

# ANALYSIS OF THREE PHASE INDUCTION MOTOR FED BY P.W.M. BOXES ALGORITHM

Dr. M.SH. NASR

Mr. E.M. ABDUL-BAKI

ELECTRICAL AND ELECTRONICS DEPARTMENT  
MILITARY ENG. COLLEGE  
P.O.BOX (478) BAGHDAD-IRAQ

## ABSTRACT

*This paper describes first Boxes algorithm as a good method to generate the pulse patterns for a pulse width modulated inverter which has a good characteristics and simple to excute. Second we present the motor analysis fed by P.W.M. inverter in steady-state operation. In this analysis we improve the MRF ( Multiple Reference Frames ) to can be easy apply to analyse all the induction motor parameters. Finally we presented all the results obtained for a 3-phase induction motor.*

## I- INTRODUCTION

Power electronic inverters that utilize pulse width modulation (P.W.M.) can control frequency and voltage maintain low harmonics content, and can be compact and high-weight, while the standard square, the particular harmonic elimination (PHE) eliminates undesirable lower order harmonics, which cannot be eliminated in three-phase system. This with respect to inverter operation. In other hand, the unequally load distribution is the main reason to generate un balanced three-phase voltages in the grid. Therefore it is very important to analyse the motor and all its parameters and steady its performance.[7]

In P.W.M. inverter applications, the inverter and the P.W.M. strategy should be properly chosen for a particular application. The inverter should be efficient and preferably less complex. In UPS systems where the output frequency is fixed and the voltage varies over a limited range, the voltage harmonic spectrum of the output voltage is an important consideration. In variable-speed AC motor drives, both voltages and frequency vary over a wide range. Because the machine impedance change with frequency, it is the current harmonic spectrum which is an important consideration in P.W.M. inverter drives.

In this paper we presented the Boxes algorithm as a method to generate firing pulses digitally to control the DC/AC inverter operation, second we used the MRF analysis method as a good tool to analyse the motor operation in steady-state fed by P.W.M. inverter. Naturally we presented the analytical obtained results.

## II- BOXES ALGORITHM.

As known in P.W.M., the fundamental and harmonic component in the output voltage are controlled by the proper choice of the pulse pattern within each half cycle in such a manner that produce the minimum harmonic losses in the AC load. Therefore, it is important to choose a modulation strategy would keep the harmonic losses low. One approach to reducing the harmonic losses would be to increase the number of pulses at the inverter output whereby the order of the harmonics is increase. The higher order harmonics are filtered by the motor leakage reactance. However, the increased number of pulses necessitates a higher commutation rate results in increasing commutation losses.[3]

The reduction in the machine losses brought about by reduction in harmonic currents can be offset by increasing the inverter losses. The overall system efficiency can decrease rather than improve. The advantages of a P.W.M. scheme depend, therefore, on the harmonic content generated, as well as on the commutation losses produced.

The output voltage wave shapes produced on P.W.M. inverters are determined by the choice of the carrier and modulating signal and the frequency ratio.

As known the ideal output of a P.W.M. inverter is the reference signal, the P.W.M. provides a series of rectangular pulses that have a constant amplitude and variable width according to the variation of the reference (output) wave.

The BOXES algorithm concept is to divide the one period of the reference wave (we considered a 1 amplitude reference wave) into K segments ( $K=P_c/F_r$ ), and the pulse width of each segment is adjustable in such a manner that the area of the rectangular pulse is equal to that of the corresponding reference wave segment. By this way the rectangular pulse (BOX) period is variable and proportional to the output voltage and frequency.

Taking the above consideration into account and Fig.(1) as a reference, the mathematical expression for Nth reference wave segments is given by :

$$AN = \int_{(N-1)*\alpha}^{N*\alpha} X * \text{SIN} (Wt) d(wt) \dots\dots (1)$$

Where :

- N = 1, 2, ..... , K
- K is the P.M.R.
- X is the modulation index.
- W is the output frequency.
- $\alpha = 2\pi/K$

The solution of equation (1) is :

$$AN = X * (\text{COS}[(N-1)*\alpha] - \text{COS}(N*\alpha)) \dots\dots (2)$$

Whenever the corresponding segment area is calculated we can represent this area as a pulse, which has ONES and ZEROS, so that its area is equal to the calculated one.

Boxes algorithm has the following advantages:[4]

1. Very small ROM is necessary to engrave the boxes ONES and ZEROS.
2. Can be operate at very low voltage and frequency output included free operation range. This facilitate the digital motor starting.

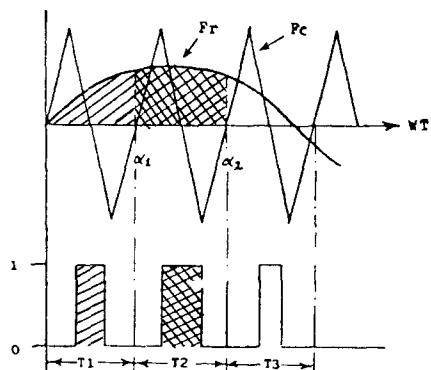


Fig.(1) Boxes generation principle.

Also we present the following notifications as the characteristics of this new algorithm :

1. Each Box has approximately one milli sec. duration and (128) points.
2. The full Box (128 ONES) represents the maximum area of the segment.
3. It exists 128 types of Boxes, the first one have 0 ONES and (128) ZEROS, while the last one have (128) ONES and (0) ZEROS.

These types of Boxes are engraved in the microprocessor memory.

4. When the No. of the correct Box is selected, its contents are readed by a counter. The readed ONES and ZEROS of the Box are available to the inverter control device (ONE is closed, ZERO is opened).

Figure (2) represents the obtained phase voltage and its harmonics analysis using boxes algorithm. Also it is important the comparison of the BOXES algorithm with the classical algorithm (here we selected the natural sampling technique). In figure (3) represents the 3rd. harmonic spectrum for variable modulation index for Boxes and natural sampling algorithms where the advantages are evedent.

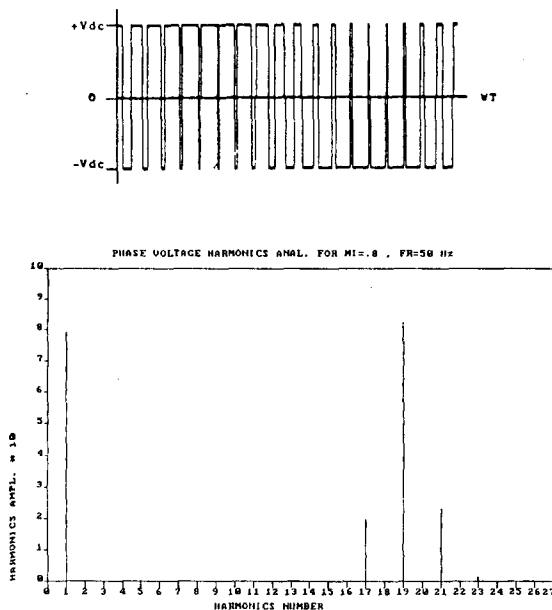


Fig.(2) Phase voltage and its harmonic analysis

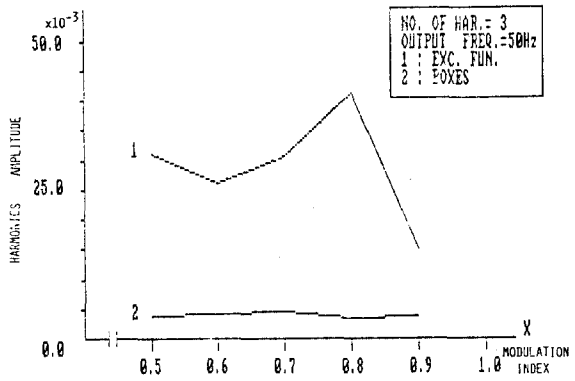


Fig. (3) Comparison of 3rd. harmonics.

III- MULTIPLE REFERENCE FRAMES THEORY.

When a nonsinusoidal voltage is used to supply an induction machine, the stator phase voltages  $V_{as}$ ,  $V_{bs}$  and  $V_{cs}$  may be related to  $ds$ - $qs$  voltages in a reference frame fixed in the stator as: [1]

$$V_{qs}^s = V_{as} \dots\dots\dots (3)$$

$$V_{ds}^s = (1/\sqrt{3}) * (-V_{bs} + V_{cs}) \dots\dots (4)$$

If the phase voltages are periodic in nature, each may be expressed in a Fourier series expansion; therefore

$$V_{qs}^s = \sum_{k=1}^{\infty} [V_{k\alpha} \cos(k\omega t) + V_{k\beta} \sin(k\omega t)] \dots (5)$$

$$V_{ds}^s = \sum_{k=1}^{\infty} [V_{k\alpha} \cos(k\omega t) + V_{k\beta} \sin(k\omega t)] \dots (6)$$

In the above equations  $\omega$  is the electrical angular velocity which is generally selected corresponding to the fundamental frequency component.  $V_{\alpha}$  and  $V_{\beta}$  denote, respectively the coefficients of the cosine and sine terms.

The applied voltage in the synchronously rotation reference frame may be expressed as follow ,

$$V_{qs}^s = V_{qs}^s \cos(\omega t) - V_{ds}^s \sin(\omega t) \dots (7)$$

$$V_{ds}^s = V_{ds}^s \sin(\omega t) + V_{qs}^s \cos(\omega t) \dots (8)$$

The superscript  $s$  denotes the synchronously rotating reference frame. By substituting (5) and (6) into (7) and (8), a long expression for  $V_{qs}$  and  $V_{ds}$  are obtained which are not given here, these expressions contain a constant term and sinusoidally varying terms which form a series of balanced set of 2-phase voltages. This means that the balanced sets will appear in the synchronously rotating reference frame regardless of the form of phase voltages.

Now, each of the balanced sets appearing in the synchronously rotating reference frame can be separately transformed to reference wherein the applied voltages are constant and hence D.C. circuit theory could be used in the analysis. The calculated quantities can then be transformed from each of these reference frames back to the synchronously rotating reference frame and combined to give the constant-speed performance of the machine.

This application of Multiple Reference Frames permits a rigorous steady-state performance analysis with nonsinusoidal stator voltages of any periodic wave form without restoring to the concept of phasor and complex impedances or the method of symmetrical component.

If each of the balanced sets are transformed to the appropriate reference frame, the following expressions are obtained for the reference frame voltages :

$$V_{qs}^{+ke} = 0.5 (V_{k\alpha} - V_{k\beta}) \dots\dots (9)$$

$$V_{ds}^{+ke} = 0.5 (V_{k\alpha} + V_{k\beta}) \dots\dots (10)$$

$$V_{qs}^{-ke} = 0.5 (V_{k\alpha} + V_{k\beta}) \dots\dots (11)$$

$$V_{ds}^{-ke} = 0.5 (V_{k\alpha} - V_{k\beta}) \dots\dots (12)$$

It is important to note that once  $V_{qs}$  and  $V_{ds}$  have been established from the phase voltages, the voltages which are to be applied in each of the reference

frames may be determined directly from (5) - (12). The superscript  $-ke$  and  $+ke$  used in (9) - (12) identify the speed and direction of the reference frames with respect to the stator.

The equations which describe the symmetrical induction machine in any reference frame may be readily obtained from the equations which describe the machine in an arbitrary reference frame from [1] and [9]. Since the multiple reference frames are selected so that the applied voltages are constant, the steady-state  $ds$ - $qs$  currents in any of the multiple reference frames may be given :

$$i_{qs}^{ne} = (A/E) V_{qs}^{ne} + (B/E) V_{ds}^{ne} \quad (13)$$

$$i_{ds}^{ne} = -(B/E) V_{qs}^{ne} + (A/E) V_{ds}^{ne} \quad (14)$$

The quantities  $A$ ,  $B$ , and  $E$  are developed from the machine equation in the arbitrary reference frame in [1]. In the above equations the index  $n$  is used to denote  $+k$  and  $-k$ . However, only the reference frames with nonzero applied voltages are to be employed.

Expression for the rotor current may be given as :

$$i_{qr}^{ne} = (C/E) V_{qs}^{ne} + (D/E) V_{ds}^{ne} \quad (15)$$

$$i_{dr}^{ne} = -(D/E) V_{qs}^{ne} + (C/E) V_{ds}^{ne} \quad (16)$$

If (13-15) are transformed back to the stationary reference frame, then

$$i_{qs}^s = \sum_{k=1}^{\infty} [(i_{qs}^{+ke} + i_{qs}^{-ke}) \cos(k\omega t) + (i_{ds}^{+ke} - i_{ds}^{-ke}) \sin(k\omega t)] \quad (17)$$

$$i_{ds}^s = \sum_{k=1}^{\infty} [(i_{ds}^{+ke} + i_{ds}^{-ke}) \cos(k\omega t) - (i_{qs}^{+ke} - i_{qs}^{-ke}) \sin(k\omega t)] \quad (18)$$

$$i_{qr}^s = \sum_{k=1}^{\infty} [(i_{qr}^{+ke} + i_{qr}^{-ke}) \cos(k\omega t) + (i_{dr}^{+ke} - i_{dr}^{-ke}) \sin(k\omega t)] \quad (19)$$

$$i_{dr}^s = \sum_{k=1}^{\infty} [(i_{dr}^{+ke} + i_{dr}^{-ke}) \cos(k\omega t) - (i_{qr}^{+ke} - i_{qr}^{-ke}) \sin(k\omega t)] \quad (20)$$

The stator and rotor currents may be obtained from (17-18) and (19-20) respectively, that is,

$$i_{as} = i_{qs}^s \quad (21)$$

$$i_{bs} = -0.5 i_{qs}^s - \sqrt{3}/2 i_{ds}^s \quad (22)$$

$$i_{cs} = -0.5 i_{qs}^s + \sqrt{3}/2 i_{ds}^s \quad (23)$$

$$i_{ar} = i_{qr}^s \quad (24)$$

$$i_{br} = -0.5 i_{qr}^s - \sqrt{3}/2 i_{dr}^s \quad (25)$$

$$i_{cr} = -0.5 i_{qr}^s + \sqrt{3}/2 i_{dr}^s \quad (26)$$

The instantaneous electromagnetic torque may be given as :

$$T = M(m/2)(p/2) (i_{qs}^s i_{dr}^s - i_{ds}^s i_{qr}^s) \quad (27)$$

It is clear that although the expression in the synchronously rotating reference frame from the basis in the derivation, these equations does not necessary used in the actual application of the method of multiple reference frames.

**IV- RESULTS.**

Boxes algorithm, FFT analysis and M.R.F. method are used to analyse a three-phase induction motor and the following results are represented :

- Torque - Time curve as shown in figure (4).
- Motor phase voltage versus the time as shown in figure (5).
- Motor phase current evaluation as shown in figure (6).
- Motor phase current harmonics analysis spectrum as shown in figure (7).

These results are calculated for a constant slip = 0.1, MI=0.8 and Fr=60 Hz. Figure (8) shows the torque-slip characteristics for V=0.8 Vs.

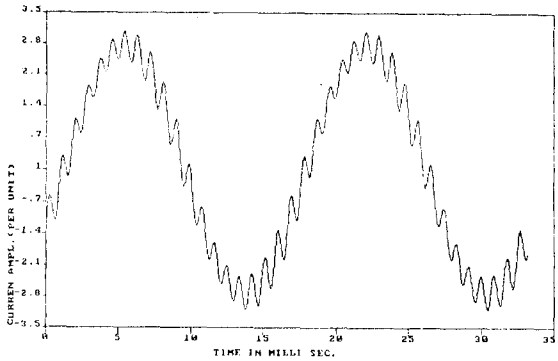


Fig.(6) Motor Phase Current.

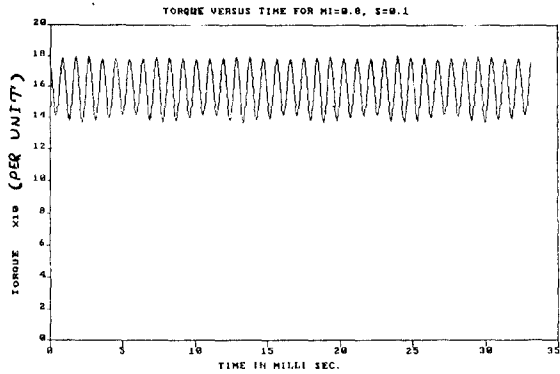


Fig.(4) Motor Torque in steady-state.

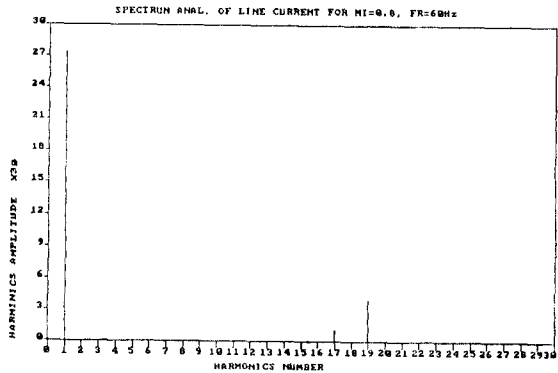


Fig.(7) Motor Phase current har. analysis

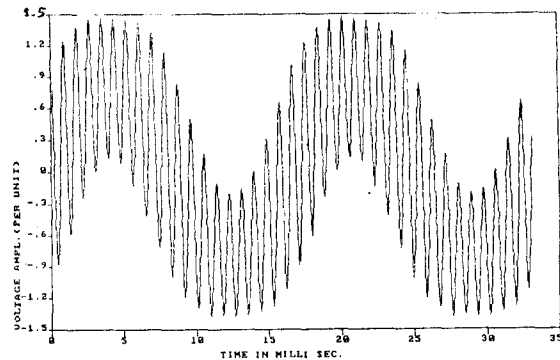


Fig.(5) Motor Phase voltage.

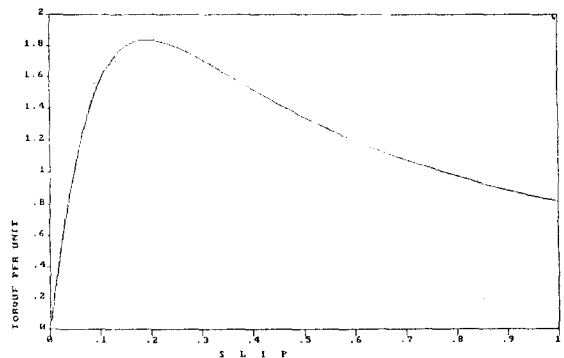


Fig.(8) Torque-slip characteristics.

## V- CONCLUSION

Digital Boxes P.W.M. as a good solution to control the DC/AC inverter operation which feed an induction motor drives is presented. From figure (3) and the characteristics in section II it is simple to note the advantages of this algorithm, in addition to its facility to practical realization.

Motor analysis in steady-state operation fed by P.W.M. inverter Boxes algorithm controlled is presented by using a very capable M.R.F. method is presented to can study with facility the motor performance for any point and condition of operation.

Finally the results obtained for balanced 3- phase induction motor is presented in this paper, which are a meaningful results.

Now we are investigated the ability of the analysis of 3-phase induction motor fed by 3-phase unbalanced sources and we have a good results. Really we have the attempt to introduces in the next paper.

## VI- REFERENCES.

- [1] Paul C. Krause "Method of Multiple Reference Frames Applied to analysis of Symmetrical Induction Machinery", IEEE Trans. on PAS-87, PP. 218-227, Jan. 1968.
- [2] Paul C. Krause "Simulation of Symmetrical Induction Machinery", IEEE Trans. on Power Apparatus and System, Vol. PAS-84, PP.1038-1053, Nov. 1965.
- [3] M.SH.Nasr "Integrated PWM Control Strategy for Inverters and Induction Motor Drives" , MIMI'1987.
- [4] M.SH.Nasr " A New Digital P.W.M. Strategy (BOXESTHEORY)", conference on applied motion control, USA, 1987.
- [5] A. Pollmann " Software Pulse Width Modulated for fp control of AC Drives", IEEE Trans. on Ind. Appl. vol. IA-22 No.4, Jul/Aug., 1987.
- [6] Bimal K. Bose "Adjustable Speed AC Drives System", IEEE press, 1980.
- [7] Mohammed T.L. & William SH. "analysis of Induction Motor Subjected to non-sinusoidal voltages containing harmonics", IEEE Trans. on Ind. Appl. Vol. IA-21, No.4, Jul/Aug., 1985.
- [8] David Finney "The Power Thyristor and its Application", McGraw-Hill, 1980.