변형 PDA 필터를 이용한 표적 추적

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Target Tracking using modified PDA filter

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Abstract

The PDA (Probabilistic Data Association) Filter proposes a new approach to the problem of tracking when the source of the measurement data is uncertain. The PDA filter shows good simulation results in a known clutter density. In this paper the PDA filter has been modified so that it can be applied when the clutter density is not known.

I. Introduction

In tracking targets there can be an uncertainty associated with the measurements in addition to their inaccuracy, which is usually modeled by some additive noise. This additional uncertainty is related to the origin of the measurements:a measurement, which is to be used in the tracking algorithm, might not have originated from the target of interest. This situation can occur in a surveillance system when a sensor such as a radar, sonar, or optical one is operating in an environment in which there is clutter, or the false-alarm rate is high.

There are two approaches in target tracking, Non-Bayesian approach based on likelihood function and Bayesian approach based on *a posteriori* probabilities. The memory and computational requirements of the non-Bayesian approach increase in time in a dense environment, but those of the Bayesian approach are slightly higher than standard Kalman filter. The PDA filter is based on Bayesian approach and no inference on the number of returns can be made from past data.[1-2] The basic assumption of the PDA filter is that tracking has been already initiated by other track split algorithms. [4]

II. The Modified Probabilistic Data Association Filter

The dynamics of the object in track are modeled by the equation

$$x_{k+1} = Fx_k + w_k, \quad k=0,1,\cdots \tag{2.1}$$

and the corresponding measurement is given by

$$z_k = Hx_k + v_k \tag{2.2}$$

with w_k and v_k being zero-mean mutually independent white Gaussian noises with known covariances Q_k and R_k, respectively.

The set of measurements lying in a neighborhood of the predicted location of the observation from the target at time k is

$$Z_k = \{z_{k,i}\}_{i=1}^{m_k} \tag{2.3}$$

and the set of measurements up to and including time k is denoted as

$$Z^k = \{Z_j\}_{j=1}^{k} \tag{2.4}$$

Measurements not having originated from a target are assumed to be i.i.d. (independent identically distributed) with uniform density C. The best estimate of the target's state is the conditional mean baed upon all the observations that with some nonzero probability are originated from the target, i.e.,

$$\begin{aligned} \hat{x} &= E\{x_k \mid Z^k\} \\ &= \sum_{i=0}^{m_k} \beta_{k,i}\, E\{x_k \mid \chi_{k,i}, Z^k\} \\ &= \sum_{i=0}^{m_k} \beta_{k,i}\, \hat{x}_{k\mid k,i} \end{aligned} \tag{2.5}$$

where $\chi_{k,i}$ denotes the event that the ith measurement, $z_{k,i}$ is correct ($i=1,\cdots,m_k$), $\chi_{k,0}$ is the event that none of them is correct and

$$\beta_{k,i} = p\{\chi_{k,i} \mid Z^k\} \qquad i=0,1,\cdots,m_k \tag{2.6}$$

Eq. (2.6) is the *a posteriori* probability of each return having orginated from the object in track. The probability density of the state conditioned upon past observations is assumed normal with mean $\hat{x}_{k\mid k-1}$ and covariance $P_{k\mid k-1}$, i.e.,

$$p(x_k \mid Z^{k-1}) = N(\hat{x}_{k\mid k-1}, P_{k\mid k-1}) \tag{2.7}$$

The set Z_k should be made up of the measurements lying in the validation region

$$\{z : v'_{k,i} S_k^{-1} v_{k,i} \le g^2\} \tag{2.8}$$

where the innovaion $\nu_{k,i}=z_{k,i}-\hat{z}_{k|k-1}$ will also be assumed to be normally distributed with mean zero and covariance S_k

$$S_k=HP_{k|k-1}H'+R \tag{2.9}$$

The gate threshold is determined by the χ^2 table because the square of the gaussian distribution becomes χ^2 distribution with M degree of freedom (the dimension of z_k) [7] and is related to P_G, the probability that the correct measurement lies within the gate. It is well known that $P_G>0.99$ whenever $g>M^{1/2}+2$. [3]

Due to the restriction of the validation region, the probability density of a measurement conditioned upon Z^{k-1}, provided that it orginated from the object in track and that it has been validated, is a truncated normal density, i.e.,

$$f(z_{k,i})=N(\hat{z}_{k|k-1}, S_k)\,/\,P_G \tag{2.10}$$

It is also assumed in the PDA derivation that the correct measurement is detected with probability P_D and that the number of false measurement is Poisson distributed with parameter CV, where V is the volume of the validation region and C is the density of the clutter. The volume at time k is given by [3]

$$V_k=c_M g^M\,|S_k|^{1/2} \tag{2.11}$$

where $c_M=\pi^{M/2}/\Gamma(M/2+1)$ is the volume of the M-dimensional unit sphere ($c_1=2$, $c_2=\pi$, etc) and $\Gamma(\cdot)$ is the standard gamma function.

The expression Eq. (2.6) can be obtained explicitly after some calculation [3]

$$\beta_{k,j}=\frac{\exp(-\frac{1}{2}\nu'_{k,i}S_k^{-1}\nu_{k,i})}{b+\sum_{i=1}^{m}\exp(-\frac{1}{2}\nu'_{k,i}S_k^{-1}\nu_{k,i})} \tag{2.12}$$

$$\beta_{k,0}=\frac{b}{b+\sum_{i=1}^{m}\exp(-\frac{1}{2}\nu'_{k,i}S_k^{-1}\nu_{k,i})}$$

where

$$b=(2\pi)^{M/2}C\,|S_k|^{1/2}(1-P_DP_G)\,/\,P_D \tag{2.13}$$

The variance of x_k is [1]

$$P_{k|k}=P^*_{k|k}+W_k\left[\sum_{i=1}^{m_k}\beta_{k,i}\nu_{k,i}\nu'_{k,i}-\nu_k\nu'_k\right]W'_k \tag{2.14}$$

where

$$\nu_k=\sum_{i=1}^{m_k}\beta_{k,i}\nu_{k,i} \tag{2.15}$$

is the weighted innovation which uses all the validated measurements,

$$W_k=P_{k|k-1}H'S_k^{-1} \tag{2.16}$$

$$P^*_{k|k}=(I-W_kH)P_{k|k-1} \tag{2.17}$$

where $P^*_{k|k}$ is the covariance of the update if we have only one return.[2] The last term of Eq. (2.14) is a positive semidefinite matrix which shows the effect of the incorrect measurements by increasing the covariance of the update. As can be seen from Eq. (2.14) the confidence on the estimate is a function of the actual number of validated returns and their location is data dependent.

Now if we try to track a target in unknown clutter environment, we must estimate the clutter density. It is assumed that no inference on the number of incorrect returns can be made from past data. Only the measurements in the validation region give any information about the present clutter density. So with the volume of the validation region V_k and the number of the false alarm m_k^F at each time, we must estimate the clutter density. In order to calculate the present clutter density, we must know the number of false alarm, m_k^F,

$$\begin{aligned} m_k^F &= (m_k-1)\times p\{\text{target is detected}\} \\ &\quad + m_k\times p\{\text{target is not detected}\} \\ &= (m_k-1)P_DP_G+m_k(1-P_DP_G) \end{aligned} \tag{2.18}$$

The present clutter density can be written as $\frac{m_k^F}{V_k}$. Then the clutter density up to time k is given by

$$\hat{C}_k=\frac{1}{k}\sum_{i=1}^{i=k}\frac{m_i^F}{V_i} \tag{2.19}$$

But the clutter density is needed at each time, Eq. (2.19) cannot be applied directly. So an recursive version of Eq. (2.19) is needed. In order to estimate the clutter density recursively, rearrange Eq. (2.19)

$$\hat{C}_{k+1}=\frac{1}{k+1}\sum_{i=1}^{k}\frac{m_i^F}{V_i}+\frac{1}{k+1}\frac{m_{k+1}^F}{V_{k+1}} \tag{2.20a}$$

$$=\frac{1}{k+1}k\hat{C}_k+\frac{1}{k+1}\frac{m_{k+1}^F}{V_{k+1}} \tag{2.20b}$$

$$=\hat{C}_k+\frac{1}{k+1}\left[\frac{m_{k+1}^F}{V_k}-\hat{C}_k\right] \tag{2.20c}$$

Now replace $\hat{C}_k$ in Eq. (2.20) with C in Eq. (2.13).

$$b=(2\pi)^{M/2}\hat{C}_k\,|S_k|^{1/2}(1-P_DP_G)\,/\,P_D \tag{2.21}$$

Then the whole modified PDA filter is given below.

Measurement Update :

$$\hat{x}_{k|k}=\hat{x}_{k|k-1}+W_k\nu_k \tag{2.22}$$

$$P_{k|k}=P^*_{k|k}+W_k\left[\sum_{i=1}^{m_k}\beta_{k,i}\nu_{k,i}\nu'_{k,i}-\nu_k\nu'_k\right]W'_k \tag{2.23}$$

$\beta_{k,i}$ is calculated by Eq. (2.12), (2.21), and (2.20c).

Time Update :

The time update of PDA filter is obtained from the stadnard Kalman filter

$$\hat{x}_{k+1|k} = F\hat{x}_{k|k} \tag{2.23}$$

$$P_{k+1|k} = FP_{k|k}F' + GQG' \tag{2.24}$$

III. Simulation Results

Simulations was carried out as nearly constant velocity objects in a tow-dimensional pane with process noise that can account for slight changes in the velocity. And it is assumed that only postion measurements are available. The state and system matrices are

$$x = [x\ \dot{x}\ y\ \dot{y}]' \tag{3.1}$$

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

$$w = [w_1\ w_2]' \tag{3.3}$$

$$G = \begin{bmatrix} T/2 & 0 \\ 1 & 0 \\ 0 & T/2 \\ 0 & 0 \end{bmatrix} \tag{3.4}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.5}$$

IV. Summary

The modified PDA filter has shown almost same results compared with the PDA filter with exact clutter density. And it has shown better results when the exact clutter density is not given. Though the nonparametric PDA filter can be used when the clutter density is not known, simulation results has shown that when there is no data in a validation region, it may blow off.

V. References

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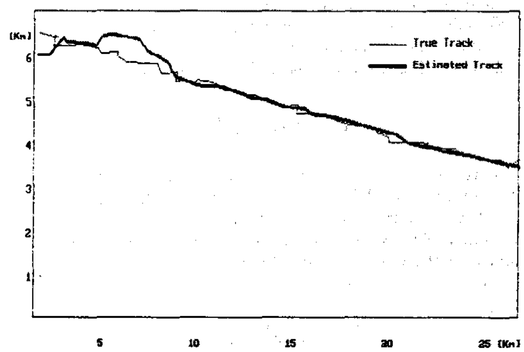

Fig. 1. PDA filter
(real clutter density : 0.3
and estimated using constant clutter density C = 0.3)

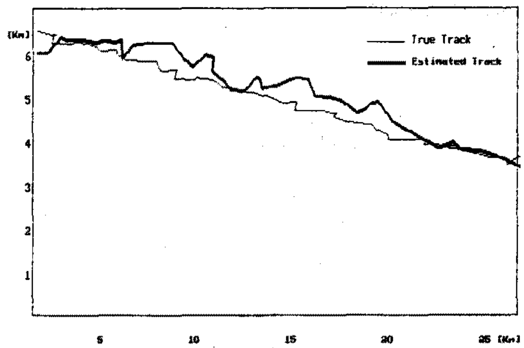

Fig. 2. PDA filter
(real clutter density : 0.3
and estimated using constant clutter density C = 0.4)

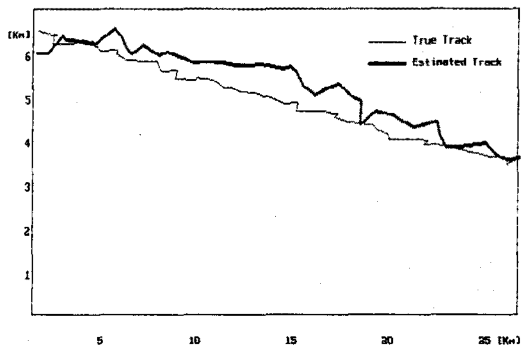

Fig. 3. PDA filter
(real clutter density : 0.3
and estimated using constant clutter density C = 0.2)

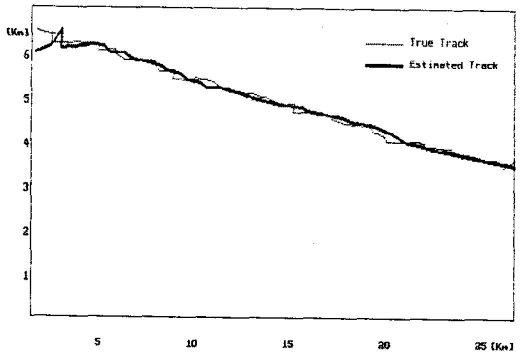

Fig. 4. Modified PDA filter
(real clutter density : 0.3
and estimated using estimated clutter density $\hat{C}$)

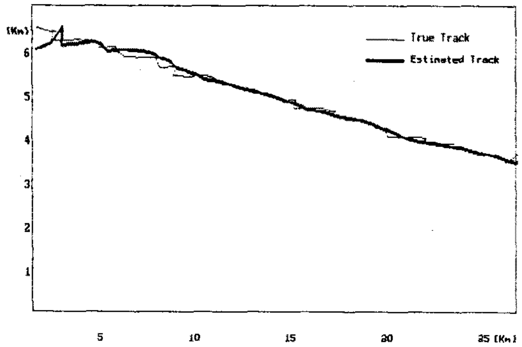

Fig. 5. Modified PDA filter
(real clutter density : 0.4
and estimated using estimated clutter density $\hat{C}$)