ANALYSIS ON A SATURATING SYSTEM WITH AN INTELLIGENT LIMITER

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All mechanical systems have saturation nonlinearity in actuators or in final control elements. When controllers have integral action, reset windup can cause instability as well as make the system performance unsatisfactory. In this study, an intelligent limiter which needs no tuning of parameters is tested with the PDF controller used for control of a second order plant. This paper presents analysis of the stability of the system using the describing function method and the Nyquist stability theorem. The improvement of the system performance by the limiter is illustrated by computer simulations.

1. INTRODUCTION

All mechanical systems have saturation in final control elements (e.g., power amplifiers and actuators). Saturation is a significant type of nonlinearity of mechanical systems. When a state feedback controller is used for a plant involving a saturation element, The actuator saturation problem can be avoided by setting the controller parameters based on the LQ design (Bryson and Ho, 1975). However, the state feedback controller results in a steady-state error. Thus, integral action is frequently added to the state feedback controller to eliminate the steady-state error.

For a control system with integral action, if the reference input is small, the controller output will stay within the linear region of the saturation element. If the reference input is large, the controller output may saturate quickly, in which case the integrator output will keep growing until the sign of the error signal changes. Because the saturation delays the sign change of the error signal, the integrator output becomes very large. Even after the error signal changes sign, the controller output may be held above the saturation value for quite some time. This can cause a large overshoot and instability in the system response and the phenomenon is called reset windup.

Saturating sampled-data systems have been investigated by Nease (1957), Kalman (1957) and Mullin (1959). Mullin suggested a digital filter which forces the systems to follow step or ramp inputs when saturation is present. To stabilize a system with a nonlinear actuator, Hsu and Meyer (1968) inserted a nonlinear element in the error path of the closed-loop system. To avoid the reset windup of an integrator in a system with actuator saturation, Krikelis (1980) used a nonlinear feedback element around the integrator. All of these control schemes involve tuning of the parameters that are in the filter or in the nonlinear elements. Hanus (1980) proposed a technique using a proportional feedback element around the integrator, but without any stability analysis. Glattfelder and Schaufelberger (1983) presented the stability analysis of systems with an antireset-windup (ARW) circuit based on the circle criterion. By using the ARW circuit, the asymptotically stable region for the system's initial state can be extended.

Another scheme to avoid reset windup is to keep the controller output to the maximum (or minimum) value of the final control element when it tends to exceed the limit, by continuously setting the output of the integrator to the value required to make the output of the controller equal to the saturation limit. This scheme has been suggested by Phelan (1977), for use with the 'pseudo derivative feedback' (PDF) controller, called intelligent PDF controller. It has been implemented successfully, with analog circuits and digitally, but no stability analysis has been made.

This paper presents a study of the stability and performance of a control system using a PDF controller, both with and without the intelligent limiter, for controlling a second-order plant. The system's stability is analyzed using the describing function method in conjunction with the Nyquist stability theorem, and its performance is illustrated by computer simulations.

2. EFFECTS OF SATURATION

The block diagram of a PDF controller controlling a second order plant is shown in Figure 1. The plant has an inertia m and linear damping coefficient c. The linear range of the input of the saturation element is $[-M \;\; M]$, and the output is limited to $\pm M$ outside the linear range. The controller is essentially a state feedback controller with only integral action in the position error feedforward path.

Before studying the effect of the saturating element on the stability of the system, analysis of the system without the saturating element will be preceded first. The characteristic equation of the system without saturation is

$$ms^3 + (c + k_d)s^2 + k_p s + k_i = 0. \tag{1}$$

By the Routh criterion, the first condition for the asymptotic stability is that all the coefficients of the characteristic equation must be positive, which is satisfied if the controller parameters are all positive. The second condition is given by

$$\frac{mk_i}{k_p(c + k_d)} < 1. \tag{2}$$

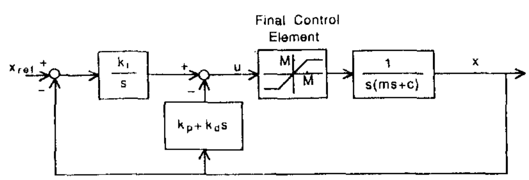

Fig. 1 The block diagram of a second-order controlled system with PDF control.

The parameters of the controller need to be tuned in consideration of performance as well as stability of the closed-loop system.

In the simulation study, the system parameters used are $m = 1$ and $c = 0.01$. The controller parameters are set as $k_p = 3.00$, $k_i = 1.00$ and $k_d = 2.99$, which satisfy the above conditions. These values are selected arbitrarily just to illustrate the responses of the control system. The system has no pole-zero cancellation. The closed-loop poles are located at -1.0 (triple). The responses of the control system and the controller outputs for various step reference inputs, without including the saturation element in the system, are shown in Figures 2 and 3. The responses have the same settling time, as expected for a linear system.

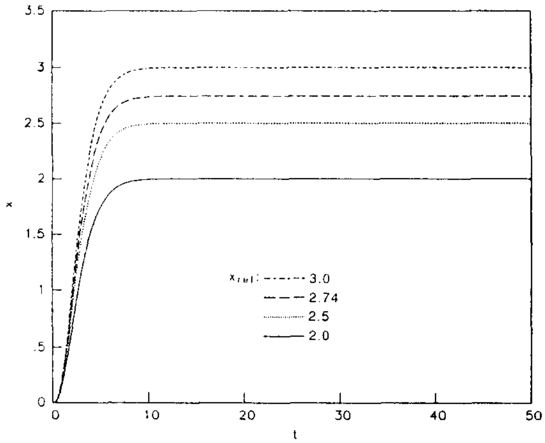

Fig. 2 The responses of the linear system (i.e., without saturation) to various step reference inputs.

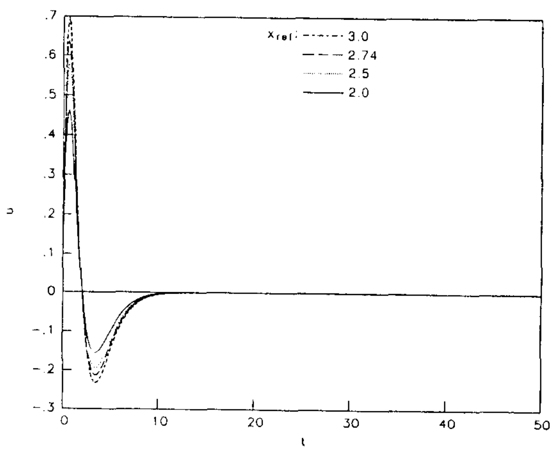

Fig. 3 The controller outputs of the linear system for various step reference inputs.

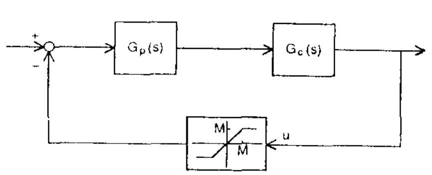

Fig. 4 The reconfigured block diagram of the control system with saturation.

To analyze the stability of the system with saturation, the control system is reconfigured to a system which has the plant, the controller in the feedforward path, and the saturating element in the feedback path; see Figure 4. (The reconfigured system has the same stability properties as the original system.) The transfer function of the plant, $G_p(s)$, and the transfer function of the controller, $G_c(s)$, of the reconfigured system are given by

$$G_p(s) = \frac{1}{s(ms + c)} \tag{3}$$

and

$$G_c(s) = k_p + k_d s + \frac{k_i}{s}. \tag{4}$$

The open-loop transfer function of the reconfigured system, $G_{op}(s)$, is

$$G_{op}(s) = \frac{k_d s^2 + k_p s + k_i}{s^2(ms + c)}. \tag{5}$$

We define a 'modified' open-loop frequency transfer function, $G^*_{op}(j\omega)$, as

$$G^*_{op}(j\omega) = \mathrm{Re}\Big\{G_{op}(j\omega)\Big\} + j\omega\mathrm{Im}\Big\{G_{op}(j\omega)\Big\}. \tag{6}$$

The Popov locus is defined as the plot of the modified frequency transfer function in the complex plane as ω is varied from zero to infinity. The nonlinear relationship in the feedback block is confined in a sector whose boundary slopes are 0 and 1. By the Popov theorem (Popov, 1962) the system is asymptotically stable if the Popov locus lies to the right of a line which passes through the negative real axis at -1 and has a slope of $1/q$, where q can be any real number. Figure 5 shows the Popov loci of the open-loop system for various parameter conditions. Because there exists no real number q which satisfies the Popov criterion, it is concluded that asymptotic stability can not be guaranteed by the Popov theorem.

The Popov criterion is only a sufficient condition for asymptotic stability. In order to further study the characteristics of the feedback system, we use the describing function method. The describing function of the saturation, $N_s(a)$, is given by (Sridhar, 1960)

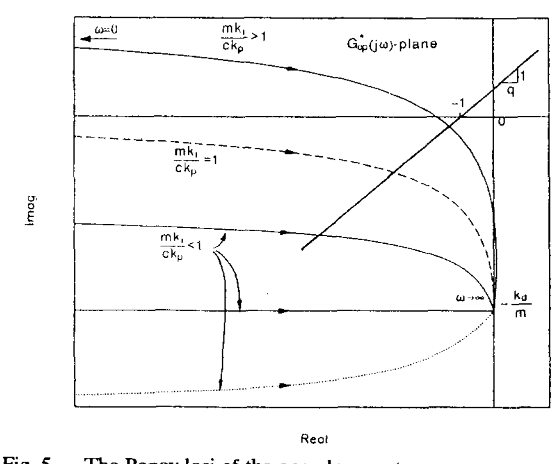

Fig. 5 The Popov loci of the open-loop system.

$$N_s(a) = \begin{cases} 1 & \text{for } a < M, \\ \dfrac{2}{\pi}\sin^{-1}\dfrac{M}{a} + \dfrac{2M}{\pi a}\left\{1 - \left(\dfrac{M}{a}\right)^2\right\}^{1/2} & \text{for } a \geq M, \end{cases} \qquad (7)$$

where a is the amplitude of the input signal (i.e. the controller output) to the saturation element. In Figure 6, the locus of $-1/N_s(a)$ is plotted for increasing a. The three curves represent the open-loop frequency responses of the system, $G_{op}(j\omega)$, for $k_i = 8.97$ (which is near the stability limit of the linear system), 1 and 0.1, with m, c, k_p, and k_d remaining unchanged from the previous study in which the saturation was not included. By substituting $j\omega$ into s in Eq. (5), we find that the frequency response curve does not cross the negative real axis if

$$mk_i - ck_p \leq 0. \qquad (8)$$

If $mk_i - ck_p \leq 0$, there exists no limit cycle, and based on the Nyquist stability theorem the system is asymptotically stable. If $mk_i - ck_p \geq k_d k_p$, the frequency response curve crosses the negative real axis between -1 and 0, and obviously, the system is unstable. When $0 < mk_i - ck_p < k_d k_p$, there exists an unstable limit cycle at the crossing point between frequency response curve and the line representing $-1/N_s(a)$. For $k_i = 1$, the frequency of the limit cycle is 0.57, and the amplitude of the limit cycle is $11.8M$. When the reference input signal is sufficiently small so that the controller output remains less than the controller output amplitude of the limit cycle, the system behaves as an asymptotically stable system. For a large reference input signal, the system's response will diverge. It should be noted that the analysis based on the describing function techniques is an approximate method. The actual frequency and amplitude of the limit cycle may be slightly different from those obtained from the analysis.

Simulations are performed for the parameters set above, which satisfy the condition $0 < mk_i - ck_p < k_d k_p$. The responses of the system and the corresponding controller outputs in the presence of the saturation to various step reference inputs are shown in Figures 7 and 8. The maximum value of the linear range, M, is 0.25 in this simulation. In Figure 7, the response of the system to $x_{ref} = 2.0$ is asymptotically stable, since the size of step reference input is not big enough to cause reset windup. The response of the system to $x_{ref} = 2.5$ shows overshoot and undershoot caused by reset windup, but it converges. When $x_{ref} = 2.74$, the response is very close to a limit cycle. (The limit cycle exists at x_{ref} between 2.74 and 2.75.) The amplitude of controller output is 3.46, which is 17 percent larger the amplitude predicted by the analysis using the describing function. The limit cycle is unstable and the response finally converges to the reference (not shown in the figure). The response of the system to $x_{ref} = 3.0$ diverges as was predicted by the analysis.

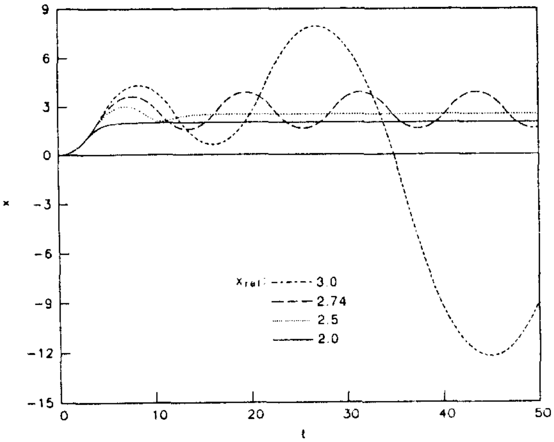

Fig. 7 The responses of the system to various step reference inputs.

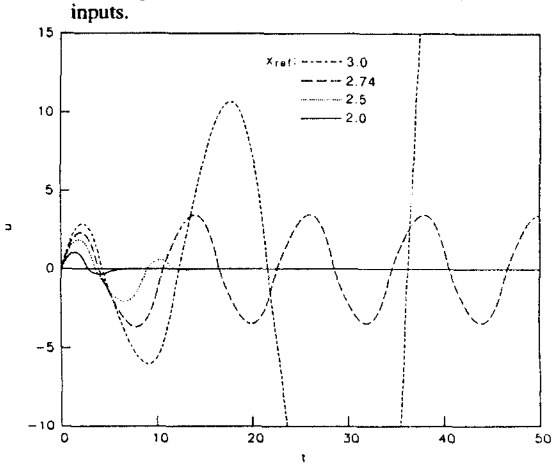

Fig. 8 The controller outputs of the system to various step reference inputs.

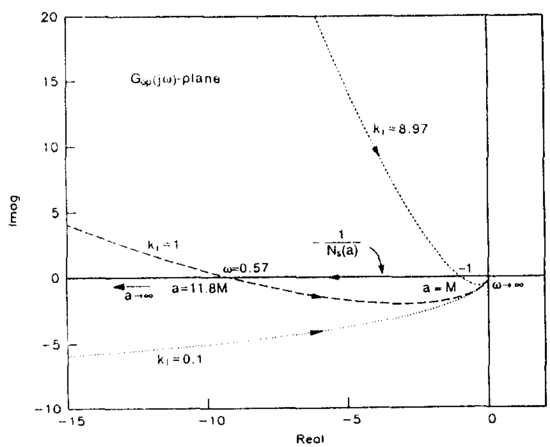

Fig. 6 The locus of $-1/N_s(a)$ and frequency response curves of the open-loop system for various values of k_i.

3. STABILIZATION USING AN INTELLIGENT LIMITER

To avoid reset windup, an intelligent limiter has been suggested by Phelan (1977) to limit the controller output. When the output of controller is within the linear range of the final control element, the intelligent is inactive. When the output instantaneously saturates the linear range, the output of the integrator is adjusted so that the controller output does not exceed the maximum or minimum value of the final control element. The controller resumes its normal operation (i.e. the intelligent limiter becomes inactive) when the output falls within the linear range. The block diagram of the previously discussed system with the intelligent limiter is shown in Figure 9.

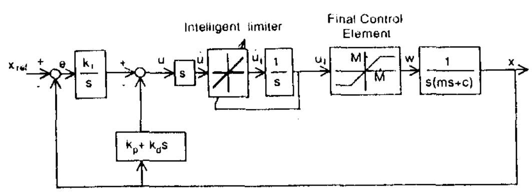

Fig. 9 The reconfigured block diagram of the control system with the intelligent limiter.

The output of the intelligent PDF controller is

$$u_l(t) = u(t) - \int_0^t n\left[u_l(\tau),\dot{u}(\tau)\right]\dot{u}(\tau)d\tau \tag{9}$$

where

$$u(t) = -k_p x(t) - k_d v(t) + k_i \int_0^t e(\tau)d\tau \tag{10}$$

and

$$e(t) = x_{ref} - x(t) \tag{11}$$

and

$$n(u) = \begin{cases} 1 & \text{if } |u_l| = M \text{ and } \dot{u} \geq 0, \\ 1 & \text{if } |u_l| = -M \text{ and } \dot{u} \leq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

When $|u_l| < M$, the saturating element is not needed in the stability analysis. To perform stability analysis for $|u_l| < M$, the block diagram of the system with the intelligent limiter is reconfigured to Figure 10. The open-loop system in Figure 10 is the same as that in Figure 4, and the nonlinear blocks of these two figures are confined in the same sector. Therefore, based on the Popov criterion we can not conclude that the intelligent integral controller improves the stability of the system. The describing function method is utilized to further study the stability of the system. The input-output relationship of the intelligent limiter is illustrated in Figure 11. The describing function, $N_l(a)$, is obtained from the input-output relationship as follows:

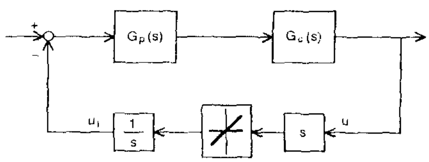

Fig. 10 The reconfigured block diagram of the intelligent (limiting) control system.

$$N_l(a) = \frac{1}{a\pi}\int_0^{2\pi} u_l \sin\theta d\theta + j\frac{1}{a\pi}\int_0^{2\pi} u_l \cos\theta d\theta, \tag{13}$$

where

$$u_l = \begin{cases} M & \text{for } \alpha < \theta \leq \frac{\pi}{2}, \\ a\sin\theta + M - a & \text{for } \frac{\pi}{2} < \theta \leq \pi + \alpha, \\ -M & \text{for } \pi + \alpha < \theta \leq \frac{3}{2}\pi, \\ a\sin\theta - M + a & \text{for } \frac{3}{2}\pi < \theta \leq 2\pi + \alpha \end{cases} \tag{14}$$

and $\sin\alpha = (2M-a)/a$. By performing the above integration, $N_l(a)$ becomes

$$N_l(a) = \frac{1}{2} + \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} + j\frac{1}{\pi}\cos^2\alpha. \tag{15}$$

The locus of $-1/N_l(a)$ is plotted in Figure 12. When $k_i = 1$ or 0.1, it is evident that the limit cycle does not exist because frequency response curves do not cross the locus of $-1/N_l(a)$. Furthermore, based on the Nyquist stability theorem, the asymptotic stability is expected because the frequency response curves do not encircle the locus clockwisely. The open-loop frequency response curves for $k_i = 8.97$ (near the stability limit of the linear system) crosses the locus of $-1/N_l(a)$ twice (not shown clearly in the figure due to figure size limitation). According to the describing function analysis, there exist one stable limit cycle and one unstable limit cycle, and the system is not asymptotically stable. However, since the two curves barely intersect and the describing function analysis is only an approximation method, stability of the system cannot be concluded cased on this method for a parameter value at or near the stability limit of the linear system.

The frequency response curves of the open-loop system for other sets of control parameters are similar to the curves in Figure 12. The only exception is that when the values of the controller parameters are at or close to the stability limit of the linear control system, the frequency response curves may or may not intersect the locus of $-1/N_l(a)$, as shown in Figure 13.

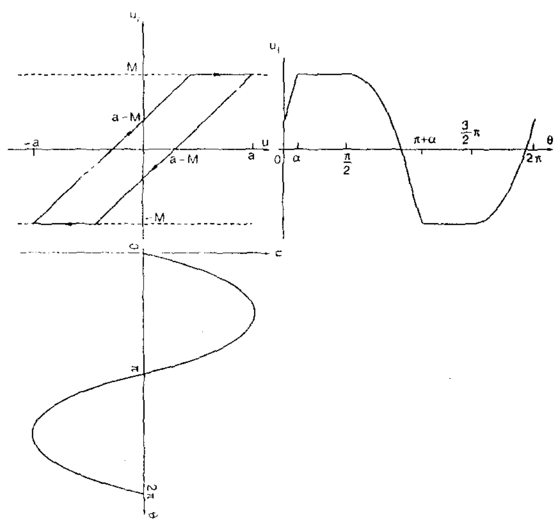

Fig. 11 Input-output relationship of the intelligent limiter.

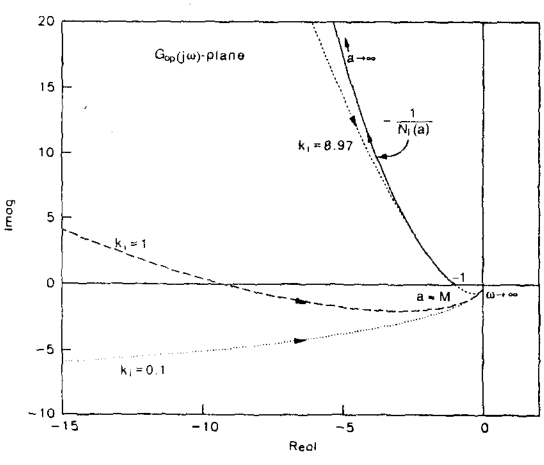

Fig. 12 The locus of $-1/N_l(a)$ and the frequency response curves of the open-loop system for various values of k_i.

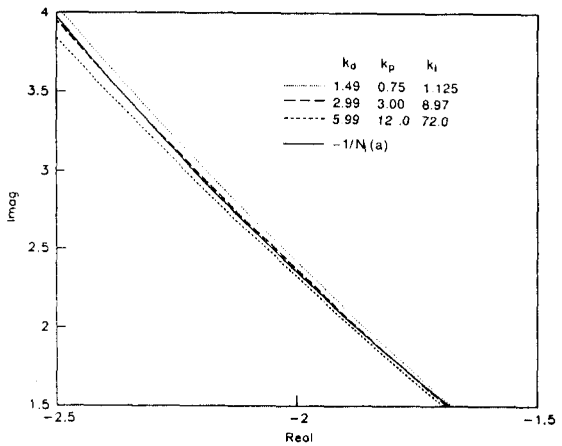

Fig. 13 The locus of $-1/N_l(a)$ and the frequency response curves of the open-loop system for various cases of stability limit.

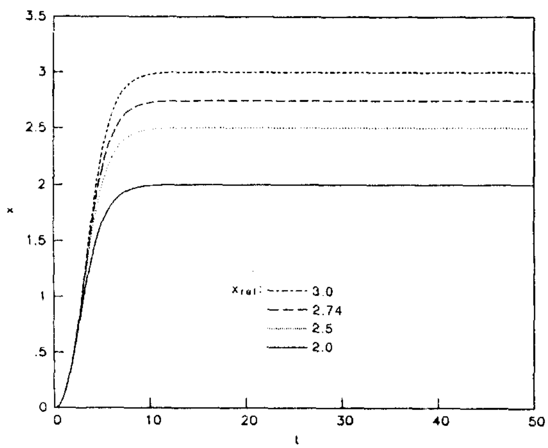

Fig. 14 The responses of the intelligent control system to various step reference inputs when $k_i = 1$.

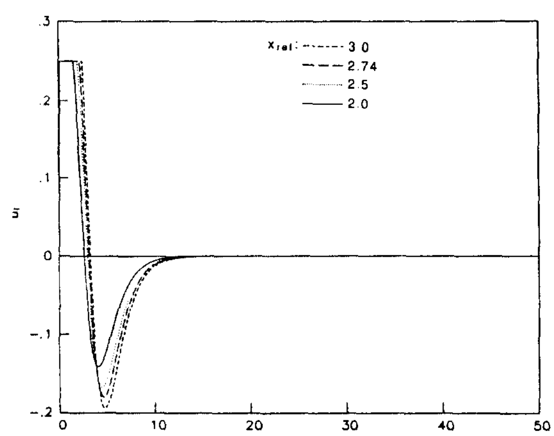

Fig. 15 The controller outputs of the intelligent control system to various step reference inputs when $k_i = 1$.

Therefore, based on the describing function analysis it is concluded that the control system with the intelligent limiter is asymptotically stable if the controller parameters are tuned based on the stability condition of the linear system given by Eq. (2), except that no conclusion on the system stability can be drawn from this analysis when parameter values are at or near the stability limit of the corresponding linear control system.

The responses of the system with the intelligent limiter to various step reference inputs for the same set of parameter values used in Figure 7 and 8 are shown in Figure 14. The corresponding controller outputs are shown in Figure 15. All the system responses converge asymptotically, and no overshoot is observed. The reset windup phenomenon shown in Figure 7 has been eliminated by the use of the intelligent limiter. The controller outputs never exceed the saturating level. The speeds of the system responses are slightly slower than those of the linear systems (Compare Figures 2 and 14) because of the existence of the saturating element.

Since the describing function analysis cannot lead to a conclusion on system stability for the nonlinear control system with parameter values near the stability limit of the corresponding linear control system, computer simulations are used to further study the stability limit condition in detail. For all simulated cases, the results show that as long as the parameter values render the corresponding linear control system stable,

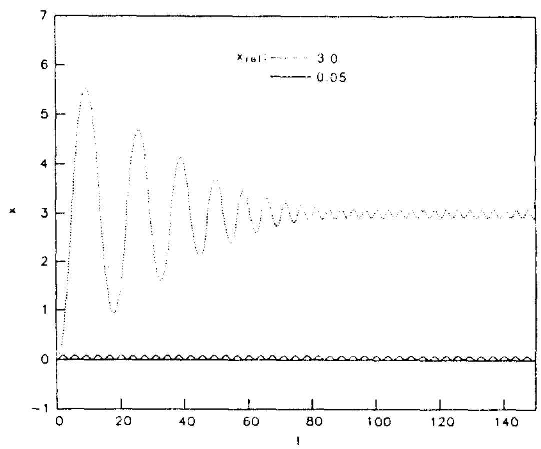

Fig. 16 The responses of the intelligent control system to various step reference inputs when $k_i = 8.97$.

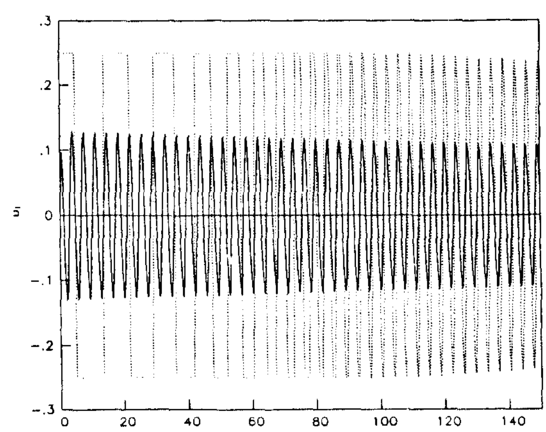

Fig. 17 The controller outputs of the intelligent control system to various step reference inputs when $k_i = 8.97$.

despite that they may be very close to the stability limit, the nonlinear control system with the limiter is always stable. As an example, Figure 16 shows the responses of the control system for two step reference inputs when $k_i = 8.97$ and k_p and k_d are the same as those used in the previous simulations. Figure 17 shows the controller outputs of the system.

4. CONCLUSIONS AND REMARKS

Integral action in a closed-loop system involving a saturating element can cause instability of the system and poor performance of the system. A intelligent limiter used with PDF controller for control of a second-order system with saturation nonlinearity has been studied. By use of the describing function method and the Nyquist stability theorem together and with the aid of digital simulation, the stability of the intelligent PDF control system is shown to be the same as that of the corresponding linear control system (i.e. without the saturating element and the intelligent limiter). The improvement in the performance of the system over that without using the intelligent limiter is confirmed by simulations.

The intelligent limiter described in also applicable to other linear controllers involving integral action. For example, Phelan (1977, 1987) has shown by both analog and digital simulations the ability of the limiter to stabilize systems having a zeroth-order or first-order plant controlled by a PDF or PI (proportional plus integral) controller. For all of the control systems the function of the limiter is to confine the controller output to the appropriate limiting value defined by the saturating element, whenever the output tends to exceed the linear range of the saturating element. Stability analysis as presented in this paper can be performed to show the stability of these and other control systems with use of the intelligent limiter.

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