Mode와 Ray해법에서의 표면파에 관한 해석

Surface Wave Analysis in Modal and Ray Solutions

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Abstract

When an electric (or a magnetic) line source is located near the surface, surface wave type field is generated and the energy associated with this field is guided very close to the impedance surface. In this paper, field strength is calculated by the exact modal and ray methods for a line source excited parallel plate waveguide. The surface wave contribution to the modal and ray solutions is anticipated very strong and must be included in both solutions.

1. Formulation of the problem

The problem of a line source excited two dimensional (2-D) parallel plate waveguide of infinite extent with impedance walls is analyzed in this section. The 2-D time harmonic wave equation for the parallel plate Green's function G due to a line source at $x = x'$ and $z = z'$ in the waveguide geometry of Figure 1 is given by

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] G = -\delta(x - x')\,\delta(z - z') \quad (1)$$

where k is the free-space wavenumber and $\delta(x)$ is the Dirac delta function. In this 2-D problem, the EM fields can be simply related to the Green's function G because one can scalarize the problem separating it into the TE_y and TM_y cases. One notes that the magnetic field has only a $\hat{y}$ component for the TE_y case and likewise, the electric field has only a $\hat{y}$ component for the TM_y case.

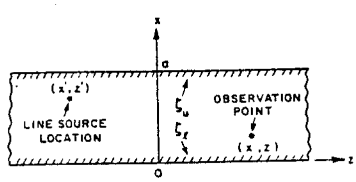

Figure 1: Infinitely long (in $\pm\hat{z}$) parallel plate waveguide excited by an interior line source.

Thus, let $\mathbf{H} = \hat{y}H_y$ represent the magnetic field in the TE_y case; likewise let $\mathbf{E} = \hat{y}E_y$ represent the electric field in the TM_y case. The excitation in the TE_y case can be a magnetic line source of strength M at (x', z'); likewise, an electric line source of strength I at (x', z') generates the TM_y fields. These line sources are of infinite extent in the $\hat{y}$ direction. It can be shown that $H_y = -jkYMG$ and $E_y = -jkZIG$ where Z (or Y) is the free-space impedance (or admittance), provided G satisfies the following boundary conditions:

$$\frac{\partial G}{\partial z} \pm jk\,G = 0 \qquad as\ |z| \to \infty \quad (2)$$

$$\frac{\partial G}{\partial x} - jk\,\varsigma_l\,G = 0 \qquad at\ x = 0 \quad (3)$$

$$\frac{\partial G}{\partial x} + jk\,\varsigma_u\,G = 0 \qquad at\ x = a \quad (4)$$

where

$$\varsigma_{l,u} = \begin{cases} Z_{l,u} & for\ TE_y\ case \\ Y_{l,u} & for\ TM_y\ case \end{cases} \quad (5)$$

and $Z_{l,u}$ (or $Y_{l,u}$) is the surface impedance (or admittance) at $x = 0$ and $x = a$ which is normalized to the free-space wave impedance (or admittance).

Using the above boundary conditions, one can obtain $G(x, x'; z, z')$ by solving Equation (1) as following: [1]

$$G = \frac{1}{2\pi j}\int_{C_z} \frac{(e^{jk_x x_<} + R_l e^{-jk_x x_<})}{2k_x\,(1 - R_l R_u)} \cdot (e^{-jk_x x_>} + R_u e^{jk_x x_>}) \cdot e^{-jk_z|z-z'|}\, dk_z \quad (6)$$

where $x_<$ and $x_>$ denote the values of x which satisfy $x < x'$ and $x > x'$, respectively. An evaluation of the above integral via the *residue theorem* yields a representation for G in terms of a summation of the conventional guided modes propagating along the z direction; namely: [1]

$$G = \sum_{n=0}^{\infty} \frac{1}{2k_z} \frac{(e^{jk_{xn} x_<} + R_l e^{-jk_{xn} x_<})}{\frac{\partial(R_l R_u)}{\partial k_x}} \cdot (e^{-jk_{xn} x_>} + R_u e^{jk_{xn} x_>}) \cdot e^{-jk_{zn}|z-z'|} \quad (7)$$

where

$$R_l = \frac{k_z - k\varsigma_l}{k_z + k\varsigma_l} \tag{8}$$

$$R_u = \frac{k_z - k\varsigma_u}{k_z + k\varsigma_u} e^{-j2k_z a} \tag{9}$$

It is noted that the modes arise from the residues of the poles in the integrand. The zeros of the denominator of the integrand in Equation (6) yields the required poles. Also these zeros yield the eigenvalues of the modes. Specifically, these eigenvalues are obtained by solving the transcendental equation $1 - R_l R_u = 0$ in the integrand of Equation (6) via a numerical '*Newton–Raphson*' iteration method.

An alternative ray expansion representation for G is obtained by expanding the resonant denominator of the integral in Equation (6) into a geometric series [2],

$$\frac{1}{1 - R_l R_u} = \sum_{n=0}^{\infty} (R_l R_u)^n \tag{10}$$

Then, the Green's function $G(x, x'; z, z')$ is expressed in the complex α plane as

$$\begin{aligned} G = & -\frac{j}{4\pi} \int_C \sum_{n=0}^{\infty} (e^{jk\cos\alpha\, z_<} + R_l e^{-jk\cos\alpha\, z_<}) \\ & \cdot (e^{-jk\cos\alpha\, z_>} + R_u e^{jk\cos\alpha\, z_>}) \\ & \cdot e^{-jk\sin\alpha\,|x-x'|} (R_l R_u)^n \, d\alpha \end{aligned} \tag{11}$$

where

$$R_l = \frac{\cos\alpha - \varsigma_l}{\cos\alpha + \varsigma_l} \tag{12}$$

$$R_u = \frac{\cos\alpha - \varsigma_u}{\cos\alpha + \varsigma_u} e^{-j2ka\cos\alpha} \tag{13}$$

After interchanging the orders of summation and integration, each of the integrals in the sum is evaluated asymptotically for large $k\sqrt{(x-x')^2 + (z-z')^2}$ term by term via the method of steepest descent to arrive at the ray expansion [1,3].

2. Surface waves analysis

In the modal expression for the configuration in Figure 1, there are two surface wave type modes in addition to the usual waveguide type modes which are excited if the impedance is ***inductively*** (or ***capacitively***) reactive when the excitation is due to a ***magnetic*** (or an ***electric***) line source within the waveguide [4,5,6,7]. The surface wave modes can be included in the modal solution by simply evaluating the integral in Equation (6) via the residue theorem for the surface wave poles as done for the other ordinary modes. The distinction between the surface wave poles from the other ordinary poles is that the real part of k_z of the surface wave pole is greater than ka.

Figure 2 shows an effect of the surface wave contribution to the field strength at an observation point in the modal solution. Despite the inclusion of the surface wave effect, the agreement between the exact modal and the approximate ray solution is not so good in Figure 2, unless the distance from the source to the observer is sufficiently large. It was found that the reason for this discrepancy between the two solutions could be traced to the need for an increased accuracy in the asymptotic approximation of the ray solution near the surface when the observation point lies within the surface wave "***transition region***" where the surface wave is not fully established. This "***transition region***" extends over a certain distance from the source depending on the value of the impedance; e.g., it becomes larger for the magnetic line source excitation of an inductively reactive impedance boundary as the inductive reactance becomes smaller. This transition region may be viewed as a "***launching***" or "***peel out***" distance required to establish the surface wave.

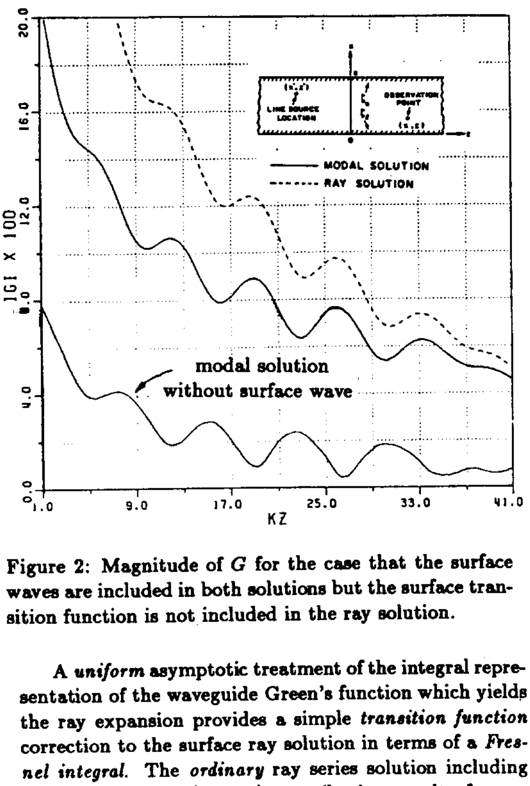

Figure 2: Magnitude of G for the case that the surface waves are included in both solutions but the surface transition function is not included in the ray solution.

A ***uniform*** asymptotic treatment of the integral representation of the waveguide Green's function which yields the ray expansion provides a simple ***transition function*** correction to the surface ray solution in terms of a ***Fresnel integral***. The ***ordinary*** ray series solution including the surface wave (or ray) contribution results from a ***non-uniform*** asymptotic treatment of the integral for the waveguide Green's function; this ordinary ray solution is accurate only outside the ***surface wave transition region***. Mathematically, the observation point lies within the surface wave transition region when the surface wave pole α_p is close to the saddle point α_s as shown in Figure 3.

Consider the integral given by

$$I(kr) = \int_{C_{SDP}} \frac{F(\alpha)}{\alpha - \alpha_p} e^{kr f(\alpha)} \, d\alpha \tag{14}$$

where α_p and α_s are a surface wave pole and saddle point, respectively and the surface wave pole is near the saddle point as shown in the above figure. Then, the integral is evaluated asymptotically using the uniform saddle point approximation [3], and is given by

$$I(kr) = e^{-jkr}\left[\sqrt{\frac{\pi}{kr}}\,T(0) \pm j2\sqrt{\pi}F(\alpha_p) \cdot e^{-krb^2}\,Q(\mp jb\sqrt{kr}\,)\right] \quad (15)$$

$$; Im(b) > 0$$

where

$$b = \sqrt{f(\alpha_s) - f(\alpha_p)} \quad (16)$$

$$T(0) = \frac{hf(\alpha_s)}{\alpha - \alpha_p} + \frac{f(\alpha_p)}{b} \quad (17)$$

$$h = \sqrt{\frac{-2}{f''(\alpha_s)}} \quad (18)$$

$$Q(y) = \int_y^{\infty} e^{-x^2}\,dx \quad (19)$$

Using the above formula, the integral in Equation (14) can be evaluated to obtain the surface ray field.

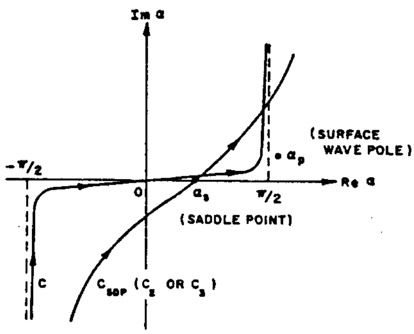

Figure 3: Integration path and a surface wave pole α_p which is near the saddle point α_s.

For convenience, let G in Equation (11) be expressed as

$$G = \sum_{n=0}^{\infty}\sum_{m=1}^{4} G_{mn} \quad (20)$$

For analytical details, consider G_{2n} in Equation (20). For $n = 0$, G_{2n} reduces to

$$G_{20} = -\frac{j}{4\pi}\int_{C_2}\frac{\cos\alpha - Z_u}{\cos\alpha + Z_u}\,e^{+jkr\cos(\alpha-\theta)} \quad (21)$$

where

$$r = \left[(x_< + x_>)^2 + (z - z')^2\right]^{1/2} \quad (22)$$

$$\theta = \tan^{-1}\left(\frac{|z - z'|}{x_< - x_>}\right) = \alpha_s \quad (23)$$

Then, the parameters in Equations (16)–(19) are expressed as

$$f(\alpha) = j\cos(\alpha - \alpha_s) \quad (24)$$

$$f''(\alpha) = -j\cos(\alpha - \alpha_s) \quad (25)$$

$$b = \sqrt{j\,(1 - \cos(\alpha - \alpha_s))} \quad (26)$$

$$T(0) = \frac{\sqrt{2}\,e^{+j\pi/4} f(\alpha_s)}{\alpha_s - \alpha} + \frac{f(\alpha)}{\sqrt{2}\,e^{-j\pi/4}\sin(\frac{\alpha-\alpha_s}{2})} \quad (27)$$

$$h = \sqrt{2}\,e^{-j\pi/4} \quad (28)$$

The analysis for G_{30} is very similar and is thus ommitted here.

A comparison of the improved or *uniform* ray solution with the exact modal solution shown in Figure 4 now indicates that they are in excellent agreement.

3. Conclusion

The surface wave modes can be included in the modal solution by simply evaluating the integral via the residue theorem for the surface wave poles as done for the other ordinary poles. In the ray method, a *uniform* asymptotic treatment of the integral representation of the waveguide Green's function provides a simple *transition function* correction to the surface ray solution in terms of a *Fresnel integral.*

An important characteristic of the surface wave type fields is that the energy associated with the fields is guided very close to the impedance surface. This surface wave contribution to the total field at an observation point is quite strong and must be considered in the field computations.

References

[1] B. Friedman, *Principles and Techniques of Applied Mathematics*, John Wiley and Sons, Inc., 1966.

[2] L.B. Felsen and A.H. Kamel, "Hybrid Ray–Mode Formulation of Parallel Plane Waveguide Green's Functions", IEEE Trans, AP, Vol. AP-29, No. 4, pp. 637–649, Jul. 1981.

[3] L.B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*, Prentice Hall, Englewood Cliffs, New Jersey, 1973.

[4] Robert E. Collin, ***Field Theory of Guided Waves***, McGraw-Hill Book Co., New York, 1960.

[5] R. Mittra and S.W. Lee, ***Analytical Techniques in the Theory of Guided Waves***, The Macmillan Company, New York, 1971.

[6] L. Lewin, ***Theory of Waveguides***, John Wiley and Sons, New York, 1975.

[7] C.S. Lee, S.L. Chuang, S.W. Lee, Y.T. Lo, "Wave Attenuation and Mode Dispersion in a Waveguide Coated with Lossy Dielectric Material", Technical Report No. 84-13, University of Illinois, Electromagnetics Laboratory, July 1984.

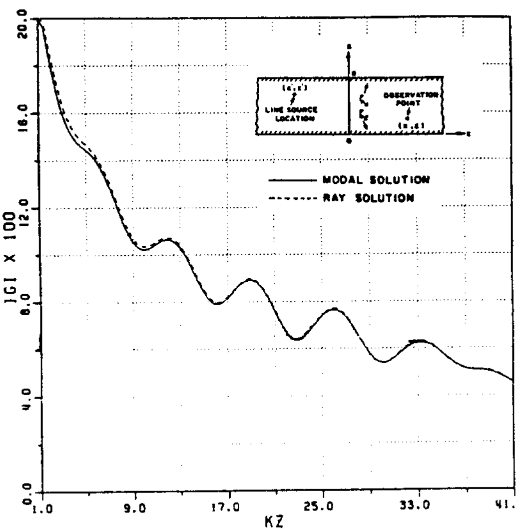

Figure 4: Magnitude of G in the case of Figure 2 except that the *uniform* transition function is included in the ray solution.