AN NECESSARY AND SUFFICIENT CONDITION FOR THE REACHABILITY OF LSFC NETS

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LSFC NETS 가 가도달성을 가지기위한 필요충분조건

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Abstract 이 論文에서는 페트리 네트의 한 서브클래스인 活性이고 安全인 자유선택 네트의 可到達性에 관해 고찰했다. 活性이고 安全인 자유선택 네트의 한 클래스가 可到達性을 가지기위한 필요충분조건을 도출했다.

1. INTRODUCTION

Petri nets are widely recognized as a powerful model for discrete event systems characterized by asynchronous and concurrent evolutions. The main advantages for a net approach in the behavioral analysis of discrete event systems exist in the structural characterization of three fundamental features, i.e, sequentiality (causality), choice (nondeterminism), and concurrency (nonsequentiality). A sequential nondeterministic system is classified as a state machine in net theory. A deterministic concurrent system is modelled by a marked graph or an event graph. State machines and marked graphs are structurally restricted subclasses of Petri nets for which many analytical results have been obtained. In general, discrete event systems are modelled by mixture of these two subclasses of Petri nets. Free choice nets proposed by Hack[1] are a subclass of Petri nets, which can represent the substantial features of systems by modelling both choice and concurrency. In [6], Thiagarajan and Voss presented structural properties satisfied in live and safe free choice (LSFC) nets.

In the application of net theory to discrete event systems such as FMS or communication systems, many control problems are formulated by the reachability problem in the net model. A reachability problem is equivalent to the existence and the executability of firing count vector S for given initial marking and target marking. Reachability for marked graphs and some other subclasses can be solved in an analytical form [3] [4]. Specifically, necessary and sufficient conditions are obtained in terms of initial and target markings and the structural distribution of an initial marking. Such conditions are not obtained for general cases. It is worth to note that a reachability problem of general Petri nets is reduced to the reachability problem of a free choice net [2].

The aim of this paper is to give an necessary and sufficient condition for the reachability of a class of LSFC net. Basic definitions and notations are shown in Section 2. In section 3, an necessary and sufficient condition for the reachability of strongly connected state machine is given. Main results for the reachability of a class of LSFC net are presented in Section 4. Section 5 is the conclusion.

2. DEFINITIONS AND NOTATIONS

A Petri net N is a 3-tuple $N=(P,T;F)$ that corresponds to a bipartite graph with two sets of nodes, P and T. P is a set of places represented by circles in a diagram, and T is a set of transitions represented by bars in a diagram. F is a set of ordered pair of a place and a transition which specifies directed arcs between these two kinds of nodes in a diagram. In this paper, we assume that all Petri nets are connected and finite, and assume that the number of places are n ($|P|=n$), and the number of transitions are m ($|T|=m$). Suppose $x \in P \cup T$, ${}^*x$ denotes a set of input nodes of x, i.e., ${}^*x \times x \in F$. Similarly x^* denotes a set of output nodes of x.

We assign an non-negative number of tokens to each place. Dynamic evolutions of systems are then represented by the changes of token distributions according to firings of transitions. Marking is a function $M{:}P \rightarrow Z^+$, where Z^+ is the set of non-negative integers, and M is n×1 column vector. M(p) denotes the number of tokens in p. Suppose $P \supseteq P'$, $M(P')$ implies $\Sigma_{p \in P'} M(p)$. A marked net is defined by 4-tuple $N_{M0}=(P,T;F,M_0)$, where $N=(P,T;F)$ is a Petri net and M_0 is the initial marking.

DEFINITION 1. Transition t is firable at marking M, denoted by M[t>, iff all the input places of transition t have at least one token, i.e., $\forall p \in {}^*t$; $M(p)>0$. New marking M' is obtained from M by the firing of transition t such that

$$\forall p \in P;\quad M'(p)=M(p)-1 \quad \text{if } p \in {}^*t \backslash t^*.$$
$$=M(p)+1 \quad \text{if } p \in t^* \backslash {}^*t.$$
$$=M(p) \quad \text{otherwise.}$$

DEFINITION 2. For a marked net N_{M0}, a sequence $\sigma=t_1 t_2 \ldots t_n$ of transitions is called a firing sequence from M_0, if there exists a sequence of markings $M_1, M_2, \ldots, M_{n+1}$ such that $M_i[t_j>M_{i+1}$ $(i=0,1,\ldots,n)$. If there exists a firing sequence $\sigma=t_1 t_2 \ldots t_n$ from M_0 to M, M is called reachable from M_0 and denoted by $M_0[\sigma>M$. The set of all markings which are reachable from M_0 is denoted by $R(M_0)$, i.e., $R(M_0) = \{M \mid M_0[\sigma>M, \sigma \in T^*\}$.

DEFINITION 3. *An incidence matrix* $A=(a_{ij})$ *is defined by*

$$a_{ij}=a_{ij}^{+}-a_{ij}^{-} \quad \text{where } a_{ij}^{+}=1 \quad ; p_i \in t_j^*.$$
$$=0 \quad ; \text{otherwise.}$$
$$a_{ij}^{-}=1 \quad ; p_i \in {}^*t_j.$$
$$=0 \quad ; \text{otherwise.}$$

DEFINITION 4. If $M_0[\sigma>M$, M can be written by $M=M_0+A\Sigma$, where $\Sigma(i)=\#(\sigma \mid t_i)$. This equation can be viewed as a state equation of a Petri net, although a control vector Σ is not completely arbitrary since each marking M must be an non-negative integral vector. m×1 non-negative integral vector Σ is called a firing count vector.

DEFINITION 5. A class of Petri net $N=(P,T;F)$ is called a state machine iff $\forall t \in T$, $|{}^*t|=|t^*|=1$. A marked graph is a class of Petri net where $\forall p \in P$, $|{}^*p|=|p^*|=1$. A class of N is *called a free choice net iff* $\forall p \in P$, $|p^*| \geq 2 \rightarrow {}^*(p^*)=\{p\}$.

DEFINITION 6. A marked net N_{M0} is live iff $\forall M' \in R(M_0)$, $\forall t \in T$; $\exists M'' \in R(M')$, $M''[t>$. N_{M0} is safe iff $\forall M' \in R(M_0)$, $\forall p \in P$; $M'(p) \leq 1$.

DEFINITION 7. Let $N_{M0}=(P,T;F,M_0)$ be a marked net and

$N_1=(P_1,T_1;F_1)$ be a subnet of N.

(a) N_1 is called a T-component of N_{M0} iff N_1 is a strongly connected marked graph and generated by T_1, i.e., $P_1={}^*T_1\cup T_1{}^*$, $F_1=F\cap((P_1\times T_1)\cup(T_1\times P_1))$.

(b) N_1 is called an S-component of N_{M0} iff N_1 is a strongly connected state machine and generated by P_1, i.e., $T_1={}^*P_1\cup P_1{}^*$, $F_1=F\cap((P_1\times T_1)\cup(T_1\times P_1))$.

(c) N_1 is SM-component of N_{M0} iff N_1 is a S-component of N_{M0} and $M_0(P_1)=1$.

3. REACHABILITY CRITERIA FOR A CLASS OF LSFC NETS

In this section, we consider some structural properties of LSFC net and an necessary and sufficient condition for the reachability of a class of LSFC nets.

An necessary and sufficient condition for the reachability for a strongly connected state machine was presented by the authors in [7].

THEREM 1. Let $N_{M0}=(P,T;F,M_0)$ be a strongly connected state machine. A marking M is reachable from nonzero marking M_0 iff $M(P)=M_0(P)$.

Note that THEOREM 1 does not hold if a state machine is not strongly connected.

A major part of structural properties of LSFC net have been investigated by Hack in [1]. The following lemma plays a central role in the reachability problem.

LEMMA 2. Let $N_{M0}=(P,T;F,M_0)$ be an LSFC net and x be an arbitrary element of $P\cup T$.

(a) There exists a T-component $N_1=(P_1,T_1;F_1)$ of N such that $x\in P_1\cup T_1$.

(b) There exists an SM-component $N_2=(P_2,T_2;F_2)$ of N such that $x\in P_2\cup T_2$.

[Proof] See THEOREM 4 and its dual version in [1].

LEMMA 2 implies that an LSFC net is covered by a set of T-components, and also covered by a set of SM-components and hence by a set of S-components. A transition which belongs to more than two S-components is called a common transition.

DEFINITION 8. Let LSFC net N_{M0} be decomposed into a set of S-components. An elementary directed circuit C in an S-component is called a minimal net circuit iff there exist no elementary directed circuits in the S-component which contain a proper subset of nodes of C.

LEMMA 3. Every elementary circuit of an LSFC net N_{M0} is contained in some S-components of N_{M0}.

[Proof] See Corollary 5.6 and its dual version in [6].

LEMMA 4. An non-minimal net circuit of LSFC net N_{M0} is contained in an SM-component.

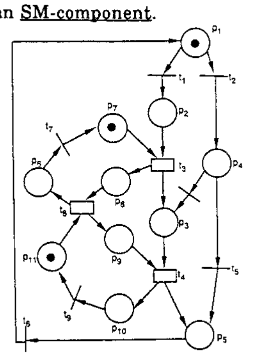

FIGURE 1. An example which is not LSFC net

THEOREM 5. A net which satisfies following conditions (1) and (2) is not LSFC net.

(1) There exists a non-minimal net circuit with more than two common transitions, say t_1 and t_2, such that t_1 and t_2 are disjointly contained in different elementary circuits in an S-component S_i, respectively.

(2) There exists a path between t_1 and t_2 which is not contained in S_i.

Formal proofs of LEMMA 4 and THEOREM 5 are omitted here, but, instead, an example is shown in Fig. 1. Directed circuit $p_1, t_1, p_2, t_3, p_3, t_4, p_5, t_6, p_1$ satisfies conditions (1) and (2) of THEOREM 5.

Suppose that an LSFC net N_{M0} is decomposed into S-components, $S_1(P_1,T_1)$, $S_2(P_2,T_2)$,...,$S_r(P_r,T_r)$, and in each S-component the condition of LEMMA 1 is satisfied for a given target marking M, i.e., $M(P_i)=M_0(P_i)$ for i=1, 2,..., r. Then a firing count vector Σ_i exists for each S_i. Assume that for any transition t of N_{M0} which belongs to more than two S-components, say S_i and S_j, the corresponding firing count $\Sigma_i(t)=\Sigma_j(t)$. We call this case as an alignment of common transitions, or in short A-condition, holds for each Σ_i. In this case we can define an non-negative vector Σ for N_{M0} such that $\Sigma(t)=\Sigma_i(t)$ for $t\in S_i$, i=1, 2,..., r. Then, Σ is a solution of state equation for N_{M0}. Now suppose that A-condition is not satisfied for a minimal firing count vector Σ_i obtained in each S_i. We need a criterion to decide whether there exists another solution Σ_i satisfying A-condition and, if there exist, a constructive algorithm to obtain such Σ_i for each S_i. In the following, we define A-graph associated with N_{M0} and a given target marking M. Suppose that N_{M0} is decomposed into S-components, $S_1,S_2,...,S_r$.

Construction of node set of A-graph

```
Repeat{
For(i=1,...,r)
      Repeat{
      For(each common transition t in Si)
            Select a directed circuit containing t; Define
            an node labelled with t and other common
            transitions on the direct circuit.
      }
      Delete a set of nodes, a label union of which is
      a true subset of a  label union of other set of nodes.
}
```

Construction of weighted arc set of A-graph

```
Repeat{
For(each pair of nodes defined in two S-components)
      For two nodes n1 and n2 containing label t, t∈Ti and
      t∈Tj, j≠i, respectively, define an arc directed form
      n1(n2) to n2(n1) if Σi(t)≥ Σj(t) ( Σj(t) ≥ Σi(t)),
      respectively. Assign non-negative integer | Σi(t) -
      Σj(t) | to the arc.
}
```

EXAMPLE 1. An LSFC net N_{M0} in Fig. 2(a) is decomposed into SM-components as shown in Fig. 2(b). For $M_0=[10000010001]^T$ and $M=[00010100010]^T$, Σ_i is determined in each component as $\Sigma_1=[01\underline{00}00]^T$, $\Sigma_2=[\underline{10}1]^T$, $\Sigma_3=[\underline{11}0]^T$, which dose not satisfy A-condition. An associated A-graph is shown in Fig. 2(c) for the target marking M.

To obtain a graph theoretical condition for the existence of a firing count vector Σ_i, i=1,...,r, satisfying A-condition, a restriction is made on the structure of an LSFC net. Let t be an arbitrary common transition in an S-component S_i. An LSFC net considered here is assumed to be in a class, called class A, where there exist no different directed circuits in S_i such that they pass through t and contain another common transition, respectively.

THEOREM 6. For an LSEC net N_{M0} in class A with a target marking M, a firing count vector Σ exists iff there exist an S-decomposition, $S_1, S_2, \ldots, S_r$, such that $M_0(p_i)=M(p_i)$ for i=1, 2,..., r and there exist an associated A-graph of N_{M0} where algebraic sum of arc weights in any circuit is zero. A sign in the summation is determined according to the direction of arcs.

A formal proof is omitted here, but, instead, a construction of Σ is shown for a LSFC net in Example 1.

EXAMPLE 2. An associated A-graph in Fig. 2(c) satisfies the condition of THEOREM 6. A solution vector Σ of $M=M_0+A\Sigma$ is constructed as follows.

```
Repeat {
For(i=1,...,r)
        Repeat{
        For(any node in SMi with an incoming arc et)
                Add Δt to Σi(t) to make the arc weight of et
                corresponding to label t zero. Accordingly,
                change arc weights associated with label t.
        }
}
```

The resulting Σ_i, i=1,..., 3, satisfies A-condition and we obtain a firing count vector Σ as $\Sigma=[111101010]^T$.

We call the condition in THEOREM 6 as a circuit condition, or, <u>C-condition</u>. To verify C-condition, only fundamental circuits need to be considered in A-graph. For the reachability of an LSFC net, the remaining problem is on the executability of a firing count vector Σ constructed by the discussion above. If we restrict an LSFC net to class A, the existence of Σ implies, at the same time, the executability of Σ. We restate the result in

THEOREM 7. For an LSFC net N_{M0} of class A with a target marking M, M is reachable from M_0 iff $M_0(P_i)=M(P_i)$, i=1, 2,..., r, for some SM-decomposition, $SM_1, SM_2, \ldots, SM_r$, and there exists an associated A-graph that satisfies C-condition.

[Proof] See THEOREM 6 in [7].

EXAMPLE 3. An LSFC net N_{M0} as shown in Fig. 3 with $M_0=[0000011]^T$ and $M=[0100100]^T$ satisfies A-condition for each Σ_i and we obtain a firing count vector $\Sigma=[1101102]^T$. As is easily seen, N_{M0} is not in class A and Σ is not executable. M is not reachable from M_0.

There can be more than one S-decompositions LSFC net and more than one A-graphs may correspond to an S-decompositon. Following two lemmas assure that C-condition dose not depend on the choice of S-decomposition nor the choice of A-graph.

LEMMA 8. For a given S-decomposition of an LSFC net N_{M0}, if there exists an A-graph which satisfies C-condition, then C-condition is satisfied irrespective to the choice of A-graph.

LEMMA 9. For a given S-decomposition of an LSFC net N_{M0} if there exists an A-graph which satisfies C-condition, C-condition is satisfied irrespective to the choice of S-decomposition.

Formal proofs of LEMMA 8 and 9 are omitted here, but an example is shown in Fig. 2.

From LEMMA 9 and 10, we can rewrite THEOREM 7 as following theorem.

THEOREM 10. For an LSFC net N_{M0} of class A with target marking M, M is reachable from M_0 iff $M_0(P_i)=M(P_i)$, i=1, 2,..., r, for some S-decomposition, $S_1, S_2, \ldots, S_r$, and an arbitrary associated A-graph satisfies C-condition.

5. CONCLUSION

Some structural properties of an LSFC net and an necessary and sufficient condition for the reachability of an LSFC net of class A have been presented. The structural restriction we made is only for deriving graphical criterion to verify A-condition. The result can be generalized to more general LSFC nets, but in this case the definition of A-graph and the existence theorem for a firing count vector S becomes more complicated.

References

[1] M. Hack, "Analysis of Production Schemata by Petri Nets," M. S. thesis, TR-94, Project MAC, MIT, Cambridge, Mass, 1972.

[2] M. Hack, "Extended State-Machine Allocation Nets(ESMA), an extension of Free Choice Petri Net Results," Computation group memo 78-1, Project MAC, MIT, Cambridge, Mass, 1974.

[3] K. Hiraishi and A. Ichikawa, "A Class of Petri Nets That a Necessary Sufficient Condition for Reachability is Obtainable," Trans. of SICE Vol. 24, No. 6, June 1988, pp.635-640.

[4] T. Murata, "Circuit Theoretic Analysis and Synthesis of Marked Graphs," IEEE Trans. circuits and systems, Vol. CAS-24, No. 7, July 1977, pp. 400-405.

[5] W. Reisig, "Petri Nets," Springer-Verlag, 1985.

[6] P. S. Thiagarajan and K. Voss, "A Fresh Look At Free Choice Nets," Information and Control 62, 1984, pp.85-113.

[7] D. I. Lee, S. Kumagai and S. Kodama, "Reachability of LSFC net," to appear in Proceeding of MTNS 89 Amsterdam, The Netherlands.

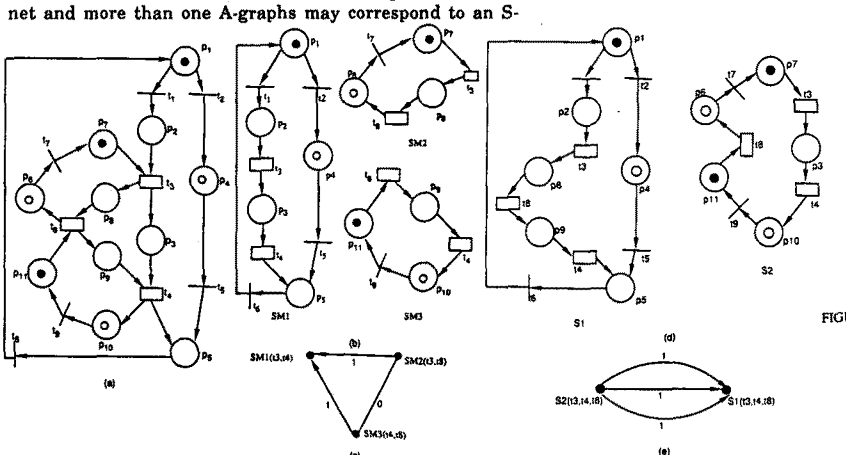

FIGURE 2 (a) An LSFC net, (b) An SM-decomposition of (a), (c) Associated A-graph of (b)
(d)An S-decomposition of (a), (e) Associated A-graph of (d)

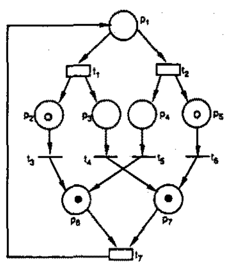

FIGURE 3. An example which is an LSFC net, but not class A.