Performance Analysis on 101 Coding Scheme

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Abstract 101 coding scheme, one of sliding block coding techniques, provides practically attractive features in some compression applications for image sources such as facsimile. This paper presents a new simple model of 101 coder. The results show that the entropy of the output of the 1υ1 coder can be reduced close to the rate distortion bound of a binary first-order markov source.

1. Introduction

101 coding scheme is regarded as one of sliding block coding techniques and provides practically attractive features in some data compression applications for image sources such as facsimile[1]. The basic idea of 101 coding is to replace an isolated "0" symbol with "1" symbol in the binary data sequence and then to perform the bit rate reduction through taking full advantage of the resultant skewed data sequence.

The mathematical models for the 101 coder were initially studied by several researchers [2],[3],[4]. The main purpose of this paper is to present a new simple model on the 101 coder. We regard the output of a facsimile source as asymmetrical first-order binary Markov process and try to evaluate the entropy of 101 coder. Finally, we compare the rate distortion bound of the facsimile source with the input entropy rate of 101 coder (i.e. the output entropy of the source), and the output entropy rate of 101 coder.

2. A new model of 101 coder

There are eighth states associated with the 101 coder We call them $q_i (i = 1, 2, \ldots, 8)$ and define $Q = \{q_1, q_2, \ldots, q_8\}$ as the set of the states. The sequences of the input and output variables of the 101 coder are denoted by $\{X_n\}$ and $\{Y_n\}$, respectively. Y_n and the next state q_j are represented as the function of q_i and X_n such as $Y_n = f(q_i, X_n)$ and $q_j = g(q_i, X_n)$, respectively. When X_{n+1} is given the state transition and the corresponding output symbol can be shown with Mealy type expression as in *Table* 1.

Let $Q_I (I = 1, 2, \ldots, N)$ be the disjoint subsets of Q where each state q_i must belong to one of the subsets.

Suppose that when the output symbol 1(or 0) is generated due to the transition from Q_i to Q_j $(i, j = 1, 2, \ldots, N)$, and the another output symbol 0(or 1) has to be generated by the transition from Q_i to $Q_k (k \neq i, j)$.

Under this requirement, we get the rearranged transition in *Table* 2.

3. The output entropy rate of the 101 coder

The transition probabilities of the input sequence are defined as

$$\left.\begin{array}{l} P(X_n = 1 \mid X_{n-1} = 0) = \alpha, \\ \text{and} \\ P(X_n = 0 \mid X_{n-1} = 1) = \beta. \end{array}\right\} \quad (1)$$

Then, the transition matrix associated with the subsets can be written in the form

$$Q = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & \frac{1-\beta+2\alpha\beta}{1+\alpha\beta} & \frac{\beta(1-\alpha)}{1+\alpha\beta} \\ 1 & 0 & 0 \end{bmatrix}. \quad (2)$$

Stationary probabilities are also given by

$$\left.\begin{array}{l} P(Q_1) = \frac{\beta(1-\alpha)}{\alpha+\beta}, \\ P(Q_2) = \frac{\alpha(1+\alpha\beta)}{\alpha+\beta}, \\ \text{and} \\ P(Q_3) = \frac{\alpha\beta(1-\alpha)}{\alpha+\beta}. \end{array}\right\} \quad (3)$$

The output entropy rate is gotten by using

$$H(Y) = \sum_{i=1}^{3}\sum_{j=1}^{3} P(Q_i, Q_j) \log P(Q_j \mid Q_i). \quad (4)$$

4. Comparison

From (1), the stationary probabilities of the input sequence are

$$\left.\begin{array}{l} P(1) = \frac{\alpha}{\alpha+\beta}, \\ \text{and} \\ P(0) = \frac{\beta}{\alpha+\beta}. \end{array}\right\} \quad (5)$$

The input entropy rate is defined by

$$H(X) = \sum_{i=0}^{1}\sum_{j=0}^{1} P(i,j) \, \log P(j \mid i). \quad (6)$$

The average run length of the symbol 1 in the input sequence has been introduced [5] by

$$EL(1) = \sum_{k=1}^{\infty} kP_k(1) = 1/\beta \quad (7)$$

where

$$P_k(1) = \beta(1-\beta)^{k-1}. \quad (8)$$

Thus, the average distortion between the input and output of the 101 coder can be obtained by

$$D = P(101) = \frac{\alpha^2\beta}{\alpha+\beta}. \quad (9)$$

The rate distortion bound of a binary first-order Markov source satisfies, in the sense of the autoregressive bound [6].

$$R(D) \geq R_L(D) = H(X) - H(D). \quad (10)$$

Where $H(\cdot)$ is an entropy function.

Figure 1 illustrates the relationships among $H(X)$, $H(Y)$ and $R_L(D)$. In this Figure, the result with respect to *i.i.d.* (independent identically distributed) source is also shown by the mark $\star$. This is completely coincident with Berger-Lau's result [4].

References

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Tabel 1 Mealy type transition table.

q_i \ X_{n+1}	0	1
$q_1(0\ 0\ 0)$	$q_1/0$	$q_2/0$
$q_2(0\ 0\ 1)$	$q_3/1$	$q_4/1$
$q_3(0\ 1\ 0)$	$q_5/0$	$q_6/1$
$q_4(0\ 1\ 1)$	$q_7/1$	$q_8/1$
$q_5(1\ 0\ 0)$	$q_1/0$	$q_2/0$
$q_6(1\ 0\ 1)$	$q_3/1$	$q_4/1$
$q_7(1\ 1\ 0)$	$q_5/0$	$q_6/1$
$q_8(1\ 1\ 1)$	$q_7/1$	$q_8/1$

Table 2 Rearranged transition table.

Q_i \ X_{n+1}	0	1
$Q_1\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$	$Q_1/0$ $Q_2/1$	$Q_1/0$ $Q_2/1$
$Q_2\begin{pmatrix} q_3, & q_4 & \\ q_6, & q_7, & q_8 \end{pmatrix}$	$Q_2/1$ $Q_3/0$	$Q_2/1$
$Q_3(q_5)$	$Q_1/0$	$Q_1/0$

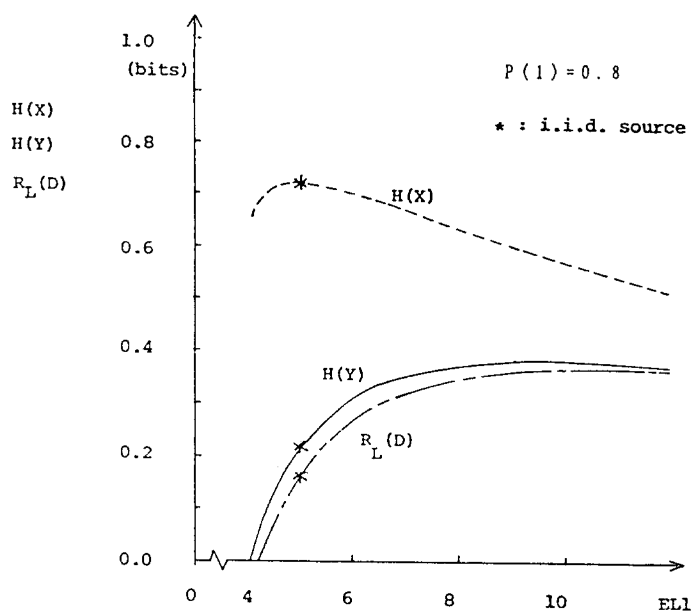

Fig.1 The relationships among $H(X)$, $H(Y)$ and $R_L(D)$ with $EL1$.