'90 KACC 1990. 10. 26~27

POSITION CONTROL OF A FLEXIBLE ROBOT ARM UNDER IMPULSIVE LOADING AT THE TIP

Seiji CHONAN and Yasuhiro YUKI

Department of Mechanical Engineering, Tohoku University
Sendai, Japan

ABSTRACT

A simulation analysis is presented for the position control of a single-link flexible manipulator whose end-effector is subjected to an impulsive force. Arm is rotated by a d.c. servomotor at the shoulder so that the end point stays precisely at its initial position even if the end effector is thumped with the impulsive loading. A gap sensor is used to measure the tip displacement. The control torque based on the PD control law is applied to the motor through the driver circuit.The control strategy is tested by means of computer simulation for the one-link flexible-arm prototype in the authers' laboratory at Tohoku University.

1.INTRODUCTION

The number of robots installed in the manufacturing industries has been increased considerably in the last few years. Under those circumstances, the requirements for faster and more precise robots, perticularly in the area of microelectronics, have made it a necessity to consider the dynamic effects of distributed link mass and flexibility in the design of the manipulator-control system, and many papers have been published on this subject during the past few years. Cannon and Schmitz(1984), Skaar and Tucker(1986), Yuh(1987), Yoshida et. al. (1988), Tahara and Chonan(1988), Yigid et. al. (1988), Wang et. al. (1989) and Chonan and Aoshima(1990) investigated the open-loop and the closed-loop position control of single-link flexible manipulators rotated by a d.c. servomotor located at the shoulder. Bailey and Hubbard(1985), Baz and Poh (1988), Lee et. al. (1989) and Jiang et. al. (1990) studied the active vibration or position control of beams and plates driven by piezoelectric actuators. As for the multi-link flexible arm, Book et. al. (1975), Ower and Van De Vegte(1987), Lee and Wang(1988) and Chonan and Umeno(1989) studied the position control of two-link robot arms with distributed flexibility. All those papers are concerned with the robot arms operated in circumstances having no external disturbances. In some cases, however, the arm has to complete the task under the influence of disturbances. It appears, to the authors' knowledge, that no study has been done on the control of robot arms working in such unfavorable condition. It is in this context that the subject discussed in this paper is that of the position control of a single-link flexible arm, whose end effector is subjected to an impulsive force. The light-weight robotic manipulator which is handed over a payload from the other manipulator is a typical example of this kind. The arm is rotated by a d.c. servomotor at the shoulder so that the end-point stays precisely at its initial position even if the effector is thumped with the impulsive load. A gap sensor is used to measure the tip displacement. The control torque based on the PD control law is then applied to the motor throuh the driver circuit. The control strategy is tested by means of simulation for the one-link flexible-arm prototype in the authers' laboratory at Tohoku University. The results show that the response of the arm controlled is satisfactory.

2. FORMULATION OF THE PROBLEM

Figure 1 shows the light-weight robotic manipulator studied in this analysis. The arm deforms in the horizontal xy-plane but not in the vertical xz-plane. The arm base is fixed on a rigid hub of radius r mounted directly on the vertical shaft of a d.c. servomotor. The other end ($x = L$) is loaded with a mass of radius r_p. M and Q are

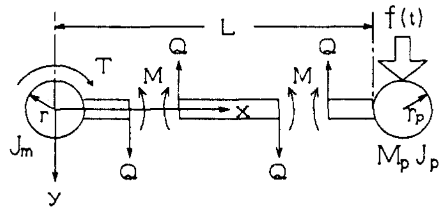

Fig.1 Geometry of problem and co-ordinates.

the bending moment and the shear force acting on the arm across-section. The problem discussed in the following is to control the motor torque T so that the arm tip stays precisely at its initial position even if it is disturbed by an impulsive loading. The tip position is measured by a gap sensor fixed in space. It is then used, together with the estimated tip velocity, as a basis for applying control torque to the other end. Denoting the displacement of the arm in the y-direction by w, the equation of motion of the arm is given by

$$EI(1 + c\partial/\partial t)(\partial^4/\partial x^4)w(x,t) + \rho A(\partial^2/\partial t^2)w(x,t) = 0, \qquad (1)$$

where E is the young's modulus, ρ is the mass density, A is the cross-sectionl area, and I is the moment of inertia of the cross section ; c is the internal damping coefficient, and t is time. One considers here the boundary conditions of the system. At one end, the arm is clamped on a rigid hub of radius r mounted directly on the vertical shaft of the motor. The geometrical consideration gives

$$w(r,t) = r(\partial/\partial x)w(r,t). \qquad (2)$$

Further, the equilibrium of moment around the motor shaft leads to

$$\begin{aligned} &J_m(\partial^3/\partial x\partial t^2)w(r,t) \\ &= EI(1 + c\partial/\partial t)(\partial^2/\partial x^2)w(r,t) \\ &- \epsilon(\partial^2/\partial t\partial x)w(r,t) \\ &- rEI(1 + c\partial/\partial t)(\partial^3/\partial x^3)w(r,t) + T, \qquad (3) \end{aligned}$$

where J_m is the polar moment of inertia of the motor shaft, and ϵ is the damping coefficient of the armature ; T is the torque applied to the arm base by the motor, which is given by

$$T = K_f i_a. \qquad (4)$$

Here, i_a is the armature current, and K_t is the torque constant of the motor. The arm tip ($x = L$) is attached with a payload of mass M_p and moment of inertia J_p. Further, it is subjected to an external impulsive force $f(t)$. The equilibria of moment and force in this case are

$$\begin{aligned} &J_p(\partial^3/\partial x\partial t^2)w(r,t) \\ &= -EI(1 + c\partial/\partial t)(\partial^2/\partial x^2)w(L,t) \\ &- r_pEI(1 + c\partial/\partial t)(\partial^3/\partial x^3)w(L,t), \qquad (5) \end{aligned}$$

$$\begin{aligned} &M_p(1 + r_p\partial/\partial x)(\partial^2/\partial t^2)w(L,t) \\ &= EI(1 + c\partial/\partial t)(\partial^3/\partial x^3)w(L,t) + f(t). \qquad (6) \end{aligned}$$

Here, r_p is the distance between the arm tip and the center of gravity of the payload.

Next, one defines the armature current i_a. The arm is driven so that the arm tip stays precisely at its initial position even if the tip is disturbed by an impulsive loading. The proportional, derivative (PD) servo loop using the tip sensing and base torquing is introdused to control the arm. One compares the initial tip position w_d(=0) with the actual position which is measured by a gap sensor fixed in space. The tip-position error ($w_d - w$) is used, together with the estimated tip velocity, as the basis for applying the control torque at the arm base. The equation governing the armature current is

$$\begin{aligned} &(L_a/R_a)(d/dt)i_a + i_a \\ &+ (K'/R_a)(\partial^2/\partial x\partial t)w(r,t) \\ &= -[G_d + G_v(\partial/\partial t)][1 + r_p(\partial/\partial x)]w(L,t). \quad (7) \end{aligned}$$

Here, L_a is the motor inductance, R_a is the circuit resistance, and K' is the back electromotive force constant ; G_d and G_v are the displacement and velocity servo loop gains.

Equations (1)-(7) are the governing equations for the problem under consideration. The solution can be obtained by applying the method of Laplace transform with respect to time. Transforming and solving equation (1), with the assumption that the arm is resting statically at t=0, one has the transformed displacement of arm $W(x,s)$ in the form

$$W(x,s) = \alpha \sin \xi x + \beta \cos \xi x + \gamma \sinh \xi x + \delta \cosh \xi x, \qquad (8)$$

where

$$\xi^4 = -\rho A s^2 / EI(1+cs).$$

Here, s is the Laplace transform parameter. The unknowns α to δ are determined as follows. One substitutes equation (8) into the transformed equations of the boundary conditions (2), (3), (5) and (6). Further, introducing the transformed current I_a derived from equation (7) into the resulted equations, one has a system of simultaneous algebraic equations of the form

$$[a_{ij}][\alpha,\beta,\gamma,\delta]^T = [0,0,0,F(s)]^T, \quad i,j = 1,...,4, \qquad (9)$$

where $F(s)$ is the transformed input loading ; a_{ij} are parameters given in the Appendix. After computing α to δ from equation (9), one has the transformed displacement in the form

$$W(x,s) = \Delta_\alpha \sin \xi x + \Delta_\beta \cos \xi x + \Delta_\gamma \sinh \xi x + \Delta_\delta \cosh \xi x)(1/\Delta)F(s). \qquad (10)$$

Here, $\Delta = det(a_{ij})$; Δ_α to Δ_δ are the Δ's with the 1st to 4th columm replaced by $[0,0,0,1]^T$, respectively. To get the final solution, one further needs to define the input forcing function $f(t)$. A typical form of the impulsive loading observed in the industrial field is

$$f(t) = F_0 \sin^2 \omega t \quad ,0 \le t \le \pi/\omega, \qquad = 0 \quad ,\pi/\omega < t, \qquad (11)$$

which is shown in Figure 2 for the case of $\omega = 20\pi$. The solution to the above input is determined as

$$W(x,s)/F_0 = (\Delta_\alpha \sin \xi x + \Delta_\beta \cos \xi x + \Delta_\gamma \sinh \xi x + \Delta_\delta \cosh \xi x) \times (1/\Delta)\{(2\omega)^2/2s[s^2+(2\omega)^2]\}[1-\exp(-\pi s/\omega)]. \qquad (12)$$

The displacement $w(x,t)$ is determined by the inverse transform of equation (12), which is finished by using the computer based on the numerical method proposed by Weeks(1966).

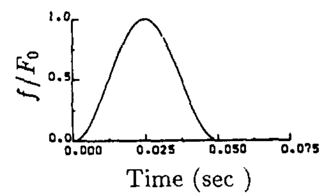

Fig.2 Input forcing function to arm tip $f(t)$. $\omega = 20\pi$, $f(t) = F_0 sin^2\omega t$ $(0 \le t \le \pi/\omega)$, $= 0$ $(\pi/\omega < t)$.

3.NUMREICAL RESULTS AND DISCUSSIONS

The single-link flexible-arm prototype which at Department of Mechanical Engineering, Tohoku University, has been chosen for developing a case study. The arm is a rectangular aluminum beam with thickness H and width B. The payload attached at the tip is a circular disk of radius r_p and mass M_p, whose moment of inertia about the diameter is given by $J_p = M_p r_p^2/4$. For the assembled control system, the physical parameters measured are given as follows.

Arm :

$E = 6.57 \times 10^{10}$Pa,
$B = 1.19 \times 10^{-2}$m,
$c = 1.19 \times 10^{-4}$s,
$H = 1.99 \times 10^{-3}$m,
$\rho = 2.67 \times 10^{3}$Kg/m^3,
$L = 5.00 \times 10^{-1}$m.

D.C. servomotor :

$J_m = 4.90 \times 10^{-6}$Kgm^2,
$K' = 1.77 \times 10^{-5}$Vs/rad,
$\epsilon = 2.60 \times 10^{-3}$Kgm^2/s,
$K_f = 6.77 \times 10^{-2}$Nm/A,
$L_a = 1.80 \times 10^{-3}$H ,
$r = 2.00 \times 10^{-2}$m ,
$R_a = 4.70\Omega$.

Tip mass :

$r_p = 2.00 \times 10^{-2}$m. (13)

Figure 3 shows the time response of the arm tip taking the tip displacement feedback gain G_d as a parameter. The velocity feedback gain is fixed

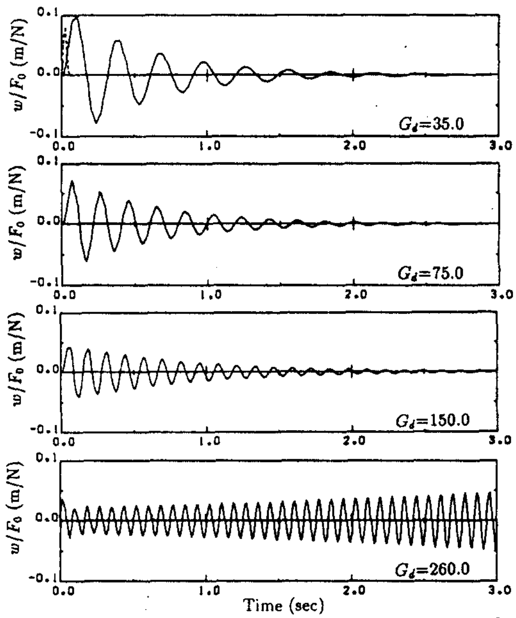

Fig.3 Effect of displacement feedback gain G_d on the arm tip responce; G_v=0.2 As/m, $\omega = 20\pi$, M_p=0; physical parameters other than M_p are as shown in equation (13).

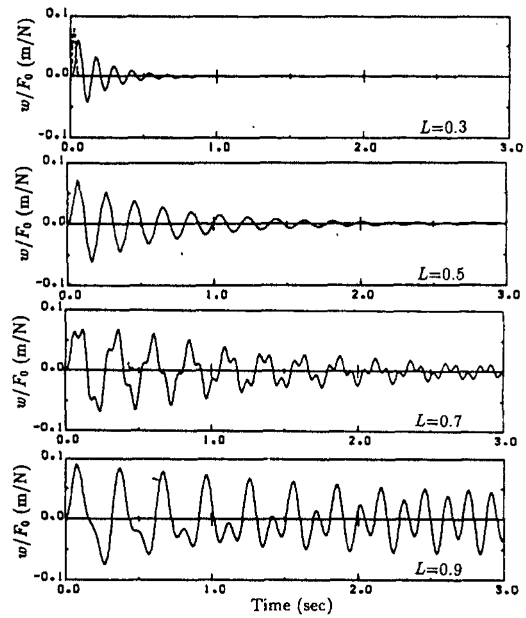

Fig.5 Effect of arm length L on the arm tip response; G_d=75.0 A/m, G_v=0.2 As/m, $\omega = 20\pi$, M_p=0; physical parameters other than M_p are as shown in equation (13).

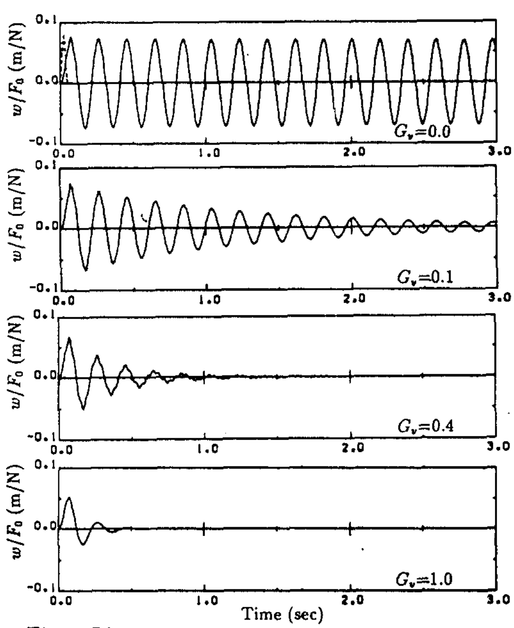

Fig.4 Effect of velocity feedback gain G_v on the arm tip response; G_d=0.75 A/m, $\omega = 20\pi$, M_p=0; physical parameters other than M_p are as shown in equation (13).

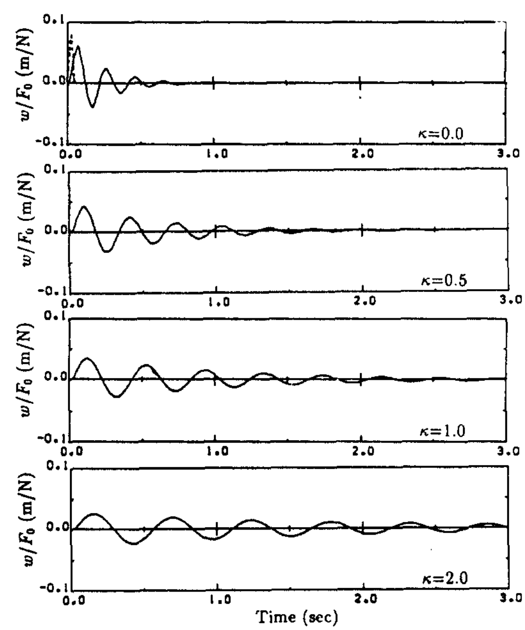

Fig.6 Effect of pay load mass on the arm tip response; G_d=75.0 A/m, G_v=0.6 As/m, $\omega = 20\pi$, $\kappa = M_p/\rho BH(L-r)$; physical parameters other than M_p are as shown in equation (13).

at G_v=0.2 As/m. The payload is not attached to the arm (M_p=0). The dotted curve in the figure indicates the form of the transient loading applied to the arm tip. The initial tip position is w=0. The tip is controlled so that it stays on the initial position precisely even if it is subjected to the disturbance. It is observed that the maximum displacement is reduced with an increase of the displacement feedback gain G_d. But the settling time of the tip to the intended position is not shortened by the increase. The figure in the bottom shows that an excess value of the gain G_d brings about a flutter type of instability on the system.

Figure 4 shows the tip response when the velocity feedback gain G_v is increased. The displacement feedback gain is fixed at G_d=75.0 A/m. It is evident that both the displacement and the duration of vibration are suppressed with an increase of G_v. Particularly its effect on the duration is remarkably observed.

Figure 5 is the variation of the tip displacement when the length of the arm is varied. The feedback gains are fixed at G_d=75.0 A/m and G_v=0.2 As/m. It is seen that an increase of the arm length significantly degenerates the rapid conversion of the tip vibration. A higher vibration mode becomes easily excited with an increase of the arm length.

Figure 6 is the results when the payload mass is increased. κ is the ratio of the tip mass to the total mass of the arm given by $\kappa = M_p/\rho BH(L-r)$. It is found that the maximum tip displacement is reduced, while the duration becomes significant with an increase of the tip mass.

4. CONCLUSIONS

PD control using the end point sensing and base torquing has been tested for the end-point position control of a one-link flexible arm under an impulsive loading. A simulation study has been presented for the flexible-arm prototype at the Department of Mechanical Engineering of Tohoku University. Results obtained can be summarized as follows.

(1) The control strategy proposed here is effective to make the arm tip stay at its initial position even if it is disturbed by an impulsive loading.

(2) Both the large tip displacement and the duration of vibration are suppressed by an increase of the velocity feedback gain, while the duration cannot be shortened with an increase of the displacement feedback gain.

REFERENCES

Bailey, T., and Hubbard, Jr., J.E., "Distributed Piezoelectric-Polymer Active Vibration Control of a Cantilever Beam", Journal of Guidance, Control & Dynamics, 1985, Vol.8, pp.605-611.

Baz, A., and Poh, s., "Performance of an Active Control System with Piezoelectric Actuators", Journal of sound and Vibration, 1988, Vol.126, pp.327-343.

Book, W. J., Maizza-Neto, O., and Whitney, D. E., "Feedback Control of Two Beam, Two Joint Systems With Distributed Flexibility", ASME Journal of Dynamic Systems, Measurement, and Control, 1975, Vol.97, pp.424-431.

Cannon, R.H.Jr., and Schmitz, E., "Initial Experiments on the End-Point Contrl of a Flexible One-Link Robot", The International Journal of Robotics Research, 1984, Vol.3, pp.62-75.

Chonan, s., and Aoshima, S., "3-Dimensional Vibration Analysis of Multi-Link Flexible Robot Arms", Transactions of the Japan Society of Mechanical Engineers, Series C, 1988, Vol.54, pp. 1170-1175 (in Japanese).

Chonan, S., and Umeno, A., "Closed-Loop End-Point Control of a Two-Link Flexible Arm with a Tip Mass", Journal of Sound and Vibration, 1989, Vol.133, pp.483-495.

Jiang, Z., Chonan, S., and Tani, J., "Control of a Flexible Arm Using Piezoelectric Bimorph Chells", Transactions of the Japanese Society of Mechanical Engineers, Series C, 1990, Vol.56, pp.416-423 (in Japanese).

Lee, C.K., and Moon, F.C., "Laminated Piezopolymer Plates for Torsion and Bending Sensors and Actuators", The Journal of Acoustical Society of America, 1989, Vol.85, pp.2432-2439.

Lee, J.D., and Wang, B.L., "Dynamic Equations for a Two-Link Flexible Robot Arm", Computers & Structures, 1988, Vol.29, pp.469-477.

Ower, J.C., and Van De Vegte, J., "Classical Control Design for a Flexible Manipulator: Modelling and Control System Design", IEEE Journal of Robotics and Automation, 1987, Vol.RA-3, pp.485-489.

Skaar, S.B., and Tucker, D., "Point control of a One-Link Flexible Manipulator", ASME Journal of Applied Mechanics, 1986, Vol.53, pp.23-27.

Tahara, M., and Chonan, S., "Closed-Loop Displacement Control of a One-Link Flexible Arm with a Tip Mass", Transactions of the Japan Society of Mechanical Engineers, Series C, 1988, Vol.54, pp.363-370 (in Japanese).

Wang, W., Lu, S., and Hsu, C., "Experiments on the Position Control of a One-Link Flexible Robot Arm", IEEE Transactions on Robotics and Automation, 1989, Vol.5, pp.373-377.

Weeks, W.T., "Numerical Inversion of Laplace Transforms Using Laguerre Functions", Journal of the Association for Computing Machinery, 1966, Vol.13, pp.419-429. Yigit, A., Scott, R.A., and Ulsoy, A.G., "Flexible Motion of a Radially Rotating Beam Attached to a Rigid Body", Journal of Sound and Vibration, 1988, Vol.121,pp.201 -210.

Yosida, K., Shimogo, T., and Inose, J., "Optimum Control of Elastic Structure System Taking Account of Spillover (Application to a Position Control of Elastic Rotating Arm)", Transactions of the Japan Society of Mechanical Engineers, Series C 1988, Vol.54, pp.201-208 (in Japanese).

Yuh, J., "Application of Discrete-Time Model Reference Adaptive Control to a Flexible Single-Link Robot", Journal of Robotic Systems, 1987, Vol.4, pp.621-630.

APPENDIX

a_{ij}=1,...,4, in equation (9) are given by :

$$a_{11} = \sin\xi r - \xi r\cos\xi r,$$
$$a_{12} = \cos\xi r + \xi r\sin\xi r,$$
$$a_{13} = \sinh\xi r - \xi r\cosh\xi r,$$
$$a_{14} = \cosh\xi r - \xi r\sinh\xi r,$$
$$\begin{aligned}a_{21} = {} & K_t(G_d + G_v s)\sin\xi L \\ & + K_t(G_d r_p + G_v r_p s)\xi\cos\xi L \\ & + \{[(L_a/R_a)s+1](J_m s^2 + \epsilon s) \\ & + K_t(K'/R_a)s\}\xi\cos\xi r \\ & + EI[(L_a/R_a)s+1](1+cs)\xi^2\sin\xi r \\ & + rEI[(L_a/R_a)s+1](1+cs)\xi^3\cos\xi r,\end{aligned}$$
$$\begin{aligned}a_{22} = {} & K_t(G_d + G_v s)\cos\xi L \\ & - K_t(G_d r_p + G_v r_p s)\xi\sin\xi L \\ & - \{[(L_a/R_a)s+1](J_m s^2 + \epsilon s) \\ & + K_t(K'/R_a)s\}\xi\sin\xi r \\ & + EI[(L_a/R_a)s+1](1+cs)\xi^2\cos\xi r \\ & + rEI[(L_a/R_a)s+1](1+cs)\xi^3\sin\xi r,\end{aligned}$$
$$\begin{aligned}a_{23} = {} & K_t(G_d + G_v s)\sinh\xi L \\ & + K_t(G_d r_p + G_v r_p s)\xi\cosh\xi L \\ & + \{[(L_a/R_a)s+1](J_m s^2 + \epsilon s) \\ & + K_t(K'/R_a)s\}\xi\cosh\xi r \\ & - EI[(L_a/R_a)s+1](1+cs)\xi^2\sinh\xi r \\ & + rEI[(L_a/R_a)s+1](1+cs)\xi^3\cosh\xi r,\end{aligned}$$
$$\begin{aligned}a_{24} = {} & K_t(G_d + G_v s)\cosh\xi L \\ & + K_t(G_d r_p + G_v r_p s)\xi\sinh\xi L \\ & + \{[(L_a/R_a)s+1](J_m s^2 + \epsilon s) \\ & + K_t(K'/R_a)s\}\xi\sinh\xi r \\ & - EI[(L_a/R_a)s+1](1+cs)\xi^2\cosh\xi r \\ & + rEI[(L_a/R_a)s+1](1+cs)\xi^3\sinh\xi r,\end{aligned}$$
$$\begin{aligned}a_{31} = {} & J_p s^2\xi\cos\xi L \\ & + EI(1+cs)(-\xi^2\sin\xi L - r_p\xi^3\cos\xi L),\end{aligned}$$
$$\begin{aligned}a_{32} = {} & -J_p s^2\xi\sin\xi L \\ & + EI(1+cs)(-\xi^2\cos\xi L + r_p\xi^3\sin\xi L),\end{aligned}$$
$$\begin{aligned}a_{33} = {} & J_p s^2\xi\cosh\xi L \\ & + EI(1+cs)(\xi^2\sinh\xi L + r_p\xi^3\cosh\xi L),\end{aligned}$$
$$\begin{aligned}a_{34} = {} & J_p s^2\xi\sinh\xi L \\ & + EI(1+cs)(\xi^2\cosh\xi L + r_p\xi^3\sinh\xi L),\end{aligned}$$
$$\begin{aligned}a_{41} = {} & M_p s^2(\sin\xi L + r_p\xi\cos\xi L) \\ & - EI(1+cs)\xi^3\cos\xi L,\end{aligned}$$
$$\begin{aligned}a_{42} = {} & M_p s^2(\cos\xi L - r_p\xi\sin\xi L) \\ & - EI(1+cs)\xi^3\sin\xi L,\end{aligned}$$
$$\begin{aligned}a_{43} = {} & M_p s^2(\sinh\xi L + r_p\xi\cosh\xi L) \\ & - EI(1+cs)\xi^3\cosh\xi L,\end{aligned}$$
$$\begin{aligned}a_{44} = {} & M_p s^2(\cosh\xi L + r_p\xi\sinh\xi L) \\ & - EI(1+cs)\xi^3\sinh\xi L.\end{aligned}$$