model. They classified the object by recursively testing the hypotheses and iteratively evaluating other boundary segments. In [6], Dubois and Glanz represented the boundary contour of an object by an autoregressive model. The model is a parametric equation that expresses each sample point of the boundary contour as a linear combination of a specified number of previous boundary sample points and an error term. Then, the object classification problem is turned into a problem to estimate and analyze the coefficients of the linear combination.

However, most of the proposed methodologies suffer from expensive computation in boundary description of an object and/or subsequent analyses for pattern classification. In this paper, a fast and systematic algorithm for classifying two dimensional occlusion-free objects is proposed, using the notion of *incremental circle transform*. As introduced in [7], the incremental circle transform describes the boundary contour of an object as a parametric vector function of incremental elements. By using determinant curve along with similarity transform and line integral it is shown that the transform classifies an object independent of position, orientation, scaling of the object and cyclic shift of the starting point for the boundary description effectively.

Section II includes the proposed theory for pattern classification, and experimental results for the theory are shown in Section III. Finally, Section IV contains conclusions.

2. A PATTERN CLASSIFICATION THEORY

Let us consider a simple closed curve **C** that is counterclockwise oriented. The curve may represent the boundary contour of an occlusion-free object in the image plane. Let the curve, **C**, be expressed by a parametric vector function,

$$\boldsymbol{\alpha}(t) \;=\; (\, x(t)\ ,\ y(t)\ )^T \ ,\quad 0 \le t \le L$$

where t is the arc length from a point $\alpha(0)$ to the boundary point $\alpha(t)$ and L denotes the total arc length of **C**.

REVIEW OF INCREMENTAL CIRCLE TRANSFORM

In [7], the notion *incremental circle transform* is introduced as follows.

Definition) Incremental Circle Transform

Let there be given a simple closed curve, $\alpha(t)$, $0 \le t \le L$ and, a feasible radius r. For each $t \in [0, L]$, let $\Delta_r \alpha(t)$ denote the vector

$$\Delta_r \alpha(t) = (\Delta_r x(t) \ , \Delta_r y(t) \ )^T$$

where

$$\Delta_r x^2(t) + \Delta_r y^2(t) = r^2 \tag{1}$$

and

$$\alpha(t + \Delta t) = \alpha(t) + \Delta_r \alpha(t) \tag{2}$$

for some Δt, $0 \le \Delta t \le L$. The $\Delta_r \alpha(t)$, $0 \le t \le L$ is called as *incremental circle transform* of $\alpha(t)$, $0 \le t \le L$.

The feasible radius r is chosen in such a way that the number of intersecting points between $\alpha(t)$, $0 \le t \le L$ and the circle represented by (1) is greater than zero. In case that there exist several Δt's satisfying (1) and (2), the minimum Δt is selected for the uniqueness of Δt in (2). In consequence, the incremental circle transform maps points on **C** onto a circle with radius r.

The incremental circle transform has the following characteristics of translation invariance and rotation matrix preservation.

Property ICT-1) Position Invariance[7]

The incremental circle transform is invariant under the translation of an object if the starting points of boundary contours for translated objects coincide with that of the original object.

Property ICT-2) Rotation Matrix Preservation[7]

The rotation matrix between two contours in the x-y plane is the same as the rotation matrix between the corresponding incremental circle transforms of the contours regardless of positions of the contours in the x-y plane.

To be specific, let four contours in Fig. 1 be expressed by

$$\begin{aligned} \mathbf{C} \ &: \ \alpha(t) \ , \quad 0 \le t \le L \ , \\ \mathbf{C}_A \ &: \ \alpha_A(t) \ , \quad 0 \le t \le L \ , \\ \mathbf{C}_B \ &: \ \alpha_B(t) \ , \quad 0 \le t \le L \ , \\ \mathbf{C}_{RT} \ &: \ \alpha_{RT}(t) \ , \quad 0 \le t \le L \quad . \end{aligned}$$

Then the relationship between **C** and each contour is

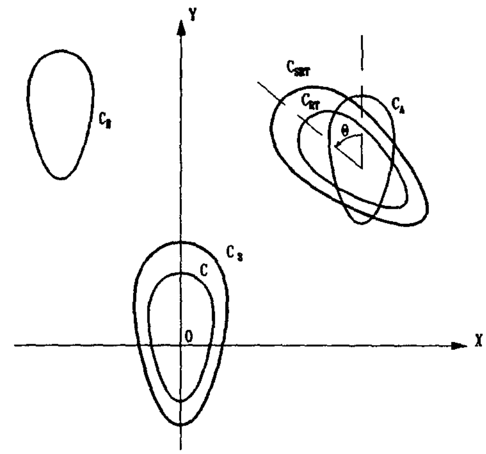

Fig. 1. Simple Closed Curves

represented in terms of the homogeneous transform as follows :

$$\begin{bmatrix} \alpha_A(t) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_{C_A} \\ 0 & 1 & y_{C_A} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha(t) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_B(t) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_{C_B} \\ 0 & 1 & y_{C_B} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha(t) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_{RT}(t) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x_{C_A} \\ \sin\theta & \cos\theta & y_{C_A} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha(t) \\ 1 \end{bmatrix}$$

where

$(x_{C_A} \ , y_{C_A} \ )^T$: center of $\mathbf{C}_A$

$(x_{C_B} \ , y_{C_B} \ )^T$: center of $\mathbf{C}_B$

θ : rotation angle in CCW-direction with respect to the z-axis at $(x_{C_A} \ , y_{C_A} \ )^T$

Referring to Fig. 1,

$$\Delta_r \alpha(t) = \Delta_r \alpha_A(t) = \Delta_r \alpha_B(t), \quad 0 \le t \le L$$

is resulted from Property ICT-1). Also, since $\alpha_{RT}(t) = \mathbf{R} \cdot \alpha_A(t)$ where $\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, Property ICT-2) implies

$$\begin{aligned} \Delta_r \alpha_{RT}(t) &= \mathbf{R} \cdot \Delta_r \alpha_A(t) \\ &= \mathbf{R} \cdot \Delta_r \alpha(t) \quad . \end{aligned} \tag{3}$$

In this section a notion called *determinant curve* will be defined as an image feature being invariant to translation, rotation, scaling of the object, and shift of starting point for object boundary coding. The determinant curve is derived from similarity transforms of incremental circle transform.

Let us consider equation (3) again, i.e.,

$$\Delta_r \boldsymbol{\alpha}_{RT}(t) = \mathbf{R} \cdot \Delta_r \boldsymbol{\alpha}(t) \quad .$$

And, for any τ, $0 \le \tau \le L$,

$$\Delta_r \boldsymbol{\alpha}_{RT}(t-\tau) = \mathbf{R} \cdot \Delta_r \boldsymbol{\alpha}(t-\tau) \quad .$$

Then,

$$\Delta_r \boldsymbol{\alpha}_{RT}(t) \cdot \Delta_r \boldsymbol{\alpha}_{RT}^T(t-\tau) = \mathbf{R} \cdot \Delta_r \boldsymbol{\alpha}(t) \cdot \Delta_r \boldsymbol{\alpha}^T(t-\tau) \cdot \mathbf{R}^{-1} \quad .$$

Let

$$\mathbf{M}_\tau(t) = \Delta_r \boldsymbol{\alpha}(t) \cdot \Delta_r \boldsymbol{\alpha}^T(t-\tau) \; ,$$
$$\mathbf{T}_\tau(t) = \Delta_r \boldsymbol{\alpha}_{RT}(t) \cdot \Delta_r \boldsymbol{\alpha}_{RT}^T(t-\tau) \quad .$$

Then,

$$\mathbf{T}_\tau(t) = \mathbf{R} \cdot \mathbf{M}_\tau(t) \cdot \mathbf{R}^{-1} \tag{4}$$

Since $\mathbf{T}_\tau(t)$ is a similarity transform, it follows immediately from equation (4) that

$$\int_{\mathbf{C}_{RT}} \mathbf{T}_\tau(t)\, dt = \mathbf{R} \cdot \int_{\mathbf{C}} \mathbf{M}_\tau(t)\, dt \cdot \mathbf{R}^{-1} \tag{5}$$

Here, we define the notion, *determinant curve*, of an object as follows.

Definition) Determinant Curve

Given an incremental circle transform $\Delta_r \boldsymbol{\alpha}(t)$, $0 \le t \le L$, let

$$\mathbf{M}_\tau(t) = \Delta_r \boldsymbol{\alpha}(t) \cdot \Delta_r \boldsymbol{\alpha}^T(t-\tau)$$

for any τ, $0 \le \tau \le L$. Then,

$$\det_{\mathbf{C}}(\tau) \overset{\Delta}{=} \det \left(\int_0^L \mathbf{M}_\tau(t)\, dt \right) , \quad 0 \le \tau \le L$$

is called as the determinant curve of the incremental circle transform $\Delta_r \boldsymbol{\alpha}(t)$, $0 \le t \le L$.

Since the translation invariance of the determinant curve is guaranteed by the translation invariance of incremental circle transform in Property ICT-1) and the line integral in (5) is invariant with respect to the shift of $\boldsymbol{\alpha}(0)$ on $\mathbf{C}$, the following properties of the determinant curve are easily established.

Property DET-1)

The determinant curve is invariant with respect to the cyclic shift of starting point for object boundary description.

Property DET-2)

The determinant curve is invariant with respect to the translation of an object.

Property DET-3)

The determinant curve is invariant with respect to the rotation of an object.

proof)

Referring to equation (5),

$$\int_{\mathbf{C}_{RT}} \mathbf{T}_\tau(t)\, dt = \mathbf{R} \cdot \int_{\mathbf{C}} \mathbf{M}_\tau(t)\, dt \cdot \mathbf{R}^{-1} \quad .$$

Then,

$$\begin{aligned} &\det \left(\int_{\mathbf{C}_{RT}} \mathbf{T}_\tau(t)\, dt \right) \\ &= \det \left(\mathbf{R} \cdot \int_{\mathbf{C}} \mathbf{M}_\tau(t)\, dt \cdot \mathbf{R}^{-1} \right) \\ &= \det(\mathbf{R}) \cdot \det \left(\int_{\mathbf{C}} \mathbf{M}_\tau(t)\, dt \right) \cdot \det(\mathbf{R}^{-1}) \quad . \end{aligned}$$

Since $\det(\mathbf{R}) = \det(\mathbf{R}^{-1}) = 1$,

$$\begin{aligned} &\det_{\mathbf{C}_{RT}}(\tau) \\ &= \det \left(\int_{\mathbf{C}_{RT}} \mathbf{T}_\tau(t)\, dt \right) \\ &= \det \left(\int_{\mathbf{C}} \mathbf{M}_\tau(t)\, dt \right) \\ &= \det_{\mathbf{C}}(\tau) \quad . \end{aligned}$$

Therefore, the determinant curve of $\boldsymbol{\alpha}(t)$, $0 \le t \le L$, is the same as that of $\boldsymbol{\alpha}_{RT}(t)$, $0 \le t \le L$.

(Q.E.D)

In the following the relationship between two determinant curves for an object and its dislocated-and-magnified object will be investigated. To be specific, let two contours in Fig. 1, $\mathbf{C}_S$ and $\mathbf{C}_{SRT}$, be expressed by

$$\begin{aligned} \mathbf{C}_S &: \boldsymbol{\alpha}_S(mt) , \quad 0 \le t \le L , \\ \mathbf{C}_{SRT} &: \boldsymbol{\alpha}_{SRT}(mt) , \; 0 \le t \le L . \end{aligned}$$

where $m = \int_{\mathbf{C}_S} dt \,/\, \int_{\mathbf{C}} dt$. And the relationship between $\mathbf{C}$ and each contour is represented as follows :

$$\begin{bmatrix} \boldsymbol{\alpha}_S(mt) \\ 1 \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\alpha}(t) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\alpha}_{SRT}(mt) \\ 1 \end{bmatrix} = \begin{bmatrix} m\cos\theta & -m\sin\theta & x_{C_A} \\ m\sin\theta & m\cos\theta & y_{C_A} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\alpha}(t) \\ 1 \end{bmatrix}$$

Then, suppose that

$$\mathbf{K} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} ,$$

$$\boldsymbol{\alpha}_S(mt) = \mathbf{K} \cdot \boldsymbol{\alpha}(t), \quad 0 \le t \le L$$

and

$$\begin{aligned} &\Delta_{mr} \boldsymbol{\alpha}_S(mt) \\ &= \boldsymbol{\alpha}_S(\, m(t+\Delta t)\,) - \boldsymbol{\alpha}_S(mt) \\ &= \mathbf{K} \cdot (\, \boldsymbol{\alpha}(t+\Delta t) - \boldsymbol{\alpha}(t)\,) \\ &= \mathbf{K} \cdot \Delta_r \boldsymbol{\alpha}(t) \end{aligned} \tag{6}$$

That is, $\boldsymbol{\alpha}_S(mt) = \mathbf{K} \cdot \boldsymbol{\alpha}(t)$ implies $\Delta_{mr}\boldsymbol{\alpha}_S(mt) = \mathbf{K} \cdot \Delta_r\boldsymbol{\alpha}(t)$.

Therefore, we can derive the following result that is very efficient for pattern classification using the determinant curve of an object.

Property DET-4)

Let $\boldsymbol{\alpha}_{SRT}(mt)$, $0 \le t \le L$ be a dislocated and magnified curve of a simple closed curve, $\boldsymbol{\alpha}(t)$, $0 \le t \le L$, where $m = \int_{C_{SRT}} dt \;/\; \int_C dt$. Then, for any τ, $0 \le \tau \le L$,

$$\det_{C_{SRT}}(\tau) = \det(\mathbf{K})^2 \cdot \det_C(\tau)$$

where

$$\mathbf{K} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} .$$

proof)

Let

$$\mathbf{M}_\tau(t) = \Delta_r\boldsymbol{\alpha}(t) \cdot \Delta_r\boldsymbol{\alpha}^T(t-\tau) ,$$
$$\mathbf{Q}_\tau(t) = \Delta_{mr}\boldsymbol{\alpha}_{SRT}(mt) \cdot \Delta_{mr}\boldsymbol{\alpha}_{SRT}^T(\, m(t-\tau)\,)$$

for an arbitrary τ, $0 \le \tau \le L$. From equation (3) and equation (6),

$$\begin{aligned}\mathbf{Q}_\tau(t) &= (\mathbf{K} \cdot \mathbf{R} \cdot \Delta_r\boldsymbol{\alpha}(t)) \cdot (\mathbf{K} \cdot \mathbf{R} \cdot \Delta_r\boldsymbol{\alpha}(t-\tau))^T \\ &= \mathbf{K} \cdot \mathbf{R} \cdot \mathbf{M}_\tau(t) \cdot \mathbf{R}^{-1} \cdot \mathbf{K}^T .\end{aligned}$$

Since the $\mathbf{Q}_\tau(t)$ is a linear transform, we have

$$\int_{C_{SRT}} \mathbf{Q}_\tau(t)\, dt = \mathbf{K} \cdot \mathbf{R} \cdot \int_C \mathbf{M}_\tau(t)\, dt \cdot \mathbf{R}^{-1} \cdot \mathbf{K}^T$$

Therefore,

$$\begin{aligned}&\det_{C_{SRT}}(\tau) \\ =\ & \det\Big(\int_{C_{SRT}} \mathbf{Q}_\tau(t)\, dt\Big) \\ =\ & \det\Big(\mathbf{K} \cdot \mathbf{R} \cdot \int_C \mathbf{M}_\tau(t)\, dt \cdot \mathbf{R}^{-1} \cdot \mathbf{K}^T\Big) \\ =\ & \det(\mathbf{K}) \cdot \det(\mathbf{R}) \cdot \det\Big(\int_C \mathbf{M}_\tau(t)\, dt\Big) \cdot \det(\mathbf{R}^{-1}) \cdot \det(\mathbf{K}^T) \\ =\ & \det(\mathbf{K})^2 \cdot \det\Big(\int_C \mathbf{M}_\tau(t)\, dt\Big) \\ =\ & \det(\mathbf{K})^2 \cdot \det_C(\tau)\end{aligned}$$

since $\det(\mathbf{R}) = \det(\mathbf{R}^{-1}) = 1$ and $\det(\mathbf{K}) = \det(\mathbf{K}^T)$.

(Q.E.D)

Therefore, we can conclude that *the determinant curve of an object can be effectively used as an image feature for shape classification* since the determinant curve of an object not only is invariant to object translation, object rotation, and variations in the starting point for incremental circle transform, but also can be normalized, being insensitive to scale variations of the object.

In section III, the validity of the above statement will be shown by experiments.

3. EXPERIMENTAL RESULTS

Let there be n kinds of model objects represented by $\{\, \boldsymbol{\alpha}_i(t),\ 0 \le t \le L_i,\ i = 1, 2, \cdots, n \,\}$. In consideration of the theory in Section II, we propose an algorithm for pattern classification as follows :

Step 1) Grab a binary image.

Step 2) Find chain codes of objects in the image.

Step 3) Determine the feasible radius of the incremental circle in consideration of the size of the perimeter of each object. For example, choose

$$r = \frac{L}{10}$$

where L denotes the perimeter of an object and r represents the radius of the incremental circle.

Step 4) Perform the incremental circle transform for each object.

Step 5) Find the normalized determinant curve for each object.

Step 6) Calculate the sum of absolute error between determinant curves of a real object denoted by $\boldsymbol{\alpha}(t)$, $0 \le t \le L$ and n model objects.

$$distance(i) = \int_0^L \mid \det_{\boldsymbol{\alpha}_i}(m_i\tau) - \det_{\boldsymbol{\alpha}}(\tau) \mid d\tau, \qquad (7)$$

$$i = 1, 2, \cdots, n \quad \text{where} \quad m_i = L_i \,/\, \mathrm{L} .$$

Step 7) Find $\mathbf{I}$, where

$$\mathbf{I} = \underset{i}{Min}\ \{\, distance(i) \mid i = 1, 2, \cdots, n \,\} .$$

Step 8) Classify the real object as being the model with $\boldsymbol{\alpha}_{\mathbf{I}}(t)$, $0 \le t \le L_{\mathbf{I}}$.

The algorithm is applied for the set of rectangles shown in Fig. 2. Determinant curves of incremental circle transform for the five rectangles are shown in Fig. 3. And a table of distance values is found as shown in Table 1. For other object set given in Fig. 4, determinant curves and a table of distance values are obtained as shown in Fig. 5 and Table 2 respectively. Accordingly it is noted that the data in Table 1 and Table 2 suggest that the determinant curve of an object is useful as an image feature for object classification.

Finally, we have compared the processing time of the proposed algorithm with that of the method using fourier descriptors[3] of an object. Determinant values at eight sample points are used for the proposed method while sixteen normalized fourier descriptors are adopted for the method in [3]. The processing time is measured for the simple rectangular objects in Fig. 2, and the time table in Table 3 is obtained. In average, the processing time is improved by the proposed method as much as a factor of 2.99. In addition, if the proposed algorithm is applied for more complex objects, the processing time will be improved still more in comparison with the method in [3].

All of the algorithms are implemented in C-language and installed in the visual processing unit[8] whose central processor is MC68000(10 MHz)-based system controller equipped with a coprocessor MC68881 for floating point arithmetic.

4. CONCLUSIONS

A fast pattern classfication algorithm for two dimensional occlusion-free objects is proposed, based on the incremental circle transform of each object boundary contour. The processing time is shortened by reducing the number of calculations for trigonometric or exponential functions in the algorithm. In addition, the algorithm is robust to noise in boundary contours since not only the correlation(auto/cross correlation) and the line integral lessen the effect of the noise, but also the incremental circle transform uses average differential rates of the boundary contours.

The algorithm, however, is yet to be improved in consideration of processing time, for the correlation of the incremental circle transforms describing an object boundary contour needs a number of multiplication.

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object	1	3	4
1	0.00	-	-
3	0.15	0.00	-
4	0.41	0.41	0.00

Table 1. Distance Values between Objects in Fig. 2

object	1	2	3	4
1	0.00	-	-	-
2	7.38	0.00	-	-
3	5.70	3.24	0.00	-
4	7.21	2.08	3.36	0.00

Table 2. Distance Values between Objects in Fig. 4

Object	Processing time (msec)	
Number	Determinant Curve	Fourier Descriptor
1	732.3	1043.4
2	750.2	5348.2
3	918.4	1217.5
4	734.8	1082.4
5	1153.7	4124.3
Average	857.88	2563.16

Table 3. Processing Time

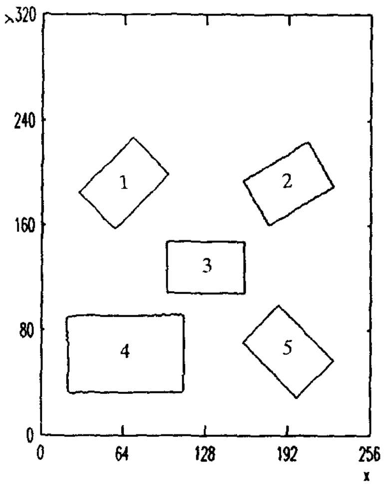
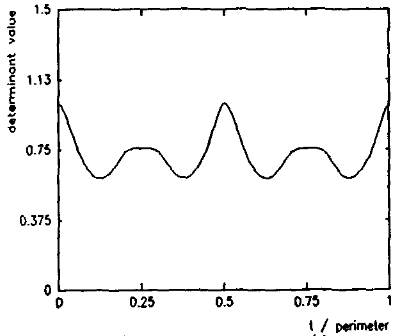
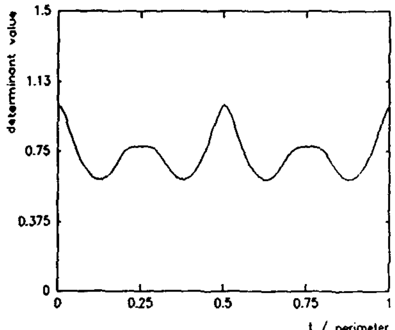


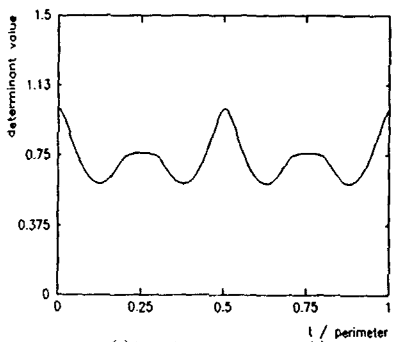
Fig. 2. Rectangles



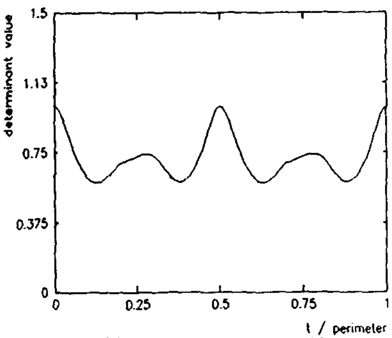
(a) Determinant Curve of Fig. 2-(1)



(b) Determinant Curve of Fig. 2-(2)



(c) Determinant Curve of Fig. 2-(3)



(d) Determinant Curve of Fig. 2-(4)

Fig. 3. Determinant Curves of Rectangles in Fig. 2.

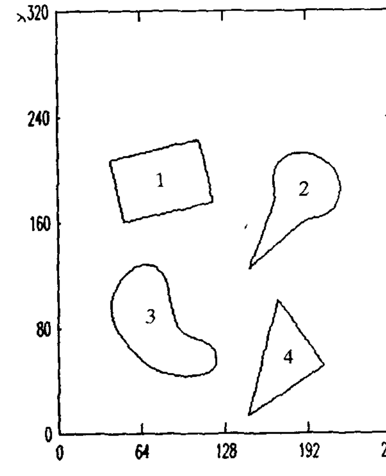
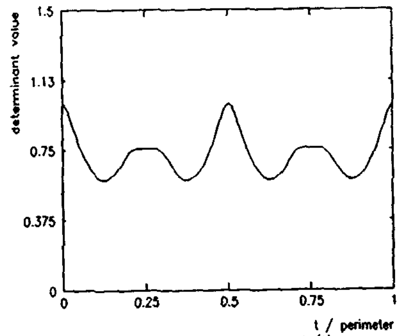
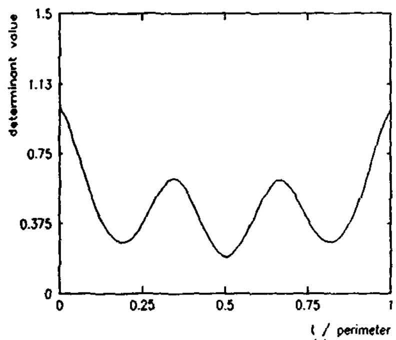


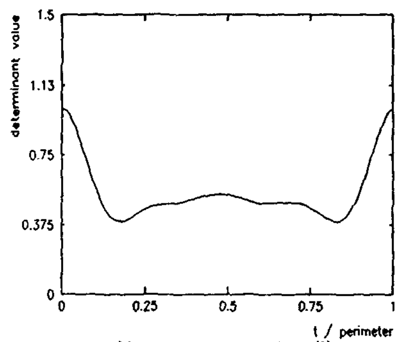
Fig. 4. Several Objects



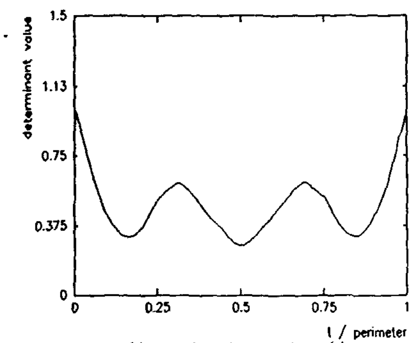
(a) Determinant Curve of Fig. 4-(1)



(b) Determinant Curve of Fig. 4-(2)



(c) Determinant Curve of Fig. 4-(3)



(d) Determinant Curve of Fig. 4-(4)

Fig. 5. Determinant Curves of Objects in Fig. 4.