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A MODIFIED SLIDING MODE CONTROLLER FOR THE POSITION CONTROL OF A DIRECT DRIVE ARM

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ABSTRACT

In this paper, a new hybrid position control algorithm for the direct drive arm is proposed. The proposed control is composed of discrete feedforward component and continuous feedback component. The discrete component is the nominal torque which approximately compensates the strong nonlinear coupling torques between the links, while the continuous control is a modified version of sliding mode control which is known to have a robust property to the disturbances of system.

For the proposed control law, we give sufficient condition which guarantees the the bounded tracking error in spite of the modeling errors, and the efficiency of the proposed algorithm is demonstrated by the numerical simulation of a three link manipulator position control with payloads and parameter errors.

1. INTRODUCTION

The dynamic equation of robotic manipulator is highly nonlinear due to the inertia and strong coupling terms among the joints[1,2]. The direct drive arm does not have backlash and friction of reducers so it is suitable for the accurate position control. However, in direct drive arm, the the load and the disturbance torque influence the motor dynamics directly, since the motor is directly linked to the load. Neglecting the nonlinear dynamic terms, it is difficult to guarantee the tracking error bound in high speed motions, since the neglected terms act as a large disturbance to the controller. In order to improve the trajectory tracking accuracy, it is necessary to take the robot manipulator dynamics into account.

Many advanced state space control methods of robot manipulators have been proposed which needs the computation of complex dynamics. The well-known Computed Torque Method (CTM) is a good robot controller, if the exact knowledge of the manipulator dynamics is available. However, it is almost impossible to obtain the complete dynamic model of robots due to modeling uncertainties, parameter variations and unknown payloads *etc.* This modeling errors, especially the error of inertia matrix, may destabilize the controlled system[3,4]. Moreover, the computation of dynamics requires relatively long sampling time and this leads to the time delay of control input which deteriorates the performance in real-time control systems[5]. These inherent modeling errors and computational time delay call for the robust robot control algorithms.

The sliding mode controller based on the Variable Structure System(VSS) control is robust to the modeling errors[6], which makes it appropriate for the manipulator control problem, which necessarily contains the modeling uncertainty and large disturbances. Therefore, many robot arm control algorithms have been proposed using the sliding mode control[7-10]. The VSS control has different structures depending on which side of the hyperplane(sliding surface) the system belongs to. In this theory, the switch of control structure is assumed to occur with infinite frequency. However, due to physical constraints, input switches not with infinite frequency but only with finite high frequency, and the motion of system will be in some neighborhood of the sliding surface with chattering, which is undesirable in practice. To avoid it, many algorithms have been proposed, which replaced the discontinuous control in the neighborhood of sliding surface by some continuous control[8-10]. These algorithms are VSS control if the system trajectory is outside the boundary of the sliding surface, but if the trajectory is inside the boundary of sliding surface, they interpolate the control by proper continuous function to avoid the discontinuity of it. However, to implement these algorithms digitally, we need the computation of robot model with feedback information at every sampling time. In spite of the efficient recursive dynamics algorithms and computing hardware, relatively long time is necessary for the computation of model[1,2,11,12].

To reduce the time delay of control, it is desirable that the feedback information is not involved in the computation of model, and the feedforward compensation using the nominal torque is a good candidate for this. The adaptive control algorithm with feedforward compensation[13] may be a robust control, but it does not contain the stability analysis and has a feedback component which needs too much computation. The hybrid controller which consists of discrete feedforward compensation and continuous P-D control gave good trajectory tracking performance[14], but the stability of system was not analyzed and the feedback component must give more robustness to the compensation error.

In this paper, a new hybrid control algorithm is proposed and the stability of the system is proved. The proposed algorithm is comprised of discrete and continuous controls. The role of discrete component is to compensate the nonlinear coupling torques approxamately. This component allows a long sampling time and can be computed off-line. The continuous component is a modified sliding mode control which gives the robust trajectory tracking property to the approximately compensated system.

2. PRELIMINARIES

The motion equations of an n d.o.f. manipulator can be derived from the Lagrange–Euler formulation and may be expressed generally as

$$D(q(t))\ddot{q}(t)+h(q(t),\dot{q}(t))=\tau(t) \qquad (1)$$

where $\tau(t)\in\mathbb{R}^n$ is joint torque, $q(t),\dot{q}(t),\ddot{q}(t)\in\mathbb{R}^n$ are joint position, velocity and acceleration, $D(q(t))\in\mathbb{R}^{n\times n}$ is inertia which is a symmetric positive definite matrix and $h(q(t),\dot{q}(t))\in\mathbb{R}^n$ is nonlinear coupling term including centrifugal, Coriolis and gravitational forces. In the following, $D(q(t))$ will be denoted as D and $h(q(t),\dot{q}(t))$ as h when necessary. If we define $x(t)=[q(t)^\top,\dot{q}(t)^\top]^\top\in\mathbb{R}^{2n}$, the state equation (1) becomes

$$\dot{x}(t)=\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}x(t)+\begin{bmatrix} 0 \\ -D^{-1}h \end{bmatrix}+\begin{bmatrix} 0 \\ D^{-1} \end{bmatrix}\tau(t) \qquad (2)$$

To compute the control torque of system (1), we need the dynamic model of robot system. But exact modeling is impossible because of the existence of parameter uncertainties, unknown frictions and varying payloads *etc.*. So we may express the available model as follows

$$\hat{D}(q(t))\ddot{q}(t)+\hat{h}(q(t),\dot{q}(t))=\tau(t). \qquad (3)$$

In this paper, the position control of the system (1) is considered when the desired trajectories $q_d(t),\dot{q}_d(t),\ddot{q}_d(t)$ and the available model (3) are given.

Let's define $s(t)\in\mathbb{R}^n$ as

$$s(t)=\dot{e}(t)+K_v e(t)+K_p\int_{t_o}^{t} e(\tau)\,d\tau \qquad (4)$$

where $e(t)=q(t)-q_d(t)\in\mathbb{R}^n$ is an error and $K_v, K_p\in\mathbb{R}^{n\times n}$ are gains. Let the trajectory (we will call **equivalent trajectory**) $q_*(t)$, $\dot{q}_*(t)$, $\ddot{q}_*(t)\in\mathbb{R}^n$ satisfy the following equation

$$\ddot{e}(t) + K_v\dot{e}(t) + K_p e(t) = 0. \qquad (5)$$

or

$$\dot{x}_*(t)-\dot{x}_d(t)=A\cdot[x_*(t)-x_d(t)] \qquad (6)$$

where $x_*(t)=[q_*(t)^\top,\dot{q}_*(t)^\top]^\top$, $x_d(t)=[q_d(t)^\top,\dot{q}_d(t)^\top]^\top$ and $A\in\mathbb{R}^{2n\times 2n}$ is

$$A=\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}. \qquad (7)$$

Since $det[\lambda I-A] = det[\lambda^2 I+\lambda K_v+K_p]$, we can find K_v and K_p which makes the system (6) exponentially stable, or equivalently which makes the condition

$$\|e^{At}\| \le g\cdot e^{-\kappa t} \quad \textit{for all } t\ge 0 \qquad (8)$$

hold for some given $g>0$ and $\kappa>0$[15]. Here, the induced Euclidean matrix norm of A is defined as

$$\|A\| = [\lambda_M(A^\top A)]^{\frac{1}{2}} \qquad (9)$$

where $\lambda_M(\cdot)$ denotes the maximum eigenvalue of matrix. It must be noted that the equivalent trajectory is only an imaginary intermediate function between $x(t)$ and $x_d(t)$ and does not exists in real system. In the following, we assume that the gain matrices K_v, K_p are chosen so that the condition (8) holds for given g,κ. Now, we state the following Lemmas as a prerequisite to main Theorems.

Lemma 1 : If $\|s(t)\|\le\gamma$ is satisfied for any $t\ge t_o$, then

$$\|x(t)-x_*(t)\|\le[\|x(t_o)-x_*(t_o)\|+2\gamma]\cdot e^{\|A\|t} \qquad (10)$$

is satisfied for all $t\ge t_o$.

Proof : Using (4) and (6), we may rewrite (2) as

$$\dot{x}(t)-\dot{x}_*(t)=A[x(t)-x_*(t)]+\begin{bmatrix} 0 \\ \dot{s}(t) \end{bmatrix} \qquad (11)$$

If we integrate and take the norm of both sides, then if we apply the Bellman–Gronwall inequality[16], we obtain

$$\|x(t)-x_*(t)\|\le[\|x(t_o)-x_*(t_o)\|+2\gamma]\cdot e^{\|A\|t} \qquad (12)$$

for all $t\ge t_o$, which proves Lemma 1.

If we take $x_*(t_o)=x(t_o)$, (10) becomes

$$\|x(t)-x_*(t)\|\le 2\gamma\cdot e^{\|A\|t}. \qquad (13)$$

The above Lemma implies that the distance of $x(t)$ from equivalent trajectory $x_*(t)$ is bounded **for finite time**. The next Lemma states the boundedness of tracking error **for the infinite time interval**. Define the set $S(\cdot;\cdot)$ as

$$S(\rho;v)=\{w\in\mathbb{R}^n;\|w-v\|\le\rho\}. \qquad (14)$$

for given $\rho>0$ and vector $v\in\mathbb{R}^n$.

Lemma 2 : Suppose $\|s(t)\|\le\gamma$ is satisfied for all $t\ge t_o$, and the system (6) satisfies (8). Then $x(t)$ converges exponentially into the set $S(\epsilon\gamma;x_d(t))$ with ϵ given by

$$\epsilon = 2\cdot(1+\mu)\cdot\left\{\frac{\mu}{g\cdot(1+\mu)}\right\}^{-\mu} \qquad (15)$$

where μ is defined as

$$\mu = \frac{\|A\|}{\kappa} \qquad (16)$$

Proof : Since the equivalent trajectory satisfies (8), there exists $T < \infty$ which satisfy

$$\| x_*(t+T) - x_d(t+T) \| \le \alpha \| x_*(t) - x_d(t) \| \qquad (17)$$

for any $\alpha \in (0,1)$ and for all t. Define $E(\alpha,\beta)$ as

$$E(\alpha,\beta) = \frac{2 \cdot e^{\|A\| \cdot T}}{\beta - \alpha} \qquad (18)$$

for any $\beta \in (\alpha,1)$. Now if $\| x(t_o) - x_d(t_o) \| > E(\alpha,\beta)\gamma$ holds for some t_o, we take $x_*(t_o) = x(t_o)$ and apply the triangle inequality

$$\| x_* - x_d \| \le \| x - x_* \| + \| x_* - x_d \| \qquad (19)$$

to (17) and use the inequalities (13) and (17) to get

$$\| x(t_o+T) - x_d(t_o+T) \| < \beta \| x(t_o) - x_d(t_o) \| \qquad (20)$$

Now if $\| x(t) - x_d(t) \| > E(\alpha,\beta)\gamma$ still holds at $t = t_o + T$, we repeat the same process with the initial condition $x_*(t_o+T) = x(t_o+T)$. Then we get

$$\| x(t_o+2T) - x_d(t_o+2T) \| < \beta^2 \| x(t_o) - x_d(t_o) \| \qquad (21)$$

If this process is repeated n times, it follows

$$\| x(t_o+nT) - x_d(t_o+nT) \| < \beta^n \| x(t_o) - x_d(t_o) \|. \qquad (22)$$

Equation (22) implies the exponential convergence of $x(t)$ into the set $S(E(\alpha,\beta); x_d(t))$. The infimum of $E(\alpha,\beta)$ with respect to β occurs as $\beta \to 1$, so if we differentiate $E(\alpha,1)$ with respect to α and equate to 0, we get

$$\alpha = \alpha^* = \frac{\frac{\|A\|}{\kappa}}{1 + \frac{\|A\|}{\kappa}}, \qquad (23)$$

and the upper bound of the trajectory error $E(\alpha^*,1)$ is given by (17) and (18). This completes the proof.

Lemma 2 says that if $\| x(t_o) - x_d(t_o) \| \le \epsilon\gamma$ is satisfied at $t = t_o$, then $\| x(t) - x_d(t) \| \le \epsilon\gamma$ holds for all $t > t_o$. In the next section, we propose a control algorithm which guarantees $\| s(t) \| \le \gamma$ for the position control.

3. CONTROL ALGORITHMS

$$\tau(t) = \tau_{ff}(t) + \tau_c(t) \qquad (24)$$

where

$$\tau_{ff}(t) = \hat{D}(\bar{q}_d(t))\,\ddot{\bar{q}}_d(t) + \hat{h}(\bar{q}_d(t), \dot{\bar{q}}_d(t)) \qquad (25)$$

$$\tau_c(t) = \hat{D}(\bar{q}_d(t)) \cdot u_c(t) \qquad (26)$$

and

$$u_c(t) = -K_v \dot{e}(t) - K_p e(t) - k_s s(t) - k_o \frac{s(t)}{\| s(t) \| + \Delta}. \qquad (27)$$

The feedforward component $\tau_{ff}(t)$ and $\hat{D}(\bar{q}_d(t))$ in feedback component $\tau_c(t)$ are step functions with large time interval T_β which must be greater than the necessary time for the computation of model (3). We used $\bar{q}_d(t), \dot{\bar{q}}_d(t), \ddot{\bar{q}}_d(t)$ to denote that these are sampled values of the desired trajectories $q_d(t), \dot{q}_d(t), \ddot{q}_d(t)$ with a sampling interval T_β. In the following, let's denote $\hat{D}(\bar{q}_d(t))$ as $\hat{D}_d$ and $\hat{h}(\bar{q}_d(t), \dot{\bar{q}}_d(t))$ as $\hat{h}_d$ for brevity. If we apply the input control torque given by (24)–(27) to the robot system (1), we get

$$\dot{s}(t) = -k_s s(t) - k_o D^{-1} \hat{D}_d \frac{s(t)}{\| s(t) \| + \Delta} + n(t) \qquad (28)$$

where the disturbance vector $n(t) \in \mathbb{R}^n$ is given by

$$n(t) = n(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}) = \delta D(t)[\ddot{\bar{q}}_d(t) - K_v \dot{e}(t)$$

$$-K_p e(t) - k_s s(t)] + [\ddot{\bar{q}}_d(t) - \ddot{q}_d(t)] + D^{-1}[\hat{h}_d - h] \qquad (29)$$

and $\delta D(t)$ is defined as

$$\delta D(t) = \delta D(q_d, q) = D^{-1} \hat{D}_d - I. \qquad (30)$$

We can rewrite (28) as

$$\dot{s}(t) = -k_s s(t) - k_o \frac{s(t)}{\| s(t) \| + \Delta} - k_o \frac{\delta D(t)\, s(t)}{\| s(t) \| + \Delta} + n(t) \qquad (31)$$

Let's define N and M as

$$N = \max_{t,z,w} \{ \| n(q_d, \dot{q}_d, \ddot{q}_d, z, w) \| ; [z(t)^T, w(t)^T]^T \in S(\epsilon\gamma; x_d(t)) \} \qquad (32)$$

$$M = \max_{t,z} \{ \| \delta D(q_d, z) \| ; z(t) \in S(\epsilon\gamma; q_d(t)) \} \qquad (33)$$

Theorem 1 : Consider the system (1) with controls (24)–(27). If $\| s(t_o) \| \le \gamma$ and $\| x(t_o) - x_d(t_o) \| \le \epsilon\gamma$ is satisfied for some $\gamma > 0$, and if the gain k_o satisfies

$$N \cdot \left(1 + \frac{\Delta}{\gamma}\right) < k_o < \frac{(k_s - \rho) \cdot (\Delta + \gamma)}{M} \qquad (34)$$

for given K_v, K_p, k_s and for some small $\rho > 0$, then the system trajectory satisfies

$$\| s(t) \| \le \gamma, \; x(t) \in S(\epsilon\gamma; x_d(t)) \qquad (35)$$

for all $t \ge t_o$.

Proof : Take $V(t) = \frac{1}{2} s(t)^T s(t)$ as a Lyapunov function and differentiate it with respect to t, then use the matrix inequality[17]

$$x^T A\, y \le \| x \| \cdot \| y \| \cdot \| A \| \qquad (36)$$

we get

$$\frac{dV}{dt}(t) \le \|s(t)\|\left\{\|n(t)\| - k_o\frac{\|s(t)\|}{\|s(t)\| + \Delta}\right\}$$
$$+ \|s(t)\|^2\left\{k_o\frac{\|\delta D(t)\|}{\|s(t)\| + \Delta} - k_s\right\}. \tag{37}$$

If we assume $x(t)\notin S(\epsilon\gamma;x_d(t))$ for some $t=t_2$, then there exists $t_1\in[t_o,t_2)$ such that $s(t)\in S(\gamma)$ for all $t\in[t_o,t_1)$ but $\|s(t_1)\|=\gamma$ and $\frac{dV}{dt}(t_1)>0$. Then $x(t_1)\in S(\epsilon\gamma;x_d(t_1))$, $\|n(t_1)\|\le N$ and $\|\delta D(t_1)\|\le M$ follow from Lemma 2, (32) and (33). Hence

$$\frac{dV}{dt}(t) \le \|s(t)\|\left\{N - k_o\frac{\|s(t)\|}{\|s(t)\| + \Delta}\right\}$$
$$+ \|s(t)\|^2\left\{k_o\frac{M}{\|s(t)\|+\Delta} - k_s\right\} \tag{38}$$

at $t=t_1$. Hence if (34) holds

$$\frac{dV}{dt}(t) < -\rho s(t)^{\mathsf{T}}s(t) \tag{39}$$

at $t=t_1$, which completes the proof by contradiction.

If we take smaller T_β and use more accurate model, k_o has smaller lower bound and larger upper bound, which ensure the existence of gain k_o. We might take a large sampling time T_β for the discrete terms τ_{ff} and $\hat{D}_d$ considering the computational burden. However, if we take large sampling time for the discrete terms, $\|n(t)\|$ and $\|\delta D(t)\|$ become so large that the condition (34) become severe. For the computation of $\hat{D}_d$, we may use the G–D algorithm[18] or CMAC[19]. Note that these terms are functions of only desired trajectories so these can be computed off–line.

If we can choose smaller γ, it is evident that $s(t)$ remains closer to the surface $s(t) = \mathbf{0}$ and the trajectory error becomes smaller. If we take smaller Δ in control algorithm (27), then the lower and upper bounds of control gain k_o is decreased, but it is difficult to expect the smooth change of control. If accurate model is available and the sampling time T_β is small, we can assume that $M<1$, which gives another sufficient condition of k_o for the system stability.

Theorem 2 : Consider system (1) with controls given by (24)–(27). Assume that $M<1$ is satisfied and for some $\gamma>0$, $\|s(t_o)\|\le\gamma$ and $\|x(t_o)-x_d(t_o)\|\le\epsilon\gamma$ is satisfied. If the gain k_o satisfies

$$\frac{N(1+\frac{\Delta}{\gamma})}{1-M} < k_o \tag{40}$$

for given K_v, K_p and k_s, then the system trajectory satisfies

$$\|s(t)\| \le \gamma \text{ and } x(t)\in S(\epsilon\gamma,x_d(t)) \tag{41}$$

for all $t \ge t_o$.

PROOF : Same as the proof of Theorem 1, we get

$$\frac{dV}{dt}(t) = \|s(t)\|\left\{\|n_1(t)\| - k_o\frac{\|s(t)\|}{\|s(t)\| + \Delta}\right.$$
$$\left.(1-\|\delta D(t)\|)\right\} - k_s s(t)^{\mathsf{T}}s(t). \tag{42}$$

If we assume that $\|s(t)\|\le\gamma$ for all $t_o\le t\le t_1$, and $\|s(t_1)\|=\gamma$, then it follows

$$\frac{dV}{dt}(t)\le\|s(t)\|\left\{N-k_o\frac{\|s(t)\|}{\|s(t)\|+\Delta}[1-M]\right\}-k_s s(t)^{\mathsf{T}}s(t) \tag{43}$$

at $t=t_1$. Hence, from the condition (40), it follows

$$\frac{dV}{dt}(t) < -k_s s(t)^{\mathsf{T}}s(t) \tag{44}$$

at $t = t_1$, which completes the proof.

Theorem 2 gives some insight of the role of the term, $-k_s s(t)$: if $\|s(t)\| > \gamma$ is satisfied, then $\|s(t)\|$ converges exponentially into the bound γ with a faster rate than $1/k_s$. For the implementation, it is efficient to take the gain matrices K_v and K_p as diagonal so that we may compute $u_c(t)$ independently for each joint. When we compute the feedback component, the inertia matrix $\hat{D}_d$ is assumed to be known since the update of it allows large sampling time. If we digitally implement the proposed algorithm, the controller may take the multirate structure. Since the feedforward component is computed using the available model, which needs $132n$ multiplications and $111n - 4$ additions using the recursive Newton–Euler algorithm. However, the needed number of multiplication and addition to compute $\tau_c(t)$ is $2n + 6$ multiplications, $2n + 5$ additions and 1 root only. So that the feedback component might have frequency of up to forty times higher than feedforward component.

4. NUMERICAL SIMULATION

The 3 d.o.f. robot model is shown in Fig.1, and the parameters are D=0.0243kg·m, $m_1 = m_2 = 0.782$ kg , and $l_1 = l_2 = 0.23$ m. We assumed the modeling errors of each mass,link and inertia to be 1%. Although the model was assumed to carry no payload, the real robot system (1) was simulated by carrying the payloads of 0 kg.,0.3 kg., and 0.5 kg.. The execution time was 2 seconds and the desired trajectory was

$$q_d(t) = q_i+\frac{(q_f-q_i)\,t}{2} - \frac{(q_f-q_i)\cdot\sin(\pi t)}{2\pi}$$

with initial position $q_i = [0.4,-0.1,0.2]^{\mathsf{T}}$rad and final position $q_f = [-0.1,0.3,0.65]^{\mathsf{T}}$rad. We assumed T_α as 1ms but two cases for T_β ; 10ms, 50ms. The simulation block diagram is shown in Fig.2. We assumed K_v=50I, K_p=100I, k_s=100, k_o=20 and γ=0.1. The simulation result is shown in Fig.3.

When the payload is 0 kg(*i.e.* no payload error exists) and T_β = 10ms, the proposed algorithm gives very good performance of tracking error. If we take the compensation interval T_β as 50ms, the error bound becomes larger than that of T_β=10ms case.

As the payload error is increased, the proposed algorithm maintains the robust trajectory tracking. In case of T_β=50ms, it must be noted that the tracking errors does not deteriorate so much as the increase of the payload error.

The simulation results show that the input torque of proposed algorithm changes smoothly, which is a merit for real implementation. To compensate for the disturbance of the payload error and input delay due to large sampling time, large input torque is necessary and chattering occurs with small magnitude.

5. CONCLUSION

In this paper, a new hybrid control algorithm is proposed. It is shown that the modified sliding mode feedback component gives stability property to the system in spite of large feedforward compensation errors. The proposed algorithm is practical for real-time digital implementation, since a relatively long sampling time is allowed for the off-line computation of the model and the simple structure of feedback control law allows a small sampling time, which leads to the multirate control. The simulation result shows that the proposed algorithm has efficient trajectory tracking property and robust property to the modeling inaccuracy and unknov payloads.

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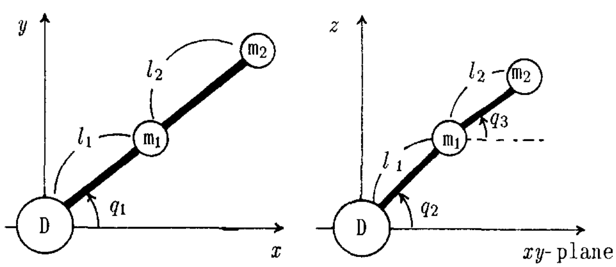


Fig.1 A Three d.o.f. Manipulator

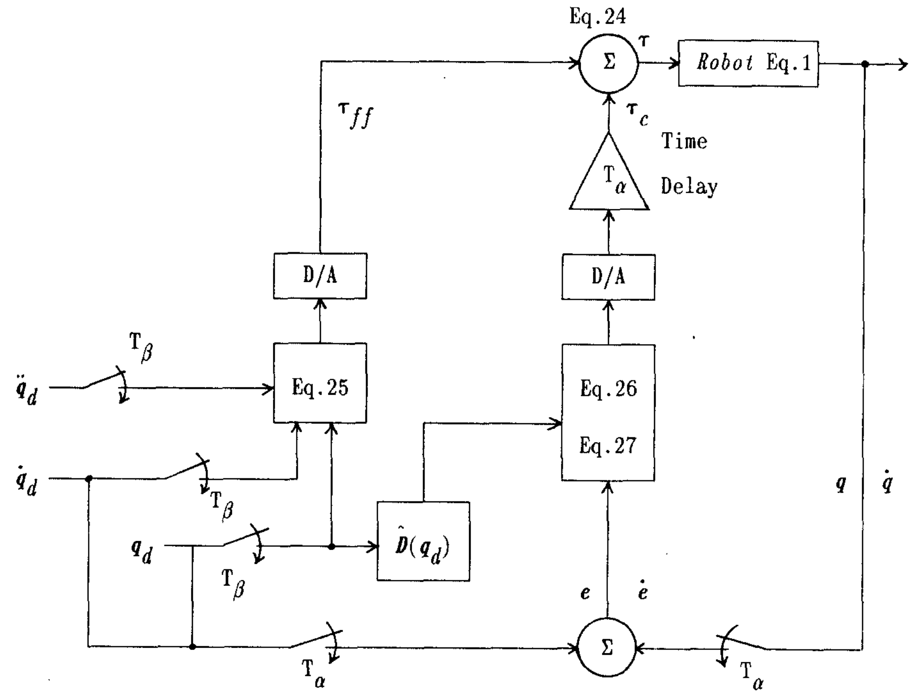
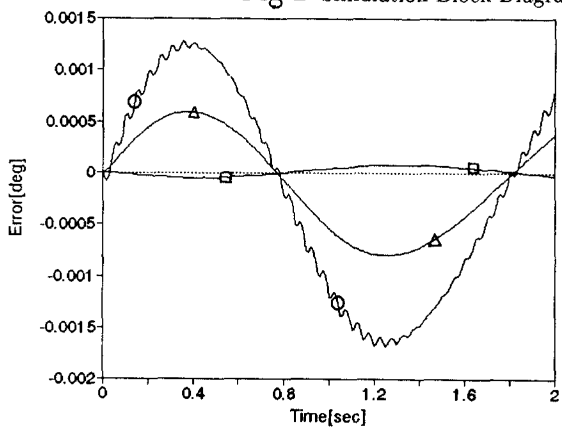
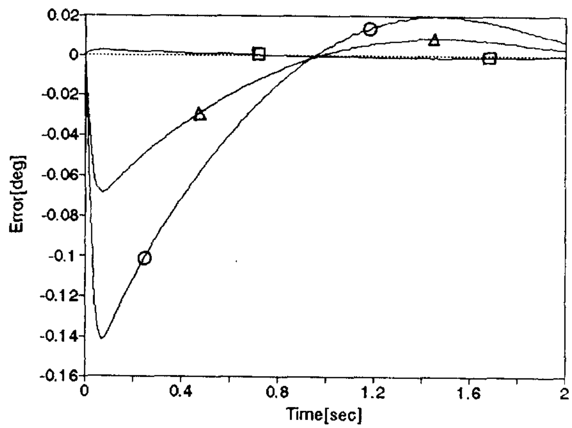


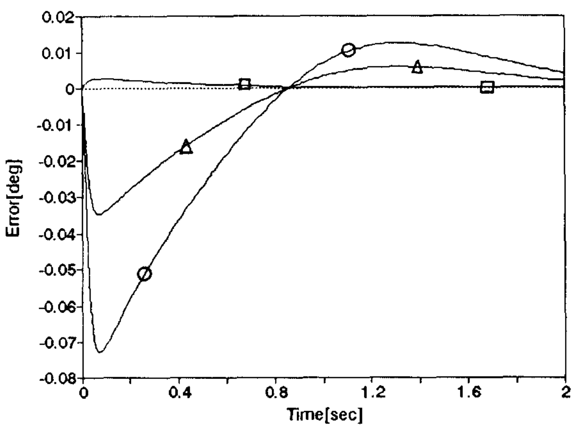
Fig.2 Simulation Block Diagram of the Proposed Control Algorithms



(a) Joint 1



(c) Joint 3



(b) Joint 2

Fig.3 Simulation result of the proposed algorithm

(□ : Load (Error) = 0 kg and T_β=10ms,
Δ : Load (Error) = 0.3kg and T_β=10ms,
○ : Load (Error) = 0.5kg and T_β=50ms)