* Adaptive Control of Uncertain Systems with Application to a Robotic Manipulator

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In this paper, an adaptive control method is presented to guarantee the ultimate boundedness of uncertain systems with partially known uncertainty bounds. This method, with a conventional linear compensator, is used to improve the performance of the trajectory tracking of a robotic manipulator with uncertainties. The proposed method is simulated under several different environments, and its performance is compared with the computed torque method. The simulation results show that the proposed method is well suited for high-performance operation of uncertain robotic systems.

1. Introduction

Recently, various approaches have been developed to control robotic manipulators. These approaches include nonlinear control[1,4,16], adaptive control [2,7-9,11,13], variable structure control[6,12], and robust control of uncertain systems[5,10,14,15]. Such control methods have at least one of the following drawbacks. These are the requirements of detailed robot models and accurate load forecasting, the complexity in implementation, chattering phenomena in control input and state variables, and the requirement of perfect knowledge of uncertainty bounds.

In this paper, an adaptive control method is presented to guarantee the ultimate boundedness of uncertain systems with partially known uncertainty bounds. The method is applied to the trajectory tracking control of a robotic manipulator which carries an unknown payload and its inertial parameters are uncertain. A linear compensator is first introduced to the tracking problem based on a nominal model of the robotic manipulator. Then, the adaptive control method is used to stabilize and to improve the tracking errors of the uncertain robotic system.

The proposed adaptive control method needs a relatively small number of parameters to be estimated, which is independent of the number of joints. It can achieve a good performance by an appropriate choice of design parameters even in the presence of a computational time-delay. The proposed method is tested in the trajectory tracking problem of PUMA 560 robot arm by computer simulations, and its performance is evaluated.

This paper is organized as follows. In Section 2, an adaptive control method for uncertain systems is stated as a preliminary result to be used later for the control of uncertain robotic systems. The implementation of the robotic controller is given in Section 3. Simulation results are presented in Section 4, and conclusions are in Section 5.

Some mathematical notations are introduced as follows. The ith eigenvalue of $A\in\mathbb{R}^{n\times n}$ is $\lambda_i(A)$; $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimum and maximum of the real parts of the eigenvalues of A, respectively; the matrix norm of $A\in\mathbb{R}^{n\times m}$ is $\|A\|:=\max\{\|Ax\|: \|x\|=1, x\in\mathbb{R}^m\}$ $=[\lambda_M(A^TA)]^{1/2}$; $f\in C^n(X,Y)$ if $f:X\rightarrow Y$ is n times continuously differentiable; $C(X,Y)$ denotes $C^0(X,Y)$; $\mathbb{R}:=(-\infty,+\infty)$ and $\mathbb{R}_+:=[0,+\infty)$.

2. Preliminary Result

Consider a class of uncertain systems described by

$$\dot{x}(t)=Ax(t)+B\{[I+E(t,x(t))]u(t)+v(t,x(t))\}, \quad (2.1)$$

where $x(t)\in\mathbb{R}^n$ is the system state, $u(t)\in\mathbb{R}^m$ is the control input, $E\in C(\mathbb{R}_+\times\mathbb{R}^n,\mathbb{R}^{n\times m})$ and $v\in C(\mathbb{R}_+\times\mathbb{R}^n,\mathbb{R}^m)$ are unknown functions, and (A,B) is a stabilizable pair.

The objective is to find a control law which guarantees that the closed-loop system has a bounded

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solution with a tolerable error bound when the bounds of the uncertainties $E(\cdot,\cdot)$ and $v(\cdot,\cdot)$ are partially known.

The following assumptions are needed for developments in this section.

Assumption 2.1: All the eigenvalues of A have negative real parts.

Assumption 2.2: There exists a constant $c\in\mathbb{R}_+$ such that

$$1+\lambda_m\{\tfrac{1}{2}[E(t,x)+E^T(t,x)]\}\geq c \quad \forall(t,x)\in\mathbb{R}_+\times\mathbb{R}^n. \tag{2.2}$$

Assumption 2.3: There is a function $\rho_v\in C(\mathbb{R}_+\times\mathbb{R}^n, \mathbb{R}_+)$ such that

$$c^{-1}\|v(t,x)\|\leq\rho_v(t,x), \quad (t,x)\in\mathbb{R}_+\times \mathbb{R}^n. \tag{2.3}$$

In particular, $\rho_v(t,x)$ has the form

$$\rho_v(t,x) = \theta^T\varphi(t,x) =: \rho(t,x,\theta), \tag{2.4}$$

where $\theta\in\mathbb{R}^p_+$ is an *unknown* constant and $\varphi:\mathbb{R}_+\times\mathbb{R}^n\rightarrow\mathbb{R}^p$ is a *known* continuous function.

If the bounding function of the uncertainty is not completely known but has the structure given in *Assumption 2.3*, an adaptive law, which estimates θ, can be incorporated in the contol.

It follows from *Assumption 2.1* that there exists a unique positive-definite solution $P\in\mathbb{R}^{n\times n}$ to the Lyapunov equation

$$A^TP + PA = -Q \tag{2.5}$$

for a given positive-definite matrix $Q\in\mathbb{R}^{n\times n}$.

The control input and parameter update equation are given as follows:

$$u(t):=-\rho(t,x,\hat{\theta})\text{sat}[\eta(t,x,\hat{\theta})], \tag{2.6}$$

$$\dot{\hat{\theta}}(t)=-\lambda\hat{\theta}(t)+L\varphi(t,x(t))\|\alpha(t,x(t))\|,\ \hat{\theta}(t_o)\in\mathbb{R}^p, \tag{2.7}$$

where

$$\alpha(t,x):=B^TPx(t), \tag{2.8}$$

$$\eta(t,x,\hat{\theta}):=\varepsilon^{-1}\rho(t,x,\hat{\theta})\alpha(t,x), \tag{2.9}$$

$$\text{sat}(\xi) := \xi,\ \|\xi\|\leq1;\quad \xi/\|\xi\|,\ \|\xi\|>1. \tag{2.10}$$

Positive numbers, ε and λ, and positive-definite matrix $L:=\text{diag}\{l_1,\cdots,l_p\}$ are design parameters which can be selected to make the state error sufficiently small.

The constants l_m, l_M, μ, $\bar{\varepsilon}$, d_o, d_1, and d_2 are defined as follows:

$$l_m:=\lambda_m(L)=\min\ l_i;\ l_M:=\lambda_M(L)=\max\ l_i, \tag{2.11}$$

$$\mu := \tfrac{1}{2}\min\{\lambda_m(P^{-1}Q),\ \lambda\}, \tag{2.12}$$

$$\bar{\varepsilon} :=c[\varepsilon/4+\lambda\|\theta\|^2/(2l_m)], \tag{2.13}$$

$$d_o:=\left[\frac{\bar{\varepsilon}}{\mu\cdot\max\{\lambda_M(P),cl_m^{-1}\}}\right]^{1/2}, \tag{2.14}$$

$$d_1:=\left[\frac{\bar{\varepsilon}}{\mu\cdot\lambda_m(P)}\right]^{1/2}, \tag{2.15}$$

$$d_2:=[l_M\bar{\varepsilon}/(c\mu)]^{1/2}. \tag{2.16}$$

The next theorem states the ultimate boundedness of the solution of the system (2.1) with the adaptive control law (2.6) and (2.7).

Theorem 2.1: Suppose that *Assumptions 2.1–2.3* are satisfied. Assume that the system (2.1), with the adaptive control law given by (2.6) and (2.7), has a solution $x:\mathbb{R}_+\rightarrow\mathbb{R}^n$ for a given initial state $x(t_o)\in\mathbb{R}^n$ with $t_o \in \mathbb{R}_+$. Then, for $t \in [t_o,\infty)$,

$$\|x(t)\| \leq \begin{cases} d_1, & \|z(t_o)\|\leq d_o, \\ d_1\{1+[\|z(t_o)\|^2/d_o^2-1]e^{-2\mu(t-t_o)}\}^{1/2}, & \|z(t_o)\|>d_o, \end{cases} \tag{2.17}$$

$$\|\tilde{\theta}(t)\| \leq \begin{cases} d_2, & \|z(t_o)\|\leq d_o, \\ d_2\{1+[\|z(t_o)\|^2/d_o^2-1]e^{-2\mu(t-t_o)}\}^{1/2}, & \|z(t_o)\|>d_o, \end{cases} \tag{2.18}$$

where

$$\tilde{\theta}(t) := \hat{\theta}(t) - \theta,\quad z(t) := [x^T(t)\ \tilde{\theta}^T(t)]^T.$$

Proof: See [19].

Ultimate bounds of the state and parameter error are determined by d_1 in (2.17) and d_2 in (2.18), respectively. To gain insights on the bounds, consider the following case. When $\lambda_m(P^{-1}Q) \geq \lambda$, $\mu = \lambda/2$ from (2.12). In this case, combining (2.13) with (2.15) and (2.16) gives

$$d_1 = \left[\frac{c}{\lambda_m(P)}\left(\frac{\varepsilon}{2\lambda} + \frac{\|\theta\|^2}{l_m}\right)\right]^{1/2} \tag{2.19}$$

and

$$d_2 = \left[\frac{\varepsilon l_M}{2\lambda} + \frac{l_M}{l_m}\|\theta\|^2\right]^{1/2}, \tag{2.20}$$

respectively.

It can be seen from (2.19) that the influence of $\|\theta\|$ on d_1 decreases as l_m becomes large. By selecting a small ε and a large $\lambda_m(P)$ and l_m, one can make d_1 as small as desired. Since $l_M \geq l_m$, it can be noted in (2.20) that d_2 must be greater than $\|\theta\|$ regardless of $L(>0)$. Moreover, the term $\varepsilon l_M/2\lambda$ in (2.20) causes d_2 to increase as L becomes large.

3. Trajectory Tracking of a Robotic Manipulator

In this section, a linear compensator is introduced to the tracking problem based on a nominal model of robotic manipulator. The linear compensator includes the conventional PD and PID controllers as special cases. To guarantee the ultimate boundedness of tracking errors for precompensated robotic system by the linear compensator, the adaptive control method proposed in the previous section is applied.

Consider the dynamics of an n-link robot manipulator described by

$$D(q(t))\ddot{q}(t) + N(q(t),\dot{q}(t)) = \tau(t) \tag{3.1}$$

where $q:\mathbb{R}_+\to\mathbb{R}^n$ are a vector of joint variables, and $\dot{q}$ ($\ddot{q}$) is the first (second) time derivative of q; $D(\cdot) \in C^1(\mathbb{R}^n,\mathbb{R}^{n\times n})$ is a symmetric positive-definite matrix which represents inertia and inertial interactions of the joints; the vector $N(\cdot,\cdot) \in C(\mathbb{R}^n\times\mathbb{R}^n,\mathbb{R}^n)$ contains terms due to Coriolis, centrifugal, and gravity effects on the manipulator; $\tau:\mathbb{R}_+\to\mathbb{R}^n$ signifies the generalized torque vector applied to joints of the manipulator.

A function $q_d \in C^2(\mathbb{R}_+,\mathbb{R}^n)$ prescribes a desired trajectory which is to be tracked by the manipulator; the object is ideally

$$q(t) = q_d(t), \quad \forall t\in[t_o,\infty),$$

where $t_o\in\mathbb{R}_+$ is an initial time.

$D(\cdot)$ and $N(\cdot,\cdot)$ can be decomposed into three parts as follows:

$$\begin{aligned} D(q) &= D_o(q) + D_u(q) + D_p(q), \\ N(q,\dot{q}) &= N_o(q,\dot{q}) + N_u(q,\dot{q}) + N_p(q,\dot{q}), \end{aligned} \tag{3.2}$$

where the subscripts $(\cdot)_o$, $(\cdot)_u$, $(\cdot)_p$ represent dynamics due to the nominal robot without payload, uncertainties of inertial parameters, and unknown payload, respectively.

In (3.2), $D_u(\cdot) \equiv 0$ and $N_u(\cdot,\cdot) \equiv 0$ when inertial parameters are exactly known; and $D_p(\cdot)\equiv 0$ and $N_p(\cdot,\cdot) \equiv 0$ if there is no payload. The things that are known to the designer are quantities represented by the subscript $(\cdot)_o$. Since the quantities with subscripts $(\cdot)_u$ and $(\cdot)_p$ are uncertainties of the robotic system, letting

$$\Delta D(q) := D_u(q) + D_p(q)$$

and

$$\Delta N(q,\dot{q}) := N_u(q,\dot{q}) + N_p(q,\dot{q})$$

enables us to write

$$\begin{aligned} D(q) &:= D_o(q) + \Delta D(q), \\ N(q,\dot{q}) &:= N_o(q,\dot{q}) + \Delta N(q,\dot{q}). \end{aligned} \tag{3.3}$$

The following assumptions are needed.

Assumption 3.1: These exists a constant $c\in(0,1)$ such that for all $q\in\mathbb{R}^n$

$$\|D^{-1}(q)\cdot\Delta D(q)\| \le 1-c. \tag{3.4}$$

Assumption 3.2: These exist constants $b_i\in\mathbb{R}_+$ (i = 1,2,3,4) such that for all $q\in\mathbb{R}^n$

$$b_1 \le \|D^{-1}(q)\| \le b_2, \tag{3.5}$$

$$\|\Delta N(q,\dot{q})\| \le b_3\|\dot{q}\|^2 + b_4. \tag{3.6}$$

Remark 3.1: For a robot arm with rotary joints, *Assumptions 3.1* and *3.2* are possible since q_i's in (3.4)-(3.6) appear only in the form of $\cos(q_i)$ and $\sin(q_i)$.

For the robot manipulator described by (3.1), a control method is presented as follows.
First, nonlinear state feedback

$$\tau(t) = N_o(q,\dot{q}) + D_o(q)\bar{u}(t) \tag{3.7}$$

is applied to (3.1) to obtain

$$\ddot{q}(t) = -D^{-1}(q)\Delta N(q,\dot{q}) + D^{-1}(q)D_o(q)\bar{u}(t) \tag{3.8}$$

Second, the following linear compensator is introduced to enhance tracking performance:

$$\bar{u}(t) = \ddot{q}_d(t)+K_v\dot{e}(t)+K_pe(t)+\gamma[\dot{e}(t)+\beta x_f(t)] - u(t), \tag{3.9}$$

where

$$e(t) := q_d(t) - q(t), \tag{3.10}$$

$$\dot{x}_f(t) = -\alpha x_f(t)+K_v\dot{e}(t)+K_pe(t), \tag{3.11}$$

with K_v and K_p positive-definite; α, β, and $\gamma \in \mathbb{R}_+$.
Substituting u(t) in (3.8) with (3.9) yields

$$\begin{aligned} \ddot{q}(t) = {} & D^{-1}(q)D_o(q)[\ddot{q}_d(t) + (K_v+\gamma I)\dot{e}(t) + K_pe(t) \\ & + \beta\gamma x_f(t)] - D^{-1}(q)D_o(q)u(t) - D^{-1}(q)\Delta N(q,\dot{q}). \end{aligned} \tag{3.12}$$

Since $D^{-1}(q)D_o(q) = I-D^{-1}(q)\cdot\Delta D(q)$ from (3.3), (3.12) can be rewritten as

$$\ddot{e} + (K_v+\gamma I)\dot{e} + K_pe + \beta\gamma x_f = (I+E)u + v, \tag{3.13}$$

where

$$E := -D^{-1}\cdot\Delta D, \tag{3.14}$$

$$v := D^{-1}\cdot\Delta N + D^{-1}\cdot\Delta D[\ddot{q}_d+ (K_v + \gamma I)\dot{e} + K_pe + \beta\gamma x_f]. \tag{3.15}$$

In (3.13)-(3.15), all the function arguments are omitted for simplicity.

Let $x(t) := [e^T(t)\ \dot{e}^T(t)\ x_f^T(t)]^T$ be the state of the trajectory tracking error for the precompensated robotic system. Then, (3.13) with (3.11) can be described as

$$\dot{x}(t) = Ax(t) + B\{[I+E(t,x)]u(t) + v(t,x)\}, \tag{3.16}$$

where

$$A := \begin{bmatrix} 0 & I & 0 \\ -K_p & -(K_v+\gamma I) & -\beta\gamma I \\ K_p & K_v & -\alpha I \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}. \tag{3.17}$$

Remark 3.2: E in (3.14) and v in (3.15) are functions of (t,x). Since $q(t) = q_d(t) - e(t)$,

$$E = -D^{-1}(q_d(t)-e(t))\cdot\Delta D(q_d(t)-e(t)) = E(t,e(t)) = E(t,x(t)).$$

Similarly, $v = v(t,x(t))$.

Lemma 3.1: If *Assumption 3.1* is satisfied,

$$1+\lambda_m\{\tfrac{1}{2}[E(t,x)+E^T(t,x)]\}\geq c \quad \forall(t,x) \in \mathbb{R}_+\times\mathbb{R}^n. \quad (3.18)$$

Proof: Omitted.

Lemma 3.2: Suppose that *Assumptions 3.1* and *3.2* are satisfied. Then, there exist a positive integer p, a constant $\theta\in\mathbb{R}_+^p$, and a function $\varphi\in C(\mathbb{R}_+\times\mathbb{R}^n, \mathbb{R}_+^p)$ such that

$$c^{-1}\|v(t,x)\| \leq \theta^T\varphi(t,x) =: \rho(t,x,\theta). \quad (3.19)$$

Proof: Let

$$\zeta(t,x):=\ddot{q}_d(t)+(K_v+\gamma I)\dot{e}(t)+K_p e(t)+\beta\gamma x_f(t). \quad (3.20)$$

Then, $\|v(t,x)\| \leq \|D^{-1}(q)\|\cdot\|\Delta N(q,\dot{q})\| + \|D^{-1}(q)\cdot\Delta D(q)\|\cdot\|\zeta(t,x)\| \leq b_2(b_3\|\dot{q}\|^2+b_4) + (1-c)\|\zeta(t,x)\|$, or,

$$c^{-1}\|v(t,x)\| \leq c^{-1}b_2b_3\cdot\|\dot{q}_d(t)-\dot{e}(t)\|^2 + c^{-1}(1-c)\cdot\|\zeta(t,x)\| + c^{-1}b_2b_4.$$

Letting p = 3, $\theta = \begin{bmatrix} c^{-1}b_2b_3 \\ c^{-1}(1-c) \\ c^{-1}b_2b_4 \end{bmatrix}$, and

$$\varphi(t,x) := \begin{bmatrix} \|\dot{q}_d(t)-\dot{e}(t)\|^2 \\ \|\zeta(t,x)\| \\ 1 \end{bmatrix} \quad (3.21)$$

implies (3.19). Note that each component of θ is *unknown*.■

Theorem 3.1: Let $K_v=2\Lambda$ and $K_p=\Lambda^2$, where $\Lambda:=\text{diag}\{\lambda_1,\lambda_2,\cdots,\lambda_n\}$ with $\lambda_i\in(0,\infty)$, $i=1,2,\cdots,n$. Suppose that *Assumptions 3.1* and *3.2* are satisfied. The state of the precompensated robotic system (3.16), with the adaptive control law given in (2.6) and (2.7), is ultimately bounded.

Proof: If $K_v=2\Lambda$ and $K_p=\Lambda^2$, it can be easily verified that matrix A in (3.17) is stable. By this fact and the results from *Lemmas 3.1* and *3.2*, the hypotheses of *Theorem 2.1* are satisfied. Application of *Theorem 2.1* to the system (3.16) completes the proof.■

Remark 3.3: The control input u(t) can be regarded as a stabilizing control which stabilizes the precompensated robotic system (3.16) in the presence of uncertainties.

Remark 3.4: Let $u(t)\equiv0$ in (3.9). Then, two types of the commonly-used controller form can be noted as follows. First, $\bar{u}(t)$ becomes the PD control when $\gamma\equiv0$. Second, taking the Laplace transform of (3.9) gives

$$\bar{U}(s) = s^2Q_d(s)+sK_vE(s)+K_pE(s)+\gamma[sE(s)+\beta X_f(s)],$$
$$sX_f(s) = -\alpha X_f(s)+sK_vE(s)+K_pE(s),$$

or, equivalently,

$$\bar{U}(s) = s^2Q_d(s) + [(K_v+\gamma I)s + (K_p+\beta\gamma K_v) + \beta\gamma(K_p-\alpha K_v)/(s+\alpha)]E(s) \quad (3.22)$$

Use $K_1:=K_v+\gamma I$, $K_2:=K_p+\beta\gamma K_v$, and $K_3:=\beta\gamma(K_p-\alpha K_v)$ to obtain

$$\bar{U}(s) = s^2Q_d(s) + [K_1s + K_2 + K_3/(s+\alpha)]E(s) \quad (3.23)$$

Then, $\bar{u}(t)$ in (3.23) with $\alpha\equiv0$ results in a PID controller for robotic manipulators.

Remark 3.5: Let $K_v:=\text{diag}\{k_{v1},k_{v2},\cdots,k_{vn}\}$ and $K_p:=\text{diag}\{k_{p1},k_{p2},\cdots,k_{pn}\}$. Put $\alpha = 0$ and $\beta = 1$ of the matrix A in (3.17). Then, the characteristic polynomial of A is $\det(sI-A) = \prod_{i=1}^{n}(s^2 + k_{vi}s + k_{pi})(s + \gamma)$. That is, an additional stable pole $-\gamma$ is introduced to the closed-loop poles which are allocated by the PD control for each joint.

4. Simulation Results

A computer simulation is conducted to evaluated the performance of the proposed adaptive control method (ACM) in the joint-variable space. The proposed method is compared with the computed torque method (CTM) under several different environments. The robot hand is operated with unknown payload, uncertain inertial parameters, computational time-delay, or suitable combinations of these three. It is assumed that the last three joints of PUMA 560 are frozen. The geometric and inertial parameters of the robot used in the simulation are based on [17] and [18].

The payload is a point mass of 5Kg attached at the end of the sixth joint of the robot. The uncertainties in inertial parameters are introduced by rounding off each value of the true inertial parameters in its second significant digit. When a computational time-delay is needed, its value is assumed to be 0.05-second, which coincides with the sampling period.

Each of the lower three joints is required to rotate 90 degrees along a preplanned 4-3-4 joint-interpolated trajectory[3] in one second ($q_d(0)$=[0° 45° 45°]^T, $q_d(0.2)$=[9° 36° 54°]^T, $q_d(0.8)$=[81° -36° 126°]^T, and $q_d(1)$=[90° -45° 135°]^T), and to retain the final position for another one second.

Table 1 shows naming convention of the two control methods, ACM and CTM, under several different environments.

Figures 1 and 2 show position errors of the first three joints for ACM/L and for CTM/L, respectively. Figure 3 shows parameter estimates in ACM/L.

When the payload is unknown, ACM gives superior tracking performance to CTM. In addition, ACM does not require larger input torque than CTM to follow the trajectory accurately.

Table 2 shows the position errors of the robot hand. Here, $P_e(t)$ denotes the position error at time t.

It can be observed that ACM gives much smaller tracking errors than CTM under uncertain environments. When there is a computational time-delay, the maximum error in ACM grows 2-3 times larger, but it is still much smaller than the error in CTM.

Control method (K_v=40I, K_p=400I)		Payload[1)]	Uncertainties in inertial parameters[2)]	Computational time delay[3)]
ACM (α=0.01, β=0.9999, γ=100)	CTM			
ACM/P*	CTM/P	A	P	A
ACM/PD+	CTM/PD	A	P	P
ACM/L*	CTM/L	P	A	A
ACM/LD+	CTM/LD	P	A	P
ACM/LP*	CTM/LP	P	P	A
ACM/LPD#	CTM/LPD	P	P	P

Table 1. Naming convention of the two control methods under several different environments (A=Absent, P= Present).

* (ε=0.01, λ=1, L=10^3I)

\+ (ε=1, λ=2.5, L=10^2I)

\# (ε=1, λ=1.5, L=10^2I)

1) point mass of 5Kg attached at the robot hand
2) nominal parameters obtained by rounding off the true parameters in their second significants
3) 0.05-second time-delay assumed to exist between the robotic system and the controller

Experiment	max $P_e(t)$	$P_e(1)$	$P_e(2)$
ACM/P	0.2275	0.1213	1.029×10^{-3}
ACM/PD	0.6029	0.1104	6.782×10^{-4}
ACM/L	1.085	0.2939	3.076×10^{-3}
ACM/LD	2.488	0.5011	5.029×10^{-4}
ACM/LP	1.012	0.2147	2.912×10^{-3}
ACM/LPD	2.140	0.4855	2.313×10^{-3}
CTM/P	8.549	6.482	7.209
CTM/PD	8.531	6.492	7.209
CTM/L	62.82	33.99	42.39
CTM/LD	63.09	33.99	42.39
CTM/LP	49.14	26.93	34.21
CTM/LPD	49.35	26.90	34.21

Table 2. Position errors of the robot hand [mm].

5. Conclusions

A trajectory tracking control method has been proposed for a robotic manipulator with revolute joints. By applying the adaptive control method of uncertain systems, a control method for the robotic manipulator is proposed to track a prescribed trajectory with good accuracy under uncertain environments. The proposed method is compared with the computed torque method.

The simulation results show that the proposed method gives good tracking performance in the presence of unknown payload and/or uncertainties in inertial parameters. The proposed method requires to estimate only a small number of parameters. This makes the controller relatively simple compared to other existing adaptive control methods. The proposed method still gives good performance. Therefore, the proposed method is expected to be a powerful method for high-performance robotic controller under uncertain environments.

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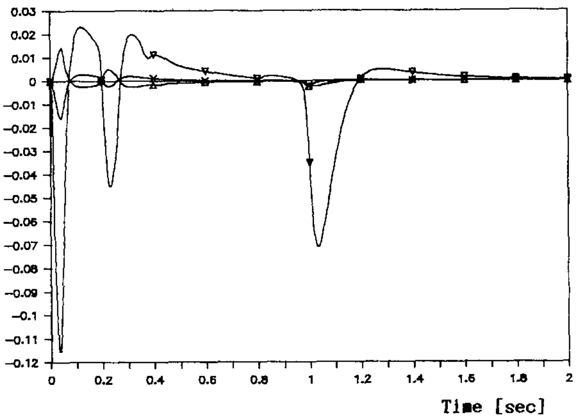

△: Joint 1 ×: Joint 2 ▽: Joint 3

Fig.1. Position errors for ACM/L [deg].

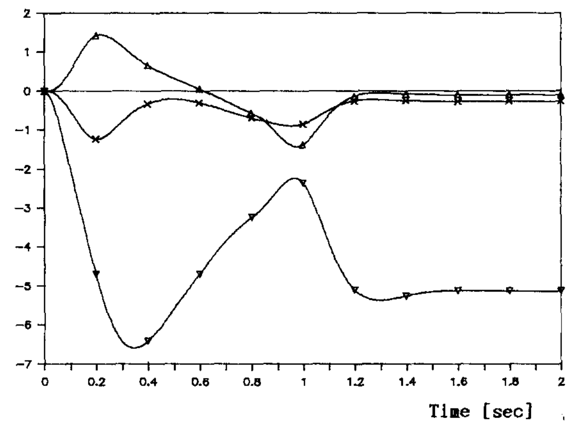

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Fig.2. Position errors for CTM/L [deg].

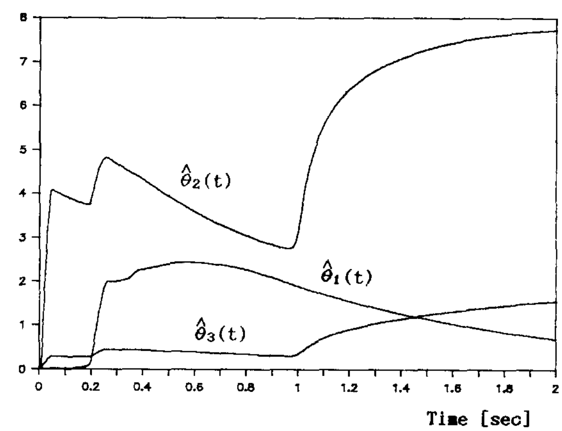

Fig.3. Parameter estimates for ACM/L.