ON A ROBUST DESIGN OF TIME-VARYING SYSTEMS WITH BOUNDED DISTURBANCE

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ABSTRACT

The purpose of this paper is to design a robust controller for a class of time-varying systems with bounded disturbance described by the differential equation. The robust designing method proposed in this paper, called "incentive design method" is different from developed designing methods in the past, and has following properties. The robust control law designed by this method can guarantee a certain value of the cost functional no matter how the disturbance vary within the given bounds. Here, the certain value of the cost functional may not be a saddle-point value, but is the value selected by a system designer. Therefore, the bounded disturbance has at least no bad effect on the value of the cost functional during finite interval of time. The method is based on the theory of incentive differential games. In addition, the form of control law is constructed by the system designer ahead of time. A numerical illustrative example is given in this paper. It is shown from this derivation and this numerical example that the approach developed in this paper is effective and feasible for some practical control problem.

1. INTRODUCTION

Robustness has been regarded as one of important properties for control systems. Robustness deals with the question whether some relevant qualitative properties are preserved if unknown perturbations are presented in the dynamical system. The basic aspect of robustness is the design of a control law which achieves pre-specified performance criteria over an entire region of operating conditions, that is, one of purpose of designing robust control is to ensure desirable closed-loop properties.

Several kinds of methods designing robust control for the system with disturbance have been proposed by many authors from different points of view. For example, servomechanism, minimax optimal method and cheap control have been proposed.[1][2][3] In addition, if the statistical properties of the disturbance are assumed, stochastic control approaches are used.[4] Furthermore, H^∞ optimal control theory based on minimax optimality has been proposed and studied recently.[5][6] It is shown that H^∞ optimal control theory has effective results for designing control system with disturbance.

The purpose of the feedback control is to coincide the state of system with desirable value under uncertain disturbance. In general, the disturbance is not measurable directly, but the effect of the disturbance can be known by observing state of system. In this case, there exists delay time until state of system is influenced by disturbance. To suppress the disturbance, the feedback control law must be regulated based on difference between state of systems and desirable value. It is possible to improve dynamical properties of system using a high-gain feedback represented by cheap control. In general, the system constructed by a high-gain feedback becomes unstable frequently and even if the systems are stable, the response has excessive peeks. Moreover, it is expected that the value of cost functional has the excessive amounts of energy.

A designing method of robust control, called "incentive design method", has been proposed by us to design the robust control law.[8] The good results have been obtained by applying the method for a class of systems containing the bounded uncertain parameters. The method is based on the theory of incentive differential games.[7]

The purpose of this paper is to provide the designing method of robust control law for a class of time-varying systems with bounded disturbance. This method is different from developed designing methods in the past and has following properties. The robust control law designed by this method can guarantee a certain value of the cost functional no matter how the disturbance vary within the given bound, that is, the value of cost functional is suppressed under a certain value during finite interval of time. Therefore, the bounded disturbance has at least no bad effect on the value of the cost functional during finite interval of time. Here, the certain value of the cost functional may not be a saddle-point value, but is the value selected by a system designer. In addition, the form of control law is constructed by the system designer ahead of time.

In this paper, firstly we illustrate the outline of this method for general systems. Then, we design concretely such a robust control law for a class of linear time-varying systems with bounded disturbance. Moreover, we give a numerical illustrative example, and show this method is effective and feasible for some kinds of practical systems. Finally, the paper will be conclude with discussions on the results obtained above and some possible futher researches.

2. INCENTIVE DESIGN METHOD

In general, the systems with bounded disturbance can be described by

$$\dot{x}(t) = f(t, x(t), u(t), v(t)), \tag{1}$$

$$x(t_0) = x_0, \qquad t \in [t_0, t_f],$$

where $x(t) \in \Re^n$ is a state vector and $u(t) \in U$ is a control input vector. $f = (f_1, \ldots, f_n)^T$ is the vector field in $\Re^n$. U is a closed and bounded subset of $\Re^m$. $v(t)$ is a scalar valued disturbance that is known to be in a closed and bounded region,

$$v(t) \in \Omega, \tag{2}$$

where Ω is the subset of $\Re$. Fig.1 shows the simplified model of the control systems for analysis.

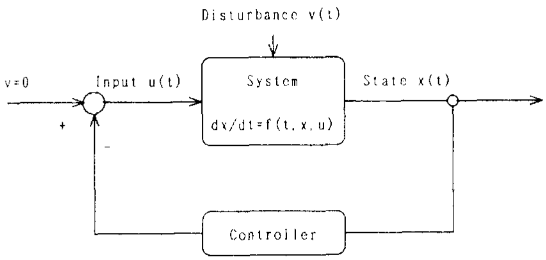

Fig.1 The Robust control of system with disturbance.

The cost functional is given as

$$J(x, u, v) = \Psi(t_f, x(t_f)) + \int_{t_0}^{t_f} L(t, x(t), u(t))dt. \tag{3}$$

A robust control law is to express $u(t)$ as a function of the state variable $x(t)$ and time t,

$$u(t) = \gamma(t, x(t)), \tag{4}$$

where $\gamma : \Re \times \Re^n \to U$, $\gamma \in \Gamma$ and Γ is the set of all admissible control law.

Now, the quation is to find a robust control law $\gamma(t, x(t))$ for the system (1) with disturbance $v(t)$ whose bound is known ahead of time. Especially, for single-input single-output (SISO) system a high-gain feedback control law is used frequently. It is expected, however, that the value of cost functional $J(x, u, v)$ increases for the control system constructed by a high-gain feedback. In this paper, we consider constructing a robust control law to guarantee a certain value of the cost functional (3). A designing method is mentioned below briefly for general system (1).

Incentive Design Method

step 1 Determine a most favorable value of cost functional (3), that is, find a pair $(u^*, v^*) \in U \times \Omega$ which is the most favorable pair for a system designer.

$$\min_u J(x, u, v^*) \tag{5}$$

$$\begin{aligned} \text{s.t.} \quad \dot{x}(t) &= f(t, x(t), u(t), v^*(t)), \\ x(t_0) &= x_0, \qquad t \in [t_0, t_f]. \end{aligned}$$

step 2 Construct a robust control law $\gamma(t, x(t))$, such that $\gamma \in \bar{\Gamma}$. $\bar{\Gamma}$ is a subset of Γ defined as

$$\bar{\Gamma} = \{\gamma \in \Gamma | \gamma(t, x^*(t)) = u^*(t)\}, \tag{6}$$

where $x^*(t)$ is an optimal state trajectory corresponding to pair (u^*, v^*).

step 3 Find a robust control law $\gamma : \Re \times \Re^n \to U$, $\gamma \in \bar{\Gamma}$, such that $v^*(t) \in \Omega$ solves the following problem uniquely.

$$\min_{v \in \Omega} \tilde{J}(v), \qquad \tilde{J}(v) = -J(x, u, v), \tag{7}$$

$$\begin{aligned} \text{s.t.} \quad \dot{x}(t) &= f(t, x(t), u(t), v(t)), \\ x(t_0) &= x_0, \qquad t \in [t_0, t_f], \\ u(t) &= \gamma(t, x(t)). \end{aligned}$$

Clearly, the robust control law $\gamma \in \bar{\Gamma}$ determined by this method sketched by the steps 1 - 3 can guarantee that

$$J(x, \gamma, v) \leq J(x^*, u^*, v^*), \forall v(t) \in \Omega, \tag{8}$$

that is, the value of cost functional (3) obteined is smaller than or equal to a certain value $J(x^*, u^*, v^*)$.

Remark 1 The method sketched by the steps 1 - 3 is different from minimax optimal method. Here, the pair (u^*, v^*) may not be a saddle-point, but is the most favorable pair selected by a system designer. In addition, the form of the robust control law $\gamma \in \bar{\Gamma}$ is constructed by the system designer ahead of time. Therefore, it is possible to design this robust control law to be linear in some practical control system.

In the next section, a robust control law will be constructed concretely by making use of the method sketched by the steps 1 - 3 for a class of linear time-varying system with bounded disturbance.

3. MAIN RESULTS

In this section, we will consider the time-varying system with bounded disturbance described by

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + D(t)v(t), \tag{9}$$

$$x(t_0) = x_0, \qquad t \in [t_0, t_f],$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ are a state vector and a control input vector respectively. $v(t) \in \Re$ is an uncertain disturbance, but it is known that v(t) vary in given bound

$$\alpha \leq v(t) \leq \beta, \tag{10}$$

where α and β are constant. In (9), $A(t)$, $B(t)$, $C(t)$ are matrices of appropriate dimensions with time-varying elements which are measurable and bounded on $[t_0, t_f]$

The cost functional will be described by

$$J(x, u, v) = \frac{1}{2}\int_{t_0}^{t_f} (x^T(t)Q(t)x(t) + u^T(t)R(t)u(t))dt, \tag{11}$$

where the matrices $Q(t) = Q^T(t) \geq 0$, $R(t) = R^T(t) > 0$ are piecewise continuous functions on $[t_0, t_f]$, and with appropriate dimensions.

Here, the maximum bounds $v^*(t) = \beta$ of the bounded disturbance and the corresponding optimal solution $u^*(t)$ will be regarded as the most favorable for the system designer. A robust control law $\gamma(t, x(t))$ can be constructed such that the equation (8) is guaranteed for all $v(t) \in \Omega$, where

$$\Omega = \{v(t) \in \Re | \alpha \le v(t) \le \beta, t \in [t_0, t_f]\}, \qquad (12)$$

For this problem, we can obtain following Theorem 1 such that the robust control law can guarantee a certain value $J(x^*, u^*, v^*)$ of cost functional (11) under Condition 1 stated below.

Theorem 1 Let Condition 1 be satisfied for the matrix $\wp(t)$ whose components are piecewise continuous functions on time interval $[t_0, t_f]$. Then, there exists a robust control law $\gamma(t, x(t))$, such that the most favorable value $J(x^*, u^*, v^*)$ of the cost functional (11) can be guaranteed, and this control law $\gamma(t, x(t))$ can be constructed as

$$\begin{aligned} \gamma(t, x(t)) &= (\wp(t) - R^{-1}(t)B^T(t)K(t))x(t) \\ &\quad - R^{-1}(t)B^T(t)M(t) - \wp(t)x^*(t), \end{aligned} \qquad (13)$$

where $x^*(t)$ is the optimal state trajectory for the following form of closed-loop system.

$$\begin{aligned} \dot{x}(t) &= (A(t) - B(t)R^{-1}(t)B^T(t)K(t))x(t) \\ &\quad + (D(t)v^*(t) - B(t)R^{-1}(t)B^T(t)M(t)), \end{aligned} \qquad (14)$$

$$x(t_0) = x_0, \qquad t \in [t_0, t_f].$$

Here, the matrices $K(t)$, $M(t)$ are the solutions of Riccati-type and linear matrix differential equations described by the following forms respectively.

$$\begin{aligned} \dot{K}(t) &= -A^T(t)K(t) - K(t)A(t) - Q(t) \\ &\quad + K\ (t)B(t)R^{-1}(t)B^T(t)K(t), \end{aligned} \qquad (15)$$

$$K(t_f) = 0, \qquad t \in [t_0, t_f].$$

$$\begin{aligned} \dot{M}(t) &= -(A^T(t) - B(t)R^{-1}(t)B^T(t)K(t))^T M(t) \\ &\quad - K(t)D(t)v^*(t), \end{aligned} \qquad (16)$$

$$M(t_f) = 0, \qquad t \in [t_0, t_f].$$

Condition 1 There exists a matrix function $\wp(t) \in \Re^{m \times n}$ such that it satisfies the relation

$$D^T(t)(P(t)x^*(t) + S(t)) < 0, \qquad (17)$$

where the matrices $P(t)$, $S(t)$ are the solutions of linear matrix differential equations of the forms

$$\begin{aligned} \dot{P}(t) &= -A^T(t)P(t) - P(t)A(t) + Q(t) \\ &\quad + P(t)B(t)R^{-1}(t)B^T(t)K(t) \\ &\quad - (\wp(t) - R^{-1}(t)B^T(t)K(t))^T \\ &\quad \times B^T(t)(K(t) + P(t)), \end{aligned} \qquad (18)$$

$$P(t_f) = 0, \qquad t \in [t_0, t_f],$$

$$\begin{aligned} \dot{S}(t) &= -A^T(t)S(t) \\ &\quad - P(t)(D(t)v^*(t) - B(t)R^{-1}(t)B^T(t)M(t)) \\ &\quad - (\wp(t) - R^{-1}(t)B^T(t)K(t))^T \\ &\quad \times B^T(t)(M(t) + S(t)), \end{aligned} \qquad (19)$$

$$S(t_f) = 0, \qquad t \in [t_0, t_f].$$

Proof In the light of the method sketched by the steps 1 - 3, we give the main steps of this proof and design concretely a robust control law for a class of linear time-varying systems with bounded disturbance.

step 1 The pair (u^*, v^*) will be assumed as the most favorable pair for the system designer, where $v^*(t) = \beta$. $u^*(t)$ is the optimal solution of the following optimal problem, called Problem A, corresponding to $v^*(t)$.

$$\min_u J(x, u, v^*) \qquad (20)$$

$$\text{s.t.} \quad \begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + D(t)v^*(t) \\ x(t_0) &= x_0, \qquad t \in [t_0, t_f]. \end{aligned}$$

For Problem A, by making use of an adjoint vector $\lambda_A(t)$, we find that Hamiltonian H_A is

$$\begin{aligned} H_A &= \tfrac{1}{2}(x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)) \\ &\quad + \lambda_A^T(t)(A(t)x(t) + B(t)u(t) + D(t)v^*(t)). \end{aligned} \qquad (21)$$

The corresponding adjoint equation is

$$\dot{\lambda}_A(t) = -Q(t)x(t) + A^T(t)\lambda_A(t), \qquad (22)$$

$$\lambda_A(t_f) = 0, \qquad t \in [t_0, t_f].$$

Let $\lambda_A(t)$ be

$$\lambda_A(t) = K(t)x(t) + M(t), \qquad (23)$$

where $K(t) \in \Re^{n \times n}$ and $M(t) \in \Re^n$, whose components are continuously differentiable on time interval $[t_0, t_f]$, are the solutions of Riccati-type matrix differential equation (15) and linear matrix differential equation (16) respectively. The optimal solution $u^*(t)$ is

$$u^*(t) = -R^{-1}(t)B^T(t)K(t)x^*(t) - R^{-1}(t)B^T(t)M(t), \qquad (24)$$

where $x^*(t)$ is the optimal state trajectory of the closed-loop system (14).

step 2 Consider constructing a subset $\bar{\Gamma} \in \Gamma$ of the robust control law. If the robust control law $\gamma \in \bar{\Gamma}$ is identical to $u^*(t)$, the state trajectory of the control system is $x^*(t)$, that is, the subset $\bar{\Gamma}$ is

$$\begin{aligned} \bar{\Gamma} &= \{\gamma \in \Gamma | \gamma(t, x^*(t)) = \\ &\quad -R^{-1}B^T(t)K(t)x^*(t) - R^{-1}(t)B^T(t)M(t)\}. \end{aligned} \qquad (25)$$

Therefore, we construct the following form of the robust control law $\gamma(t, x(t))$, which belongs to $\bar{\Gamma}$,

$$\begin{aligned} \gamma(t, x(t)) &= -R^{-1}(t)B^T(t)K(t)x(t) \\ &\quad - R^{-1}(t)B^T(t)M(t) \\ &\quad + \wp(t)(x(t) - x^*(t)). \end{aligned} \qquad (26)$$

step 3 Then, we consider the following optimal control problem, called Problem B, to seek the condition which should be satisfied by $\wp(t)$.

$$\min_v \tilde{J}(v), \qquad \tilde{J}(v) = -J(x, \gamma, v), \qquad (27)$$

$$\text{s.t.}\quad \begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)\gamma(t, x(t)) + D(t)v(t)\\ x(t_0) &= x_0, \qquad t \in [t_0, t_f].\end{aligned}$$

For this problem, by applying Pontryagin's minimum principle, we find that Hamiltonian H_B is

$$\begin{aligned}H_B &= -\tfrac{1}{2}(x^T(t)Q(t)x(t) + u^T(t)R(t)u(t))\\ &+ \lambda_B^T(t)(A(t)x(t) + B(t)\gamma(t, x(t)) + D(t)v(t))\end{aligned} \tag{28}$$

where the robust control law $\gamma(t, x(t)) \in \bar{\Gamma}$ is given by (26) on time interval $[t_0, t_f]$. Then the adjoint equation corresponding to H_B (28) is

$$\begin{aligned}\dot{\lambda}_B(t) &= (\wp(t) - R^{-1}(t)B^T(t)K(t))^T\\ &\quad\times(R(t)\gamma(t, x(t)) - B^T(t)\lambda_B(t))\\ &\quad+Q(t)x(t) - A^T(t)\lambda_B(t),\end{aligned} \tag{29}$$

$$\lambda_B(t_f) = 0, \qquad t \in [t_0, t_f],$$

where $K(t)$ is the solution of Riccati-type matrix differential equation (15).

Minimization of Hamiltonian H_B (28) with respect to $v(t)$ yields that

$$v^*(t) = \begin{cases}\alpha, & \text{if} \quad \eta(t) < 0\\ \mu, (\alpha < \mu < \beta) & \text{if} \quad \eta(t) = 0\\ \beta, & \text{if} \quad \eta(t) > 0\end{cases} \tag{30}$$

Here, the switching function $\eta(t)$ are defined by the following form.

$$\eta(t) = \lambda_B^T(t)D(t). \tag{31}$$

In order to find the condition which are satisfied by $\wp(t)$, such that a certain value $J(x^*, u^*, v^*)$ of cost functional (11) corresponding to the most favorable pair (u^*, v^*) can be guaranteed by the robust control law $\gamma(t, x(t))$, we must ensure that $x^*(t)$, $u^*(t)$ and $v^*(t)$ which is given by upper bound β are the solution of Problem B. Since the optimal state trajectory $x^*(t)$ of Problem A and the state trajectory $x(t)$ of Problem B will coincide, we can obtain $x(t) \equiv x^*(t)$ in (29) by Theorem 2. From (29), we obtain the differential equation of the form

$$\begin{aligned}\dot{\lambda}_B(t) &= (\wp(t) - R^{-1}(t)B^T(t)K(t))^T\\ &\quad\times(R(t)\gamma(t, x^*(t)) - B^T(t)\lambda_B(t))\\ &\quad+Q(t)x^*(t) - A^T(t)\lambda_B(t),\end{aligned} \tag{32}$$

$$\lambda_B(t_f) = 0, \qquad t \in [t_0, t_f].$$

Let

$$\lambda_B(t) = P(t)x^*(t) + S(t), \tag{33}$$

where $P(t) \in \Re^{n\times n}$, $S(t) \in \Re^n$ are matrix and vector functions respectively whose components are continuously differentiable on time interval $[t_0, t_f]$.

Therefore, from (32) and (33), we can obtain linear matrix differential equations for $P(t)$ and $S(t)$ which are given by (18) and (19). The condition with respect to switching function $\eta(t)$ obtained by substituting (33) into (31) represents the following form.

$$(P(t)x^*(t) + S(t))^T D(t) < 0 \qquad t \in [t_0, t_f]. \tag{34}$$

From the discussion above, we can obtain this theorem under Condition 1. *Q.E.D.*

<u>Theorem 2</u> If $v^*(t)$ is determined uniquely, we have $x(t) \equiv x^*(t)$ no matter how $\wp(t)$ is selected.

<u>Proof</u> The differential equations of the closed-loop systems with respect to the optimal state trajectory $x^*(t)$ and the state trajectory $x(t)$ are described by the following forms respectively.

$$\begin{aligned}\dot{x}^*(t) &= (A(t) - B(t)R^{-1}(t)B^T(t)K(t))x^*(t)\\ &+ (D(t)v^*(t) - B(t)R^{-1}(t)B^T(t)M(t)),\end{aligned} \tag{35}$$

$$\begin{aligned}\dot{x}(t) &= (A(t) - B(t)R^{-1}(t)B^T(t)K(t)\\ &\quad -B(t)\wp(t))x(t)\\ &+ (D(t)v^*(t) - B(t)R^{-1}(t)B^T(t)M(t)\\ &\quad -B(t)\wp(t)x^*(t)),\end{aligned} \tag{36}$$

$$x^*(t_0) = x(t_0) = x_0, \qquad t \in [t_0, t_f].$$

Let $z(t)$ denote a difference between $x^*(t)$ and $x(t)$, that is,

$$z(t) = x^*(t) - x(t). \tag{37}$$

Then, we can readily obtain the differential equation of the form

$$\dot{\Phi}(t, t_0) = \Lambda(t)\Phi(t, t_0), \tag{38}$$

$$\Phi(t_0, t_0) = I, \qquad t \in [t_0, t_f],$$

where $\Phi(t, t_0)$ is the transition matrix for $z(t)$ and I is identity matrix. $\Lambda(t)$ is the following matrix,

$$\Lambda(t) = A(t) - B(t)R^{-1}(t)B^T(t)K(t) + B(t)\wp(t). \tag{39}$$

$z(t)$ is obtained by the solution of the differential equation (38) as

$$z(t) = \Phi(t, t_0)z(t_0), \tag{40}$$

$$z(t_0) = x^*(t_0) - x(t_0) = 0.$$

Therefore, $z(t)$ which presents the difference between $x^*(t)$ and $x(t)$ is identically zero, that is, it is satisfied that $x^*(t)$ is identically equal to $x(t)$. *Q.E.D.*

<u>Remark 2</u> For Theorem 1, the upper bound $v^*(t) = \beta$ is assumed to be the most favorable value for the system designer. Under this assumption, the condition (17) which is satisfied by the robust control law $\gamma(t, x(t))$ are derived. This theorem is also effective for the low bound α. However, for other most favorable value $v^*(t) = \mu$ $(\alpha < \mu < \beta)$, we may use the condition to seek singular control, that is,

$$\frac{\partial H_B}{\partial v} = 0, \qquad \frac{d}{dt}\left(\frac{\partial H_B}{\partial v}\right) = 0.$$

Then, we may determine some conditions to be satisfied for the robust control law $\gamma(t, x(t))$ in this case.

In next section, a numerical illustrative example is given. It will be shown from this numerical example that the method is effective and feasible for practical control problems.

4. NUMERICAL EXAMPLE

A system considered in this section is described by

$$\dot{x}(t) = 2x(t) + u(t) + v(t), \qquad x(0) = 10, \qquad t \in [0, 2]. \quad (41)$$

The cost functional is given by

$$J(x, u, v) = \frac{1}{2}\int_0^2 (5x^2(t) + u^2(t))dt. \qquad (42)$$

In addition, there exists a closed bound Ω for a disturbance $v(t)$, and Ω is given by

$$\Omega = \{v(t) \in \Re | -1 \le v(t) \le 1, t \in [t_0, t_f]\}. \qquad (43)$$

step 1 Determine the most favorable value $J(x^*, u^*, v^*)$ of the cost functional (42). Firstly, we assumed that $v^* = 1$ which is upper bound, and find the optimal solutions $u^*(t)$ and $x^*(t)$ corresponding to $v^*(t)$. Then, the Riccati-type and linear matrix differential equations are given by

$$\dot{K}(t) = K^2(t) - 4K(t) - 5, \qquad K(2) = 0, \qquad t \in [0, 2], \qquad (44)$$

$$\dot{M}(t) = -(2 - K(t))M(t) - K(t), \quad M(2) = 0, \quad t \in [0, 2], \qquad (45)$$

and their solutions are shown in Fig.2.

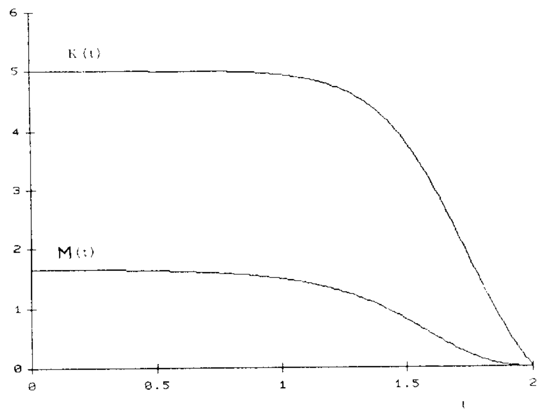

Fig.2 The solutions $K(t)$ of (44) and $M(t)$ of (45).

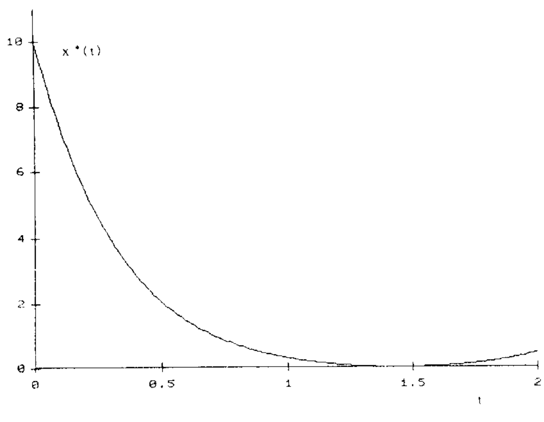

Fig.3 The optimal state trajectory $x^*(t)$.

Fig.3 and Fig.4 show the optimal solutions $u^*(t)$ and $x^*(t)$ respectively.

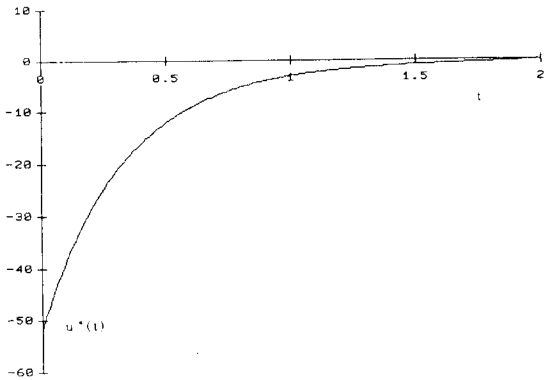

Fig.4 The optimal control input $u^*(t)$.

The most favorable value of cost functional (42) is computed as

$$J(x^*, u^*, v^*) = 268.81. \qquad (46)$$

step 2 The robust control law $\gamma(t, x(t))$ is constructed by the following form from (26).

$$\gamma(t, x(t)) = (\wp(t) - K(t))x(t) - \wp(t)x^*(t). \qquad (47)$$

step 3 The linear matrix differential equations (18) and (19) are given by

$$\begin{aligned} \dot{P}(t) &= -4P(t) + P(t)K(t) + 5 \\ &\quad -(\wp(t) - K(t))(K(t) + P(t)) \end{aligned} \qquad (48)$$

$$P(2) = 0, \qquad t \in [0, 2].$$

$$\begin{aligned} \dot{S}(t) &= -2S(t) - P(t)(1 - M(t)) \\ &\quad -(\wp(t) - K(t))(M(t) + S(t)) \end{aligned} \qquad (49)$$

$$S(2) = 0, \qquad t \in [0, 2].$$

If we select

$$\wp(t) = -2K(t), \qquad (50)$$

then the solutions $P(t)$ and $S(t)$ of differential equations (48) and (49) respectively are shown in Fig.5.

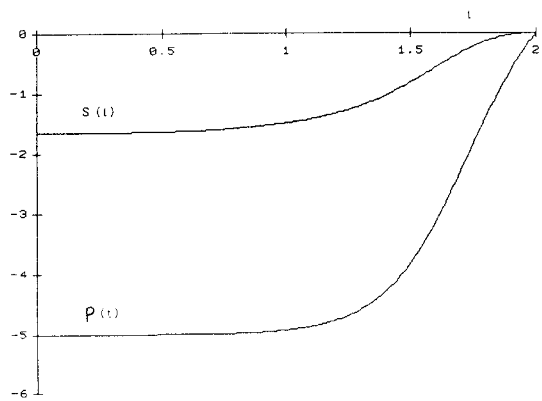

Fig.5 The solutions $P(t)$ of (48) and $S(t)$ of (49).

We confirm that the following condition is satisfied.

$$M(t)x^*(t) + S(t) < 0. \tag{51}$$

Therefore, we have the robust control law $\gamma(t, x(t))$ described by

$$\gamma(t, x(t)) = -3K(t)x(t) - M(t) + 2K(t)x^*(t). \tag{52}$$

Here, in order to confirm numerically that the robust control law $\gamma(t, x(t))$ (52) can guarantee $J(x, \gamma, v) \leq J(x^*, u^*, v^*)$, we will have a simulation with respect to the following disturbance.

$$v(t) = \sin(\pi t). \tag{53}$$

The disturbance $v(t)$ assumed by this form is belongs to the set Ω (43). The state trajectory $x(t)$ and the robust control law $\gamma(t, x(t))$ (52) corresponding to (53) are shown in Fig.6 and Fig.7 respectively.

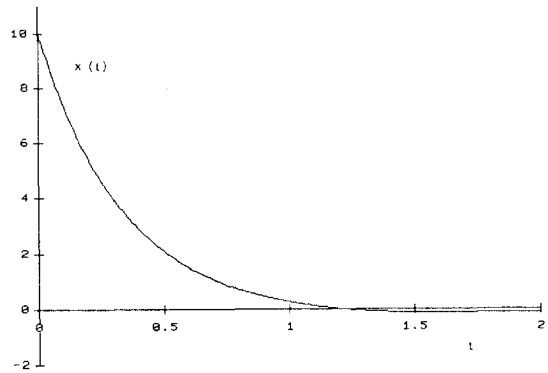

Fig.6 The state trajectory $x(t)$
with the disturbance (53).

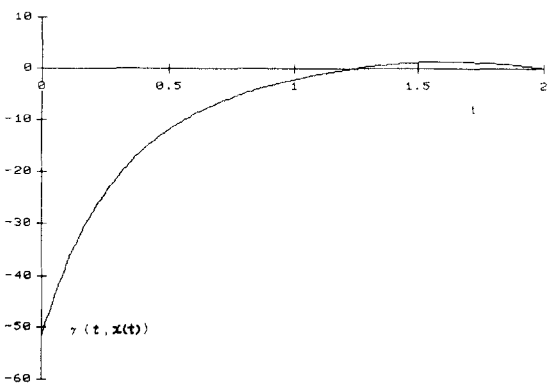

Fig.7 The robust control law $\gamma(t, x(t))$
with the disturbance (53).

The value of the cost functional $J(x, \gamma, v)$ is

$$J(x, \gamma, v) = 258.54 < J(x^*, u^*, v^*). \tag{54}$$

5. CONCLUSION

In this paper, we consider the robust control problem for a class of linear time-varying systems with bounded disturbance. For this problem, we present an approach to design a robust control law. This approach is based on such a consideration that a most favorable value of cost functional for the system designer can be guaranteed no matter how the disturbance vary within given bound, that is, the value of cost functional is suppressed under a certain value. Therefore, the bounded disturbance has at least no bad effect on the value of cost functional.

Here, for a numerical example, the disturbance described by (53) are used only to simulate, because $v(t)$ is unknown for the system designer except its bound. Therefore, it is shown from this derivation and this example that the approach developed here is effective and feasible for some practical control problems of systems with bounded disturbance.

For the systems with bounded disturbance, our further research is to guarantee the other properties, such as stability, by making use of the robust control law.

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