'89 KACC 1989. 10. 27～28

Discrete-Time Learning Control for Robotic Manipulators

TATSUYA SUZUKI, MASANORI YASUE, SHIGERU OKUMA, YOSHIKI UCHIKAWA

Department of Electronic Mechanical Engineering,
School of Engineering, Nagoya University
Furo-cho, Chikusa-ku, Nagoya 464, Japan
Tel. (052) 781-5111 (ext. 2793)

ABSTRACT – A discrete-time learning control for robotic manipulators is studied using its pulse transfer function. Firstly, discrete-time learning stability condition which is applicable to single-input two-outputs systems is derived. Secondly, stability of learning algorithm with position signal is studied. In this case, when sampling period is small, the algorithm is not stable because of an unstable zero of the system. Thirdly, stability of algorithm with position and velocity signals is studied. In this case, we can stabilize the learning control system which is unstable in learning with only position signal. Finally, simulation results on the trajectory control of robotic manipulators using the discrete-time learning control are shown. This simulation results agree well with the analytical ones.

1. INTRODUCTION

A learning control based on repetitive operations of robotic manipulators is one of the most promising methods to realize high speed and high precision control for robotic manipulators[1]-[11]. The learning control system is a discrete-time system essentially. Therefore, Ishihara et.al proposed a design method for the discrete-time learning control system[12]. But this method is based on the inpulse response of the system, and they did not study the learning control system using its pulse transfer function.

In this paper, we assume the control system of each joint of the robotic manipulator to be a discrete-time linear system. Then, to discuss the learning stability, we derive the learning stability condition of the discrete-time learning control system, which is applicable not only to the system of which input number is the same as output number but also to the single-input two-outputs system. Based on the derived condition, we study the learning stability of the following two learning algorithms in the robotic manipulator control system. One is the algorithm with the position signal, and the other is the one with the position and velocity signals.

All continuous-time systems with pole excess being larger than two will always give discrete-time systems with unstable zeros provided that the sampling period is small[13]. Although this is a serious problem for the learning algorithm using only the position signal, we can stabilize the learning control systems by adding the velocity learning.

Finally, we examine the learning stability for the non-linear robotic manipulators by numerical experiments.

2. STABILITY CONDITION FOR DISCRETE-TIME LEARNING CONTROL SYSTERMS

2. 1 Discrete-time learning control systems

We consider the following single-input r-outputs continuous system which is controllable and observable:

$$\dot{x}(t) = A_c \cdot x(t) + b_c \cdot u(t) \qquad (1-a)$$

$$y(t) = C \cdot x(t), \qquad (1-b)$$

where

$$A_c : n \times n \ , \ b_c : n \times 1 \ , \ C : r \times n$$
$$x(t) : n \times 1 \ , \ y(t) : r \times 1 \ , \ u(t) : 1 \times 1.$$

By sampling and zero order holding the system, the following equation is obtained:

$$x(i+1) = A \cdot x(i) + b \cdot u(i) \qquad (2-a)$$

$$y(i) = C \cdot x(i), \qquad (2-b)$$

where

$$A = exp(A_c \cdot T) \ : n \times n \ , \ b = \int_0^T exp(A_c \cdot t) dt \ : n \times 1,$$

T : sampling period.

From (2), the pulse transfer function of the system $G(z)$ is given by

$$G(z) = C \cdot (z \cdot I - A)^{-1} \cdot b \quad : r \times 1 \qquad (3)$$

$$Y(z) = G(z) \cdot U(z), \qquad (4)$$

where

$Y(z)$: z-transform of output signal
$U(z)$: z-transform of input signal.

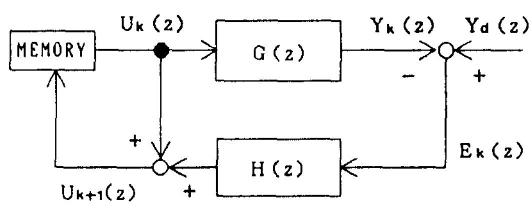

Fig. 1 Block diagram of learning control systems

Now, we consider the learning control systems shown in Fig. 1. $H(z)$ denotes the learning operetor. The input signal for $(k+1)$ th trial $U_{k+1}(z)$ is given as follows:

$$U_{k+1}(z) = U_k(z) + H(z) \cdot E_k(z), \quad H(z): 1 \times r \tag{5}$$

$$E_k(z) = Y_d(z) - Y_k(z), \tag{6}$$

where subscripts $k, k+1$ denote the trial numbers, $Y_d(z)$ the z-transform of the desired output, and $E_k(z)$ the z-transform of the error signal of the k th trial. Then, the pulse error transition function matrix $S(z)$ is given by

$$\begin{aligned} E_{k+1}(z) &= S(z) \cdot E_k(z) \\ &= (I - G(z) \cdot H(z)) \cdot E_k(z). \end{aligned} \tag{7}$$

2. 2 Learning stability condition

In this section, we show the learning stability condition, that is, the condition for the output error to converge to zero in a sense of L_2 norm. The convergence of the L_2 norm of the output error is expressed by the following equation:

$$\sum_{i=0}^{N-1} e_{k+1}^T(iT) \cdot e_{k+1}(iT) < \sum_{i=0}^{N-1} e_k^T(iT) \cdot e_k(iT). \tag{8}$$

From (8) and the Parseval's equation, we obtain the learning stability condition as follows:

$$\sup_{0 \leq \omega \leq \omega_N} \sqrt{\lambda_{max}\{S^H(e^{j\omega T}) \cdot S(e^{j\omega T})\}} < 1, \tag{9}$$

where

ω_N: Nyquist frequency

H denotes the conjugate transpose.

The left side of (9) expresses the L_2 gain of $S(z)$. So, the learning stability condition is that the L_2 gain of $S(z)$ is less than one. The L_2 gain of the system is equal to the H_∞ norm of its transfer function[14]. Then, (9) is rewritten as:

$$\| S(z) \|_\infty < 1, \tag{10}$$

where $\| \cdot \|_\infty$ denotes H_∞ norm.

2. 3 Learning stability condition for the single-input two-outputs systems

If we regard the applying voltage to a direct current motor (D.C.M.) as an input signal and the position and velocity as output signals for the control system of each joint of the robotic manipulator, we obtain a single-input two-outputs system. In this case, $G(z)$ is 2×1 vector, $H(z)$ is 1×2 vector and $S(z) = I - G(z) \cdot H(z)$ is 2×2 matrix, respectively. Because $G(z) \cdot H(z)$ is singular, $I - G(z) \cdot H(z)$ should have an eigen value 1, even if the learning control system is stable. The learning stability condition (10) is not applicable to the single-input two-outputs systems. Then, we derive a condition which is applicable to the single-input two-outputs systems as follows.

The eigen values $\lambda_1(z)$, $\lambda_2(z)$ and eigen vectors $V_1(z)$, $V_2(z)$ of $S(z)$ are given as follows:

$$\begin{aligned} \lambda_1(z) &= 1 - H(z) \cdot G(z) & V_1(z) &= G(z) \\ \lambda_2(z) &= 1 & V_2(z) &\perp H^T(z). \end{aligned} \tag{11}$$

The desired output signal is given by

$$Y_d(z) = G(z) \cdot U_d(z). \tag{12}$$

Then, the output error of the k th trial is given by

$$E_k(z) = Y_d(z) - Y_k(z) = G(z) \cdot (U_d(z) - U_k(z)). \tag{13}$$

In (13), $U_d(z)$ and $U_k(z)$ are scalar valued functions, so $E_k(z)$ is parallel to $G(z)$. As the direction of vectors $E_k(z)$ and $E_{k+1}(z)$ are restricted to that of $V_1(z)$, we need not consider the convergency along $V_2(z)$. The equation(7) is rewritten as:

$$\begin{aligned} E_{k+1}(z) &= S(z) \cdot E_k(z) \\ &= \lambda_1(z) \cdot E_k(z). \end{aligned} \tag{14}$$

As the result, the learning stability condition is obtained as follows:

$$\| \lambda_1(z) \|_\infty = \sup_{0 \leq \omega \leq \omega_N} |1 - H(e^{j\omega T}) \cdot G(e^{j\omega T})| < 1. \tag{15}$$

If the output number is one, (15) is equivalent to (9).

This idea can be expanded to the systems of which input number is less than output number[15].

3. LEARNING WITH THE POSITION SIGNAL

Now we consider a case that the controller of each joint is designed independently (including the learning controller) and that the learning algorithm is realized with only the position signal. The control system of each joint is regarded as a single-input single-output (SISO) system. If the sampling period is small, the pulse transfer function of the system often has unstable zeros. Especially, when the pole excess of its continuous-time transfer function is larger than two, the corresponding discrete-time pulse transfer function has unstable zeros[13]. We examine relationships between the unstable zeros of the system and the learning stability in the following section.

3. 1 Relationship between the unstable zero and the learning stability in the SISO systems

We assume that $G(z)$ and $H(z)$ have stable poles, and $G(z)$ has an unstable zero z_0 in a region $|z| \geq 1$ on a complex z-plane($G(z)$ and $H(z)$ are scalar valued functions). If $G(z)$

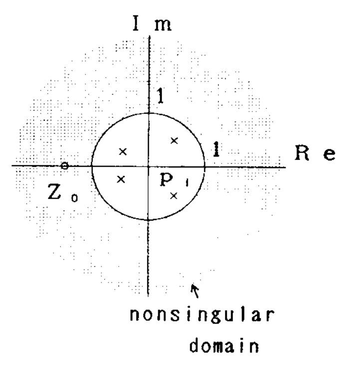

Fig. 2 p_i and z_0 on z-plane

and $H(z)$ have stable poles, $\lambda_1(z)$ in (15) has also stable poles p_i ($|p_i| < 1$), and $\lambda_1(z)$ is nonsingular in the region $|z| \geq 1$ (Fig. 2). For the unstable zero z_0 ($|z_0| \geq 1$), we obtain the following equation:

$$\lambda_1(z_0) = 1 - H(z_0) \cdot G(z_0) = 1 \quad (G(z_0) = 0) \qquad (16)$$

By the maximum modulus principle[16], the maximum value of $|\lambda_1(z)|$ in the region $|z| \geq 1$ is given on the unit circle $|z| = 1$. The left side of (15) expresses the maximum value of $|\lambda_1(z)|$ on $|z| = 1$, and we obtain the following equation:

$$\sup_{0 \leq \omega \leq \omega_N} |\lambda_1(e^{j\omega T})| \geq |\lambda_1(z_0)| = 1. \qquad (17)$$

From (17), if $G(z)$ has an unstable zero, the learning stability condition (15) is never satisfied for all $H(z)$.

3. 2 An example of the unstable learning caused by the unstable zero

If the current minor loop in the control system of each joint operates ideally, the pole excess of the continuous-time transfer function from the applied voltage to the joint position should be two. The corresponding discrete-time pulse transfer function does not have unstable zeros. If the motor has a large inductance and the electrical time constant is large, the pole excess of its transfer function is three. Then, its pulse transfer function should have an unstable zero with a small sampling period.

Figure 3 shows a block diagram of the control system of each joint. The feedback loop using the position θ and the velocity $\dot{\theta}$ is a conventional one. The feedforward voltage $v_k(i)$ is modified by the learning algorithm. This learning method is equivalent to modifying the input signal $u_k(i)$ as follows:

$$u_k(i) = K_P \cdot \theta_d(i) + K_V \cdot \dot{\theta}_d(i) + v_k(i). \qquad (18)$$

z-transform of $u_k(i)$ is $U_k(z)$ in Fig. 1.

Here we approximate the dynamics of each joint of the robotic manipulator by the one of the D.C.M. of each joint. In other words, we assume each joint of the robotic manipulator to be a single-input single-output linear system. The constants of the D.C.M. of each joint are listed on Table 2. We do not neglect the armature inductance and set the value of the inertia J at $3 \cdot J_m$ to take the arm inertia into

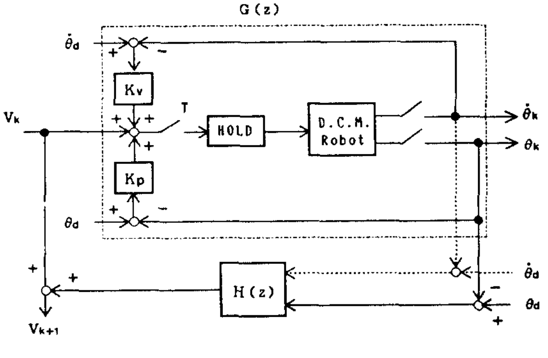

Fig. 3 Block diagram of control system

account. The transfer function from the applied voltage to the joint position is given as follows:

$$G_c(s) = \frac{b_c}{s \cdot (s - a_{c1}) \cdot (s - a_{c2})}, \qquad (19)$$

where $b_c = 2.70 \times 10^3$, $a_{c1} = -54.2$ and $a_{c2} = -156$. The corresponding pulse transfer function of the system including the feedback loop is given as follows:

$$G(z) = \frac{b_0 \cdot (z - b_1) \cdot (z - b_2)}{(z - c_1) \cdot (z - c_2) \cdot (z - c_3)}, \qquad (20)$$

where b_0, b_1, b_2, c_1, c_2 and c_3 are listed on Table 3.

It is an important problem how to decide the learning operator $H(z)$. If we use the inverse system of $G(z)$, we will have a high convergence rate[8] (because such $H(z)$ reduce the absolute value in (15)). But, if $G(z)$ has an unstable zero like (20), we can not use its inverse system. So, instead of (19), we consider a second order system of which the armature inductance is neglected, and calculate its pulse transfer function $G'(z)$ including the feedback loop like (20). Then, we use the inverse system of $G'(z)$ as the learning operator $H(z)$. $H(z)$ is given as follows:

$$H(z) = \frac{(z - d_1) \cdot (z - d_2)}{e_0 \cdot (z - e_1)}, \qquad (21)$$

where d_1, d_2, e_0 and e_1 are listed on Table 3, and the value of the inertia in $G'(z)$ is set at J_m (Note that there exist modeling errors).

Figure 4(a) shows numerical results of the learning stability with $G(z)$ and $H(z)$ expressed in (20) and (21), respectively. The vertical axis expresses the absolute value in (15), and the horizontal axis frequency. When the sampling period T is small ($2(ms)$), the learning stability condition (15) is not satisfied in high frequency domain. This result corresponds to the analytical result in 3.1.

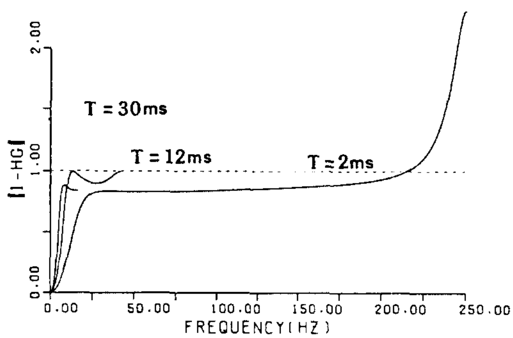

Fig. 4 $|1 - H(e^{j\omega T}) \cdot G(e^{j\omega T})|$ — ω plots

(a) learning with position

4. STABILIZATION OF THE LEARNING ALGORITHM BY ADDING THE VELOCITY LEARNING

4. 1 Learning stability of the single-input two-outputs ststems

Adding the velocity learning leads to an increase of the output number of the system to two. Here, we assume that two-outputs system does not have common unstable zeros. This means that system does not have unstable invariant zeros, because the system (2) is controllable and observable for almost any sampling period T. Then, $G(z)$ and $H(z)$ are given as follows:

$$G(z) = \frac{g_n(z)}{d_g(z)} \cdot \begin{pmatrix} g_1(z) \\ g_2(z) \end{pmatrix} \tag{22}$$

$$H(z) = \frac{(h_1(z)\ h_2(z))}{d_h(z)}, \tag{23}$$

where $g_n(z)$ is a stable polynominal, and $g_1(z)$ and $g_2(z)$ are relatively coprime. When $G(z)$ like (22) is given, we can acquire $H(z)$ which satisfies the learning stability condition (15).

By substituting (22) and (23) into (11), we obtain the following equation:

$$\begin{aligned} \lambda_1(z) &= 1 - H(z) \cdot G(z) \\ &= 1 - g_n(z) \cdot (h_1(z) \cdot g_1(z) \\ &\quad + h_2(z) \cdot g_2(z))/(d_h(z) \cdot d_g(z)). \end{aligned} \tag{24}$$

When $g_1(z)$ and $g_2(z)$ are relatively coprime, there exist $h_1(z)$ and $h_2(z)$ which satisfy the following equation for any $a(z)$[17]:

$$h_1(z) \cdot g_1(z) + h_2(z) \cdot g_2(z) = a(z). \tag{25}$$

From (25), (24) is rewritten as:

$$\lambda_1(z) = 1 - \frac{g_n(z) \cdot a(z)}{d_h(z) \cdot d_g(z)}. \tag{26}$$

If we choose $a(z)$ in (25) to be a stable polinominal, the second term of the right side in (26) does not have unstable zeros. In other words, we can realize the stable learning by choosing $h_1(z)$ and $h_2(z)$ to stabilize $a(z)$ in (25). Thus, in the case of the learning algorithm with the position and velocity signals, we can place the zeros of $H(z) \cdot G(z)$ anywhere on z-plane. This is the major difference comparing with the learning algorithm with only the position signal.

4. 2 Stabilization of the learning algorithm by adding the velocity learning

When we regard the system in Fig. 3 as two-outputs system, $G(z)$ is given by

$$G(z) = \begin{pmatrix} G_1(z) \\ G_2(z) \end{pmatrix},$$

where

$G_1(z)$: same as (20)

$$G_2(z) = \frac{f_0 \cdot (z - f_1) \cdot (z - 1)}{(z - c_1) \cdot (z - c_2) \cdot (z - c_3)}. \tag{27}$$

and f_0, f_1 are listed on Table 3. Like(21), based on the model of the D.C.M. (A_m B_m C) of which the armature inductance is neglectd, we determine $H(z)$ as follows:

$$H(z) = B_m^{\dagger} \cdot (z \cdot I - A_m) \cdot C^{-1}, \tag{28}$$

where

$$B_m^{\dagger} = (B_m^T \cdot B_m)^{-1} \cdot B_m^T.$$

Generally, there exist various inverse systems for single-input two-outputs systems. Among these inverse systems, $H(z)$ in (28) is easy to construct the learning algorithm. $H(z)$ in (28) is rewritten as:

$$H(z) = (g_0 \cdot (z - g_1)\ \ g_2 \cdot (z - g_3)), \tag{29}$$

where g_0, g_1, g_2 and g_3 are listed on Table 3. $H(z)$ in (29) stabilizes $a(z)$ in (26). Figure 4(b) shows the numerical results of the learning stability with $G(z)$ and $H(z)$ expressed in (27) and (29), respectively. In this case, the learning stability condition (15) is satisfied for any sampling period. In the case of the learning with the position and velocity signals, we can realize stable learning unless $G(z)$ has a common unstable zero. Moreover, as the pole excess of the transfer function from the applied voltage to the joint velocity is two, $G_2(z)$ does not have an unstable zero. Then, $G(z)$ does not have a common unstable zero (it may have a common stable zero). After all, we can realize the stable learning by using the position and velocity signals.

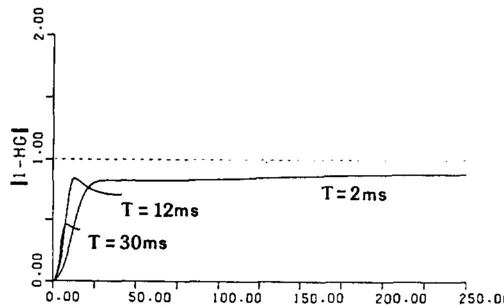

Fig. 4 $|1 - H(e^{j\omega T}) \cdot G(e^{j\omega T})|$ — ω plots

(b) learning with position and velocity

5. SIMULATION RESULTS FOR ROBOTIC MANIPULATORS

The actual robotic manipulator has nonlinear dynamics. In this chapter, we examine the effect of the nonlinear term, such as the centrifugal force, the Coriolis force and so on, to the learning stability. Figure 6 shows the simulation results about the learning stability for the robotic manipulator in Fig. 5. The constants of the robotic manipulator are listed on Table 1. In Fig. 6, the vertical axis expresses the integral of the position error of the link1, and the horizontal axis the trial number. The desired trajectory is a circle with center $(0.2\ ,\ 0.2)(m)$, and radius $0.1(m)$ on the cartesian coordinates.

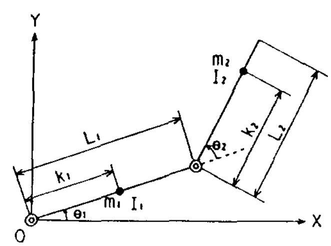

Fig. 5 Robotic manipulator of 2 d.o.f.

5. 1 Learning with the position signal

In the case of the learning with the position signal, the position error diverges when the sampling period T is small(Fig. 6(a)). The learning stability condition is not satisfied in high frequency domain as mentioned in chapter 3. Although the desired trajectory does not have any high frequency component, the error signal begins to contain high frequency components as the learning goes on. So, the learning algorithm diverges.

5. 2 Learning with the position and velocity signals

In the case of the learning with the position and velocity signals, the position error converges for any sampling period (Fig. 6(b)). The error, however, does not converge to 0 but to a certain value. This is explained as follows.

The desired velocity trajectory is made from the difference of the desired position trajectory. The desired velocity trajectory is not always accurate, in other words, $U_d(z)$ satisfying $Y_d(z) = G(z) \cdot U_d(z)$ does not always exist. Especially, when the sampling period is large, the inaccuracy is enlarged. In this case, the error converges to a large value. This happens often in systems of which input number is less than output number[15].

As shown in this chapter, when the sampling period is small, we had better use the algorithm with the position and velocity signals for the stability, and when the sampling period is large, we had better use the learning algorithm with the position signal for the convergency. These simulation results agree well with the analytical ones shown in chapter 3 and 4, which is based on the linear approximation.

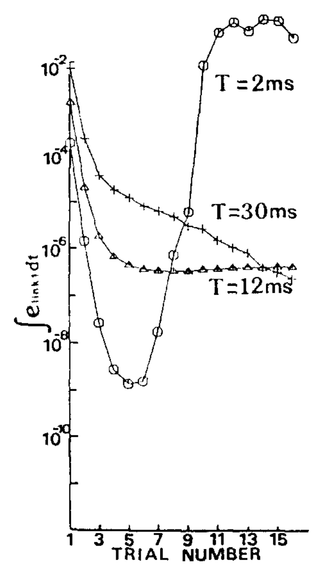

Fig. 6 Convergence of position error of link1

(a) learning with position

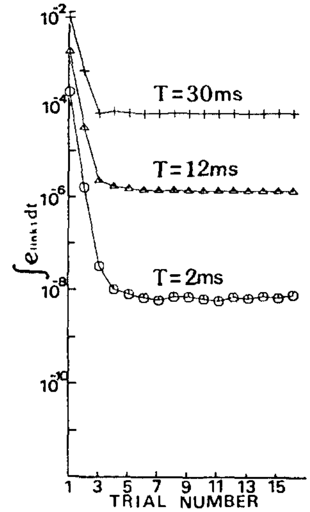

Fig. 6 Convergence of position error of link1

(b) learning with position and velocity

6. CONCLUSION

In this paper, we have shown the learning stability condition which is applicable to the single-input two-outputs systems. And, we have studied the learning stability of the following two algorithms in the robotic manipulator control system. One is the algorithm with the position signal, and the other is the one with the position and velocity signals. When the electric time constant of the D.C.M. is not negligible, the learning algorithm is not stable for small sampling period in the learning algorithm with the position signal. This is caused by the unstable zero of its pulse transfer function. In the learning algorithm with the position and velocity signals, the learning algorithm is stable for any sampling period. This is because we can place the zeros of $H(z) \cdot G(z)$ anywhere on z-plane. But, when the sampling period is large, the output error does not converge to 0 but to a certain value. Therefore, when the sampling period is small, we had better use the algorithm with the position and velocity signals for the stability, and when the sampling period is large, we had better use the learning algorithm with the position signal for the convergency.

Table 1 Constants of robotic manipulator

Link No.	Mass m(kg)	Inertia I(kg·m²)	Link length L(m)	Link center of gravity k(m)
1	9.5	4.30×10^{-2}	0.25	0.20
2	5.0	6.10×10^{-3}	0.16	0.14

Table 2 Constants of D.C.M.

Link No.	Inertia J_n ($kg\cdot m^2$)	Damping coefficient D_m ($N\cdot m\cdot s/rad$)	Electro-motive force coefficient K_E ($V\cdot s/rad$)	
1	4.61×10^{-5}	3.84×10^{-3}	0.188	
2	2.65×10^{-5}	1.39×10^{-3}	0.153	
Link No.	**Torque constant K_T ($N\cdot m/A$)**	**Armature resistance R_a (Ω)**	**Armature inductance L_a (H)**	**Gear ratio N**
1	0.188	10.0	6.3×10^{-3}	80
2	0.153	17.0	10.0×10^{-3}	60

Table 3 Parameters of system and learning operator

T	b_0	b_1	b_2			
2ms	1.86×10^{-6}	-2.05	-0.103			
12ms	9.19×10^{-6}	-0.981	-0.00425			
30ms	4.68×10^{-4}	-0.642	-4.81×10^{4}			
T	**c_1**	**c_2**	**c_3**			
2ms	0.865+0.149j	0.865-0.149j	0.0968			
12ms	0.546+0.449j	0.546-0.449j	0.0156			
30ms	0.279+0.511j	0.279-0.511j	0.00378			
T	**d_1**	**d_2**	**e_0**	**e_1**		
2ms	0.475	0.807	9.19×10^{-6}	-0.899		
12ms	0.0114	0.276	2.12×10^{-4}	-0.536		
30ms	1.61×10^{-5}	0.040	7.58×10^{-4}	-0.250		
T	**f_0**	**f_1**	**g_0**	**g_1**	**g_2**	**g_3**
2ms	0.00230	-0.363	0.121	4.81×10^{4}	115	-0.335
12ms	0.0146	-0.0409	0.287	7.64×10^{3}	36.8	-0.247
30ms	0.0254	-0.00888	0.759	1.33×10^{3}	31.6	-0.192

REFERENCE

[1] M. Uchiyama., "Formation of High Speed Motion Pattern of Mechanical Arm by Trial.", *Trans. Soc. Instr. Contr. Eng., 1978, Vol.19, No.5, pp.706-712*

[2] S. Arimoto, S. Kawamura and F. Miyazaki, "Bettering Operation of Robots by Learning.", *J. Robot. Syst., 1984, Vol.1, No.2, pp.123-140*

[3] S. Arimoto, S. Kawamura and F. Miyazaki, "Learning Control Theory for Dynamical Syatems.", *Proc. of 24th C.D.C., December, 1985*

[4] S. Kawamura, F. Miyazaki and S. Arimoto, "Applications of Learning Method for Dynamic Control of Robot Manipulators.", *Proc. of 24th C.D.C., December,1985*

[5] T. Mita and E. Kato "Iterative Control and its Application to Motion Control of Robot Arm–A Direct Approach to Servo Problems.", *Proc. of 24th C.D.C., December,1985*

[6] K. Furuta and M. Yamakita, "A Design of Learning Control System for Linear Multivariable Systems.", *Trans. Soc. Instr. Contr. Eng., 1986, Vol.22, No.12, pp.1248-1255*

[7] S. Kawamura, F. Miyazaki and S. Arimoto, "System Theoretic Study on Learning Control Method.", *Trans. Soc. Instr. Contr. Eng., 1985, Vol.21, No.5, pp.445-450*

[8] M. Takeda, S. Kawamura and S. Arimoto et.al., "Learning Control for Robots based on Linear Approximation of Inverse System.", *J. Robot. Soc. Japan, 1987, Vol.5, No.2, pp.139-149*

[9] M. Togai, O. Yamano, "Analysis and Design of an Optimal Learning Control Scheme for Industrial Robots: A Discrete Time Approach.", *Proc. of 24th C.D.C., December,1985*

[10] C. G. Atkeson, and J. Mcintyre, "Robot Trajectory Learning through Practice.", *Proc. of IEEE Conf. Robot. Automa.,1986*

[11] T. Suzuki, S. Okuma and Y. Uchikawa et.al., "Trajectory Control of Robot Arms with Current Learning.", *J. Robot. Soc. Japan, 1989, Vol.7, No.3, pp.185-194*

[12] T. Ishihara, K. Abe and H. Takeda, "A Design of Discrete-Time Repetitive Control Systems.", *Trans. Soc. Instr. Contr. Eng., 1986, Vol.22, No.1, pp.43-49*

[13] K. J. Astrom, P. Hagander and J. Sternby, "Zeros of Sampled Systems.", *Automatica, 1984, Vol.20, No.1, pp.31-38*

[14] B. A. Francis and J. C. Doyle, "Linear Control Theory with an H_∞ Optimality Criterion.", *S.I.A.M., J. Control and Optimization, 1987, Vol.25, No.4, pp.815-844*

[15] T. Suzuki, S. Okuma and Y. Uchikawa, "A Learning Stability Condition for Linear Systems of which Input Number is less than Output Number.", *Proc. of 28th S.I.C.E., July, 1989*

[16] P. L. Duren, "Theory of H_p Spaces.", *Academic Press, New York and Lond, 1970*

[17] W. A. Wolovich, "Linear Multivariable Systems.", *Springer-Verlag, 1974*