'90 KACC 1990. 10. 26~27

On-line Robust Control of a System with Dead Time

*Wen-Teng Wu
Yu-Jen Jang

Department of Chemical Engineering
National Tsing Hua University
Hisnchu, Taiwan, R.O.C.

ABSTRACT

On-line robust control based on a stability index for time-delay systems has been developed. The purpose of the proposed design algorithm is to on-line tune a filter in the control loop. The problem of robust control with an incorrect given bound on the modeling error is investigated. Illustrative examples are presented to show the promise of the proposed method.

1.INTRODUCTION

Design of a control system with model uncertainty has been investigated by many researchers. Adaptive control [1,2,3] and robust control [4,5,6] are the most common approaches to solve this problem. An adaptive control system automatically adjusts the controller parameters to compensate for changes in the process or the environment. System identification is an important step for adaptive control. A robust control system, on the other hand, requires a nominal plant. The design algorithm determines a controller which will stabilize the resulting control system, usually under performance specifications that have specific prescribed bounds of perturbations.

In industrial processes, control of a system with dead time is always required. Adaptive control is ineffective, especially for a system with a large amount of dead time. Although robust control is applicable, most are off-line approaches. Thus, when the predescribed bounds are smaller or larger than that of the actual perturbations, the robust control systems exhibit poor stability and related performance problems.

In the present study, a two-step on-line robust control is proposed. Step one deals with the system's nominal design. Step two handles the tuning of an on-line filter. The tuning method is based on a stability index which is on-line determined. Parameter variations including those with large amounts of dead time were investigated. The proposed method provides a systematic approach to solve the problem of robust design with improper given bounds of the modeling error.

2.THE CONTROL SYSTEM

Consider a control system shown in Fig.1. G(s) is a nominal plant of the actual system $G_a(s)$. The transfer function of G(s) is expressed as

$$G(s)=\frac{B(s)}{A(s)}e^{-\tau s} \tag{1}$$

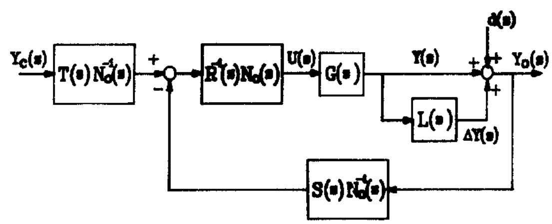

Fig.1. A two-parameter control system.

The actual system with model uncertainty in the multiplicative form is shown as

$$G_a(s)=G(s)[1+L(s)] \tag{2}$$

with

$$|L(jw)|\le lm(w),\ \forall w>0$$

The function lm(w) is a positive scalar one. R(s), S(s), T(s), and $N_d(s)$ in Fig.1 are polynomials of s. The control system can be modified to an internal model control (IMC) structure (Fig.2). The block diagram is equivalent to the original system with a Smith predictor. The transfer function of the controller $G_c(s)$ is given as

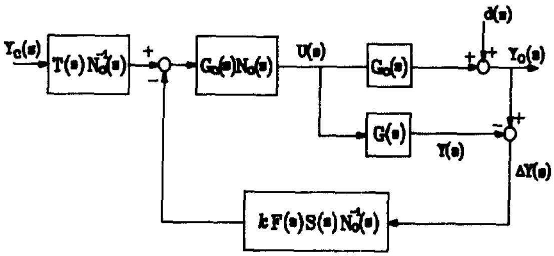

Fig.2. A complete IMC structure.

$$G_c(s)=\frac{A(s)}{A(s)R(s)+B(s)S(s)} \tag{3}$$

A filter $k \cdot F(s)$ is added in the feedback loop. The parameter, k, is on-line determined, The function F(s) which is the same as that of Morari and Zafiriou [7] is expressed as

$$F(s)=\frac{1}{\lambda s+1} \tag{4}$$

A desired closed-loop transfer function is given as

$$H_m(s)=\frac{B_m(s)}{A_m(s)}e^{-\tau s} \tag{5}$$

Control system design based on the nominal plant gives

$$\frac{B(s)T(s)}{A(s)R(s)+B(s)S(s)}=\frac{B_m(s)}{A_m(s)} \tag{6}$$

$N_o(s)$ is chosen as [8]

$$N_o(s)=(s+1)^n \tag{7}$$

where n equals the degree of R(s).
Detailed information about the design approach can be found in ref. [9].

3.ROBUST CONTROL

Based on the block diagram of Fig.2, a robust stability index is defined as

$$RSI=C_1\frac{\|\triangle y\|_2}{\|y(t)\|_2} \tag{8}$$

where $\triangle y(t)$ is the difference between the actual output $y_o(t)$ and the model output y(t). The L_2 norm is defined as

$$\|y(t)\|_2=\left[\int_0^\infty y(t)^2dt\right]^{1/2} \tag{9}$$

The constant C_1 in Eq.(8) is given as

$$C_1=\sup_w |H_m(jw)T^{-1}(jw)S(jw)F(jw)| \tag{10}$$

The functions, $H_m(jw)$, $T(jw)$, $S(jw)$, and $F(jw)$ are off-line determined, RSI is then used for stability indication. The criterion is given as the following theorem.

Theorem. The closed loop of the actual system is stable, if the following conditions hold:
(i) The controller $G_c(s)$ stabilizes the nominal plant G(s).
(ii) G(s) and $G_o(s)$ have the same number of poles on the imaginary axis and in the right half plane.
(iii) RSI<1. (11)

(proof) Based on the block diagram of Fig.2, the output variable from the nominal plant is written as

$$Y(s)=T(s)G_c(s)G(s)Y_c(s)-kF(s)S(s)G_c(s)G(s)\triangle Y(s) \tag{12}$$

From the linear system, $Y(s)=G(s)u(s)$, Eq.(13) can be derived [7]

$$\|y(t)\|_2\leq\sup_w|G(jw)|\ \|u(t)\|_2 \tag{13}$$

Hence, Eq.(12) gives

$$\begin{aligned}\|y(t)\|_2 &\leqq \sup_w|T(jw)G_c(jw)G(jw)|\ \|y_c(t)\|_2\\ &\quad -k\ \sup_w|F(jw)S(jw)G_c(jw)G(jw)|\ \|\triangle y(t)\|_2\\ &=C_2\|y_c(t)\|_2-kC_1\|\triangle y(t)\|_2\end{aligned} \tag{14}$$

where

$$C_2=\sup_w|H_m(jw)|.$$

By using Eq.(8), Eq.(14) can be written as

$$\|y(t)\|_2<C_2\|y_c(t)\|_2+k\cdot RSI\|y(t)\|_2 \tag{15}$$

The relationship between $\|y(t)\|_2$ and $\|y_c(t)\|_2$ then becomes

$$\|y(t)\|_2<\frac{C_2}{1-k\cdot RSI}\|y_c(t)\|_2 \tag{16}$$

Under the condition of

$$k\cdot RSI<1 \tag{17}$$

if $\| y_c(t) \|_2$ is hounded, then $\| y(t) \|_2$ is bounded. The definition of the RSI gives

$$\| \triangle y(t) \|_2 = \frac{RSI}{C_1} \| y(t) \|_2 \tag{18}$$

Since $\| y(t) \|_2$ is bounded, then $\| \triangle y(t) \|_2$ is also bounded. Bounded $\| y(t) \|_2$ and $\| \triangle y(t) \|_2$ indicate that $\| y_0(t) \|_2$ is also bounded. Hence, the control system is stable. Since RSI<1 implies k・RSI<1 with $k \in (0,1]$, the proof is complete.

For on-line applications, an index, RSI(t), is defined as

$$RSI(t) = C_1 \frac{\left[\int_0^t \triangle y(t)^2 dt\right]^{1/2}}{\left[\int_0^t y(t)^2 dt\right]^{1/2}} = C_1 \frac{\| \triangle y_T \|_2}{\| y_T \|_2} \tag{19}$$

where the subscript T denotes the truncation of the function.

Corollary: The closed loop of the actual system is stable, if the following conditions hold.
(i) The conditions (i) and (ii) of the theorem hold.
(ii) $RSI(t) < 1$ (20)

<proof> It is easy to verify that satisfying Eq.(20) will mean Eq.(11) must he satisfied, too. Thus, the proof is complete.

Tuning of the parameter, k, is the next step. Theoretically, for robust performance, the closed-loop system must meet the following sepcification [7].

$$\left| [1-k\, F(jw)G_c(jw)G(jw)]W_d(w) \right| + \left| F(jw)G_c(jw)lm(w) \right| < 1 \tag{21}$$

where $W_d(w)$ is a frequency-dependent weighting function. The method for determination of F(s) is the same as that of Morari and Zafiriou [7]. If RSI° is defined as

$$RSI^\circ = 1 - \left| [1-k \cdot F(jw)G_c(jw)G(jw)]W_d(w) \right|_{w=w_m} \tag{22}$$

then w_m is the frequency where the left-hand side of Eq.(21) is at its maximum value under the specification of Eq.(21). The first calculated RSI° is obtained with k=1. Since the condition of k・RSI<1 implies that the control system is stable, determination of k is based on k・RSI(t) being no larger than RSI°. The value of k is on-line calculated by

$$k = \begin{cases} \dfrac{RSI^\circ}{C_1 \| \triangle y_T \|_2 / \| y_T \|_2}, & RSI(t) > RSI^\circ \\ 1, & RSI(t) \leqq RSI^\circ \end{cases} \tag{23}$$

Thus, on-line robust design is achieved.

4.ILLUSTRATIVE EXAMPLES

Two examples are given to illustrate the proposed algorithm. The first considers the modeling error of dead time. The second investigates parameter variations, including differing amounts of dead time.

Example 1. With a second order system, the transfer function is $G_0(s)$

$$G_0(s) = \frac{1}{s^2+2.5s+1} e^{-2s} \tag{24}$$

in which the nominal plant is a model without dead time.

$$G(s) = \frac{1}{s^2+2.5\ s+1} \tag{25}$$

The closed-loop transfer function is given as

$$H_m(s) = \frac{1}{s^2+1.4\ s+1} \tag{26}$$

Based on section 2, the controller is designed. The polynomials, R(s), S(s), and T(s), are determined as follows:

$$R(s) = 0.49\ s^2 + 0.861 \tag{27}$$

$$S(s) = 0.8075\ s^2 + 1.939\ s + 1 \tag{28}$$

$$T(s) = (0.7\ s+1)^2 \tag{29}$$

The robust design is based on the given modeling error, lm(w).

$$lm(w) = \begin{cases} \left| e^{-2jw} - 1 \right| & w < \dfrac{\pi}{2} \\ 2 & w \geqq \dfrac{\pi}{2} \end{cases} \tag{30}$$

The parameter, λ, of the filter is determined when $W_d{}'(jw) = 2.0$.

$$\lambda = 4.77 \tag{31}$$

RSI° is then obtained as

$$RSI^\circ = 0.405 \tag{32}$$

The output responses and input variables are shown in Figs.3 and 4, respectively. Based on the proposed method, better performance was obtained.

Example 2. Consider the transfer function of the system $G_0(s)$.

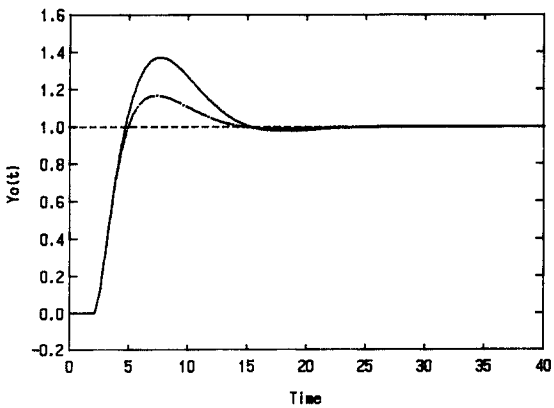

Fig.3. Output responses (Example 1).
—·— On-line adjustment of k
—— k = 1

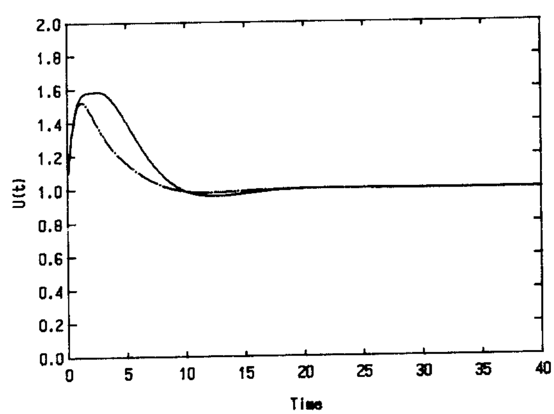

Fig.4. Input variables (Example 1).
—·— On-line adjustment of k
—— k = 1

$$G_0(s)=\frac{1.1}{s^2+s+1}e^{-4s} \tag{33}$$

If the nominal plant and the desired closed-loop transfer function are the same as those of example one, the off-line design will be identical. The output responses and input variables are shown in Figs.5 and 6, respectively. Again, the proposed method improves perfocmance significantly.

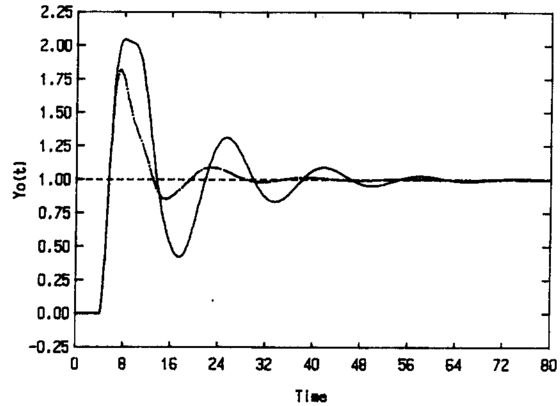

Fig.5. Output response (Example 2).
—·— On-line adjustment of k
—— k = 1

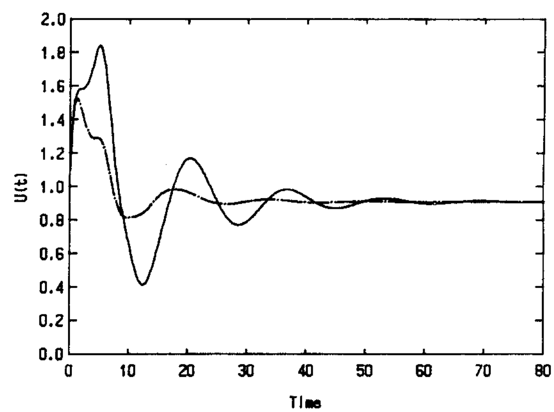

Fig.6. Input variables (Example 2).
—·— On-line adjustment of k
—— k = 1

5.CONCLUSIONS

An on-line method for robust control of a system with dead time has been developed. A parameter of the filter in the feedback loop is on-line tuned. The problem of robust design with an incorrect given bound on the modeling error is solved.

The proposed method is a general approach, in which the tuning parameter is a variable. Simulation showed that the proposed method improved the performance of the control system significantly.

REFERENCES

[1] K.J.Astrom, and B. Wittenmark, "On self-tuning regulators", Automatica, 9, pp.195-199, 1973.

[2] D.W.Clarke, and P.J.Gawthrop, "Self-tuning Controller", Proc.IEE. 122, pp.929-934, 1975.

[3] G.C.Goodwin, and K.S.Sin, Adaptive filtering prediction and contnel, Prentice-Hall Information and System Science Series, 1984.

[4] G.Zames, "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms and approximate in reverses", IEEE Trans. Automat., Contr. AC-26, pp.301-320, 1981.

[5] J.C.Doyle, "Analysis of feedback system with structured uncertainty", Pro. IEE, 129, pp.242-250, 1982.

[6] Y.Arkun and C.O.Morgan Ⅲ, "On the use of the structured singular value for robustness analysis of distillation column control", Comput. Chem. Engng., 12, pp.303-306, 1988.

[7] M.Morari, and E.Zafiriou, Robust Proceso Control Englewood Cliffs, N.J.: Prentice-Hall, 1989.

[8] M.Vidyasagar, Control System Synthesis: A Factorization Approach, Cambridge, MIT Press, 1985.

[9] K.J., Astrom, and B. Wittenmark, "Self-tuning controllers based on pole-zero placement", Proc. IEE., 127, pp.120-130, 1980.