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On a Pole Assignment of Linear Discrete Time System

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Abstrack : In this paper, a new procedure for selecting weighting matrices in linear discrete time quadratic optimal control problem (LQ-problem) is proposed. In LQ-problems, the quadratic weighting matrices are usually decided on trial and error in order to get a good response. But using the proposed method, the quadratic weights are decided in such a way that all poles of the closed loop system are located in a desired region for good responses as well as for stability and values of the quadratic cost function are kept less then a specified value.

1. Introduction

The closed-loop system constructed by utilizing an LQ-problem has some merits (Safonov and Athans, 1977;Kobayashi and Shimemura, 1981). But, when we construct a closed-loop system by utilizing the LQ-problem, the weighting matrices of the quadratic cost function must be decided by trial and error to get the good responses, because only very little is known about the relation between the quadratic weights and the dynamical characteristics of the closed-loop system (Harvey and Stein , 1978 ; Stein, 1979 ; Francis , 1979). The dynamical characteristics of a linear system are influenced by the location of poles of the system. Therefore to get good responses, it is necessary to locate all poles in the desired positions. But we know that it is sufficient to place all poles in a suitable region instead of placing them in their respective desired positions.

In this paper, we give a new method of selesting the quadratic weights in discrete time LQ-problems by which all poles of the closed-loop system can be located in the specified region for good response as well as for stability. In regard to this subject, there have been published many papers (Mori and Shimemura, 1980 ; Furuta and Kim, 1987 ; Solheim, 1974 ; Fujinaka, Sugimoto, Yamamoto and Katayama, 1988). But, these methods have the problem that we do not know for the resulting optimal closed-loop system the cost function. In this paper, we propose a method to design a closed-loop system with all poles placed in the specified region, as well as to keep the value of a given quadratic cost function less than the specified value. The system constructed by this method has the merits of an LQ-problem as well as a pole assignment problem and holds down the value of a given quadratic cost function. Conceptually this decision method may be considered to be derived from the so-called inverse optimal control problems. In continuous time case, similar methods of determining quadratic weights have been reported by Kawasaki and Shimemura (1981, 1983, 1988) in which only pole locations of the closed-loop system are considered.

2. Problem formulation

Now we consider a linear discrete time multivariabe system (1) and a quadratic cost function (2).

$$x(k+1)=Ax(k)+Bu(k) \tag{1}$$

$$J(u)=\sum_{k=0}^{\infty}\{x(k)^T Q x(k)+u(k)^T R u(k)\} \qquad (2)$$

where A and B are nXn, nXr constant matrices, Q and R are nXn, rXr positive definite symmetric matrices respectively, $x(k)$ is an n-dimensional state vector, $u(k)$ is an r-dimensional input vector, and pair (A,B) is a controllable pair. Then it is well known(Kwakernaak and Sivan,1972) that the optimal control which minimizes J(u) subject to system (1) is given by the feed-back control law

$$u(k)=-Kx(k) \qquad (3)$$

with the optimal feedback gain

$$K=(R+B^T PB)^{-1}B^T PA \qquad (4)$$

where P is the maximal solution of the algebraic Riccati equation

$$P=A^T PA-A^T PB(R+B^T PB)^{-1}B^T PA+Q\ . \qquad (5)$$

Our problem is to decide quadratic weights Q which give the optimal feedback gain K satisfying the following condition :

1) $-K \in \mathbb{K}$
2) $\lambda(A-BK) \in \Gamma$
3) $J_B(u) \leq M$

where $\mathbb{K}$ is a set of optimal feedback gain, Γ is the specified region for a good response as well as for stability (Fig.1), $J_B(u)$ is the quadratic cost function with $(Q,R)=(Q_B,R)$, and M is some realizable positive number to satisfy the condition 2) and 3). $\lambda(A)$ is a set of eigenvalues of matrix A. In the next section we propose the method of deciding quadratic weights Q in the auxiliary performance index (2), which guarantees the condition 1), 2) and 3).

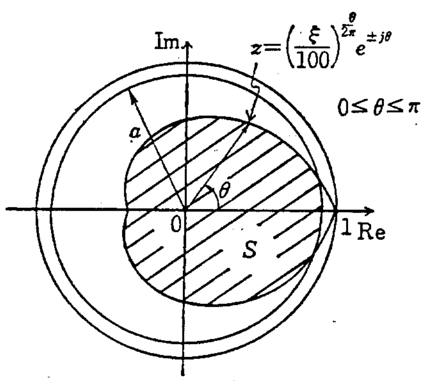

Fig.1 A desired region of closed-loop eigenvalue locations

3.Preliminaries.

3.1 Some preliminary Lemmas.

In this section, a new method of deciding quadratic weights of LQ-problem is given. Before showing the result, we prepare some preliminary lemmas. First, we consider solutions of Riccati equation (5) and of Liapunov equation

$$P_{\#}=A^T P_{\#}A+Q_{\#}\ . \qquad (6)$$

Lemma 1 (Kodama and Suda, 1978)

Let P_1 be a solution of equation (5) with $Q=Q_1$ and P_2 is one with $Q=Q_2$. If $Q_2\geqq Q_1$, then $P_2\geqq P_1$. □

Lemma 2 (Kodama and Suda, 1978)

Let $Q=Q_{\#}\geqq 0$ in equation (5) and (6). If A is asymptotically stable, then equation (6) has a real symmetric positive semidefinite solution $P_{\#}$ which satisfies $P_{\#}\geqq P\geqq 0$. □

Lemma 3 (Amin, 1984)

Let $A_1=A-B(R_1+B^T P_1 B)^{-1}B^T PA$, where P_1 is a solution of the equation(5) with $(Q,R)=(Q_1,R_1)$:

$$P_1=A^T P_1 A-A^T P_1 B(R_1+B^T P_1 B)^{-1}B^T P_1 A+Q_1$$

and, $A_2=A_1-B(R_2+B^T P_2 B)^{-1}B^T P_2 A_1$, where P_1 is a solution of the equation (5) with $(Q,R)=(Q_2,R_2)=(Q_2,R_1+B^T P_1 B)$:

$$P_2=A_1{}^T P_2 A_1-A_1{}^T P_2 B(R_2+B^T P_2 B)^{-1}B^T P_2 A_1+Q_2$$

Furthere $A_c=A-B(R_1+B^T PB)^{-1}B^T PA$, where P is a solution of the equation (5) with $(Q,R)=(Q_1+Q_2, R_1)$:

$$P=A^T PA-A^T PB(R_1+B^T PB)^{-1}B^T PA+Q_1+Q_2$$

Then $A_c=A_2$ □

Lemma 4 (Shin , Kawasaki and Shimemura , 1988)

If $P=\begin{bmatrix} P_1 & P_2 \\ P_2{}^T & P_3 \end{bmatrix}$ is a positive semidefinite symmetric matrix , it satisfies the following relation :

$$\mathrm{Range}(P_3)\supseteqq \mathrm{Range}(P_2{}^T).$$ □

Lemma 5 (Shin , Kawasaki and Shimemura , 1988)

If $P=\begin{bmatrix} P_1 & P_2 \\ P_2{}^T & P_3 \end{bmatrix}$ is a positive semidefinite symmetric matrix , it satisfies the following inequality :

$$P_1-P_2 P_3{}^+ P_2{}^T\geqq 0$$

where $P_3{}^+$ is a pseudoinverse matrix of P_3. that is , when $P_2\neq 0$, it satisfies the following equations:

$P_2P_3^+P_3=P_2$

$P_3P_3^+P_3=P_3$. □

Lemma 6

Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigenvalues of A inside the unit disk, and $\zeta_1, \zeta_2, \cdots, \zeta_n$ be the corresponding eigenvector. If a symmetric positive semidefinite matrix Q of equation (5) satisfies the following equation

$$Q\zeta_i = 0,\ i=1,2,\cdots,n \quad (7)$$

The closed loop system matrix $A_c = A-B(R+B^TPB)^{-1}B^TPA$ formed with the maximum solution P has the eigenvalues λ_i and the corresponding eigenvector ζ_i.

Proof: Let H be a discrete type Hamiltonian matrix given by

$$H=\begin{bmatrix} A+BR^{-1}B^TA^{T-1}Q & -BR^{-1}B^TA^{T-1} \\ -A^{T-1}Q & A^{T-1} \end{bmatrix} \quad (8)$$

from eqation(8) and the definition of eigenvalue and eigenvector, it follows that

$$H\begin{bmatrix} \zeta_i \\ 0 \end{bmatrix} = \lambda_i \begin{bmatrix} \zeta_i \\ 0 \end{bmatrix} \quad (9)$$

It is well known (Kimura and Inoue, 1978 ;Pappas et al., 1980) that the optimal closed loop poles are given by the eigenvalues of H inside the unit disk. If the absolute value of λ_i is less than 1, the equation (9) shows that λ_i is an eigenvalue of the closed loop matrix A_c and ζ_i is the corresponding eigenvector. □

3.2 Property of quadratic matrix Q with pole assignment problem.

In this section, we will give some proports quadratic weights about pole assignment. Let λ_i, (i = 1,$\cdots$,n) be eigenvalues of matrix A and ζ_i, (i = 1,$\cdots$,n) be the corresponding right eigenvectors and ξ_i, (i = 1,$\cdots$,n) be the corresponding left eigenvectors. Therefore, we consider the quadratic weighting matrix Q which realize the closed loop system transforming only p poles. Then, from Lemma 6, we get the forrowing relation:

$$(A-B(R+B^TPB)^{-1}B^TPA)\zeta_i = \lambda_i\zeta_i \quad ,(i=p+1,\cdots,n) \quad (10)$$

Let σ_i,(i=1,$\cdots$,p_1), be real eigenvalues of the matrix A, and $\alpha_i \pm j\beta_i$, (i=1,$\cdots$,p_2, $p_1+2p_2=p$), be complex conjugate pair of eigenvalues of the matrix A. And let ζ_i and $v_i \pm jw_i$ be the corresponding right eigenvectors. And ξ_i and $s_i \pm jt_i$ be the corresponding left eigenvectors. And we defind the matrix T as follow:

$$T=[\xi_1, \xi_2, \cdots, \xi_{p1}, s_1, t_1, s_2, t_2, \cdots, s_{p2}, t_{p2}] \quad ,(p_1+2p_2=p) \quad (11)$$

Then, we obtain the following Lemma

Lemma 7 (Solheim, 1974)

Let quadratic weight R be given. If we choose the quadratic weights Q as follow:

$$Q=TQ_\#T^T\ ,\ Q_\# \in R^{p\times p} > 0 \quad (12)$$

then it is possible to constitute the closed-loop system which transform only p poles. And p poles of being transformed are equal to poles of the optimal closed-loop system (Γ_D,TB) with the quadratic weights ($Q_\#$,R) ,where

$$\Gamma_D=\mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_{p1}, \Lambda_1, \Lambda_2, \cdots, \Lambda_{p2}),$$

$$\Lambda_i = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \alpha_i \end{bmatrix}. \quad (13)$$ □

Lemma 7 gives a special type of quadratic weights Q, which transform only p poles. However we can consider other type of quadratic weights Q which transforms only p poles. If there exists other type of quadratic weights Q, we can find the weights of type (12) to obtain the same transfer of p poles. This is fact will be shown in the following theorem

[Theorem 1]

Let Ψ be the set of the feedback gain F,

$F=(R+B^TPB)^{-1}B^TPA$,

which satisfies equation (10), that is to say

$$\Psi=\{F \mid A_1\zeta_i=(A-BF)\zeta_i=\lambda_i\zeta_i \quad ,(i=p+1,\cdots,n)$$
$$\text{s.t. } P\geqq 0\ ,P=A^TPA-A^TPB(R+B^TPB)^{-1}B^TPA+Q\} \quad (14)$$

And, let Ω be the set of the feedback gain $F_\#$,

$F_\#=(R+B^TP_\#B)^{-1}B^TP_\#A$,

which satisfies equation (4) with Lemma 7 type quadratic weights Q, that is to say

$$\Omega=\{F_\# \mid F_\#=(R+B^TP_\#B)^{-1}B^TP_\#A\ ,\ P_\#=\min(P)$$
$$\text{s.t. } P\geqq 0\ ,$$
$$P=A^TPA-A^TPB(R+B^TPB)^{-1}B^TPA+TQ_\#T^T\} \quad (15)$$

Then

$\Psi=\Omega$

The proof is given by the same method in (16) □

4.A design method of the closed-loop system with Pole assignment

In this chapter, we consider the design method a closed-loop system which satisfies the conditions 1), 2) and 3). Fundamentally, this method is the repeated application of the result of Lemma 7 as p=1 (real pole) or p=2 (complex conjugate pair poles) until the condition 1), 2) and 3) are satisfied.

Let σ be one of the transforming real eigenvalues of the matrix A, and $\alpha \pm j\beta$ be a pair of complex conjugate eigenvalues of thematrix A. And let ζ and $v \pm jw$ be the corresponding right eigenvectors. And η and $s \pm jt$ be the correspond ing left eigenvectors. From Lemma 7, we obtain the following results. These results are generalization of the results of Solhelm(1974) and of Fujinaka, Sugimoto, Yamamoto and Katayama(1988).

[Theorem 2]

a) For the quadratic weights Q defined by equation (16), the feedback gain (4) transforms the real eigenvalue σ to the realeigenvalue λ:

$$Q = q\eta\eta^T \tag{16-a}$$

$$q = \frac{\sigma}{W}\{(\lambda + \frac{1}{\lambda}) - (\sigma + \frac{1}{\sigma})\} \tag{16-b}$$

$$W = \eta^T B R^{-1} B^T \eta \tag{16-c}$$

b) For the quadratic weights Q defined by equation (17), the feedback gain (4) transforms the complex conjugate pair of eigenvalues $\alpha \pm j\beta$ to the complex conjugate pair of eigen-values λ and λ^*:

$$Q = [\,s\ t\,]\ \hat{Q}\,[\,s\ t\,]^T \tag{17-a}$$

$$\hat{Q} = \begin{bmatrix} q_1 & q_2 \\ q_2 & q_3 \end{bmatrix} \tag{17-b}$$

And if we consider W defined by

$$W = \begin{bmatrix} w_1 & w_2 \\ w_2 & w_3 \end{bmatrix} = [\,s\ \ t\,]^T BR^{-1}B^T\ [\,s\ t\,] \tag{18}$$

then k_1, k_0, q_i and w_i (i=1,2,3) satisfy the following relation:

$$k_1 = \{q_1(w_1\alpha + w_2\beta) + q_3(w_3\alpha - w_2\beta) + q_2(w_3\beta - w_1\beta + 2w_2\alpha) + 2\alpha(1+\alpha^2+\beta^2)\} * (\alpha^2+\beta^2)^{-1} \tag{19}$$

$$k_0 = [(q_1q_3 - q_2^2)(w_1w_3 - w_2^2) + q_1\{(1+\alpha^2)w_1 + \beta^2 w_3 + 2\alpha\beta w_2\} + q_3\{(1+\alpha^2)w_3 + \beta^2 w_1 - 2\alpha\beta w_2\} - 2q_2\ \{\{\beta^2 - (1+\alpha^2)\}w_2 + (w_1 - w_3)\alpha\beta\} + \{(1+\alpha^2)^2 + \beta^4 + 2\beta^2(\alpha^2-1)\}(w_1w_3 - w_2^2)] * (\alpha^2+\beta^2)^{-1} \tag{20}$$

where k_1 and k_0 are coefficient of a quadratic equation $z^2+k_1z+k_0=0$ which has a solution $\lambda + \lambda^{-1}$ and $\lambda^* + \lambda^{*-1}$.

Proof: Let λ_i be the eigenvalues of matrix $A-B(R+B^TP_+B)^{-1}B^TP_+A$ and ζ_i be the corresponding eigenvectors. Then we obtain

$$\{A-B(R+B^TP_+B)^{-1}B^TP_+A\}\zeta_i = \{A-BR^{-1}B^TA^{T-1}(P-Q)\}\zeta_i = \lambda_i\zeta_i \tag{21}$$

and from equation (5)

$$(P-Q)\zeta_i = A^TP\{A-B(R+B^TP_+B)^{-1}B^TP_+A\}\zeta_i = \lambda_i A^TP\zeta_i \tag{22}$$

If λ_i is not equal to eigenvalues of matrix A, then, from (21) and (22), we obtain

$$\zeta_i = -(\lambda_i I - A)^{-1}BR^{-1}B^T(\lambda_i^{-1}I - A^T)^{-1}Q \tag{23}$$

We multiply η_i or $[\,s_i\ t_i\,]^T$ left hand side of eq.(12) , and from Lemma 6

$$\{1+\eta_i^T(\lambda_i I-A)^{-1}BR^{-1}B^T(\lambda_i^{-1}I-A^T)^{-1}\eta_i q\}\ \eta_i^T\zeta_i = 0 \tag{24}$$

or

$$\{I_2+[s_i\ t_i]^T(\lambda_i I-A)^{-1}BR^{-1}B^T(\lambda_i^{-1}I-A^T)^{-1}[s_i\ t_i]\hat{Q}\}[s_i\ t_i]^T\zeta_i = 0 \tag{25}$$

We can obtain the result of Theorem 2 to calculate the det{·}=0 of equation (24) or (25) which is

$$\det\{1+(\lambda_i-\sigma_i)^{-1}W(\lambda_i^{-1}-\sigma_i^T)^{-1}\}=0,$$

$$\det\{I_2+(\lambda_i I-\Lambda_i)^{-1}W(\lambda_i^{-1}I-\Lambda_i^T)^{-1}\}=0$$

where

$$\Lambda_i = \begin{bmatrix} \alpha_i & -\beta_i \\ \beta_i & \alpha_i \end{bmatrix}$$

□

We obtained the method of deciding the quadratic weights Q which can transform the closed-loop pole via the solution of equation (5). Now we give the formula to evaluate the inrement of the cost function $J_0(u)$ by a result of the transform of a pair of eigenvalues. We denote $u_0=-K_0x$ be the optimal control for $J_0(u)$, with $Q = Q_0$, i.e. $K_0 = (R+B^TP_0B)^{-1}B^TP_0A$ and $u=-Kx$ be the optimal control (Q, R) = (Q_0+Q_1, R), where Q_1 is defined by equation (17). There we can obtain the following main Theorem.

[Theorem 3]

The value of quadratic cost function $J_0(u)$ satisfies the following inequality:

$$J_0(u) \leq J_0(u_0) + \|x_0\|^2\ \frac{\lambda\max(\hat{Q})}{1-(\alpha^2+\beta^2)} \tag{25}$$

where x_0 is an initial value of the state vector x(k).

Proof: First, we solve the matrix equation

$$P_1=A_1{}^TP_1A_1-A_1{}^TP_1B(R_1+B^TP_1B)^{-1}B^TP_1A_1+Q_1 \ . \quad (26)$$

where A_1 is a closed loop matrix $A-BK_0$ and Q_1 is the weight in Theorem 2 and R_1 is the weight in Lemma 3. We compose the feedback input

$$u(k)=-K_1x(k)$$
$$=-(R_1+B^TP_1B)^{-1}B^TP_1A_1x(k) \quad (27)$$

From Lemma 3, we can obtain the following relation:

$$J_0(-Kx(k))\leqq J_1(-Kx(k))$$
$$=J_0(-K_0x(k))+x_0{}^TP_1x_0$$
$$\leqq J_0(-K_0x(k))+\|x_0\|^2P_1 \quad (28)$$

Then from Lemma 2, selecting the norm of left eigenvector $[\ s_i\ \ t_i\ ]=1$, and substituting the matrix Q of Theorem 2 into the equation (5), we obtain

$$\lambda\max(P)\leqq\frac{\lambda\max(\hat{Q})}{1-(\alpha^2+\beta^2)} \quad (29)$$

This establishes Theorem 3 □

Here, $\hat{Q}$ of Theorem 2 is a 2X2 symmetric positiv edefinite matrix. Specifically, if $q_2=0$ the nTheorem 3 is

$$J_0(u)\leqq J_0(u_0)+\|x_0\|^2\frac{\lambda\max(q_1,q_3)}{1-(\alpha^2+\beta^2)} \quad (30)$$

From Theorem 3, we obtained the value of incre-

$$J_0(u)\leqq J_0(u_0)+\|x_0\|^2\ \frac{q}{1-\sigma^2}$$

This inequality is derived similarly to Theorem 3.

In Theorem 2, we have given a quadratic weights to transform the selected eigenvalue(s) to the ment of the quadratic cost function. In case of transforming a real eigenvalue, the corresponding formula to eq.(25) is

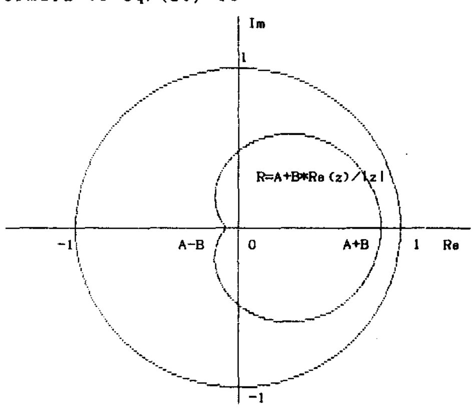

Fig. 2 The substituting heart region.

specified position(s). From this result, we can derive the condition under which the selected eigenvalue(s) can be transformed into the specified region. For that purpose we substitute the hatched region of Fig.1 by the "heart" region shown in Fig.2 ($R = A+B*(Re(z)/|z|)$, A,B aresome real numbers). For this case, we can obtain the following Lemma.

Lemma 8 (Shin, Shimemura and Kawasaki, 1989)

a) The condition to transform a real pole from σ into the region of Fig.2 is

$$q\geqq\frac{\sigma}{W}\ \{(A+B+\frac{1}{A+B}\ )-(\ \sigma+\ \frac{1}{\sigma}\ )\} \quad (31)$$

where σ, q and W are given in Theorem 2-a).

b) The condition to transform a pair of complex conjugate pole from $\alpha\pm j\beta$ into the region of Fig.2 is

$$(R^2+\frac{1}{R^2})^2-k_0\ (R^2+\frac{1}{R^2})+(k_1{}^2-2k_0-4)\leqq 0 \quad (32)$$

where, $R = A+B*(Re(z)/|z|$, $z=\lambda$ or λ^*, and k_0 and k_1 are given in Theorem 2-b). □

Inconcluding the above discussions, we can summarize the decision method of the quadratic weights as follows.

[Decision method]

Step 1.(May be skipped in case of $|\lambda(A)|<1$)

Solve an LQ-problem for arbitrary quadratic weights (Q_1,R_1) selected from the demand for the system's dynamical characteristics

$$P_1=A^TP_1A-A^TP_1B(R_1+B^TP_1B)^{-1}B^TP_1A+Q_1 \quad (33)$$

and obtain a closed-loop system matrix $A_1 = A-B(R_1+B^TP_1B)^{-1}B^TPA$ and calculate a quadratic cost function $J_0(u(k_0))$.

Step 2. Choose a real eigenvalue or a pair of complex conjugate eigenvalue which are outside the heart region of Fig.2, and obtain the quadratic weights Q utilizing Theorem 3 and Lemma 8.

Step 3. Utilizing the quadratic weights Q obtained in Step 2, obtain an optimal closed-loop system matrix $A_i-BK_i= A_{i-1}-B(R_i+B^TP_iB)^{-1}B^TP_iA_{i-1}$ where $A_0=A$. P_i is the solution of the equation

$$P_i=A_{i-1}{}^TP_iA_{i-1}-A_{i-1}{}^TP_iB(R_i+B^TP_iB)^{-1}B^TP_iA_{i-1}+Q_i \quad (34)$$

and $R_i=R_{i-1}+B^TP_{i-1}B$. And calculate a quadratic cost function $J_0(u(k_i))$ and a value of increment ΔJ_i.

Step 4. Repeat Step 2 and Step 3 so that the all poles enter the heart region of Fig.2. The least upper bound value of quadratic cost function is then

$$J_\theta(u(k)) \leqq J_\theta(u(k_\theta)) + \Sigma \Delta J_i \quad (35)$$

There are maybe cases, where the condition 2) and 3) can not be satified simultaneously. In such cases, the uppert bound M should be increased.

5. Numerical example

In this section the design procedures given in the previous sections are illustrated by a numerical example. We consider a discrete-time system given by

$$x(k+1)=Ax(k)+Bu(k)$$

$$A = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & -2.2 \\ 0 & 1 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (36)$$

The open-loop poles of the system are {0.5, 0.8 $\pm$ j0.4}. And we omit the Step 1, because the open-loop system satisfies the condition $|\lambda(A)| < 1$ viz., quadratic costfunction $J_\theta(u(k_\theta))=0$, and objecting quadratic cost function $J_\theta(u(k))=6$. And the heart region which we desired is given by

$$R=0.1+0.3*Re(z)/|z| \quad (37)$$

Then all poles are outside of the heart region in Fig.2. First, we consider transforming a real pole, and a value of increment ΔJ_1 being less than 3 and quadratic weights R = 1. Then from Theorem 3 and Lemma 8, we can obtain q=3. Then utilizing q=3 at Step 2 and Step 3, we can obtain the closed-loop matrix A_1. The closed-loop poles of the system A_1 are located {0.1342, 0.8 $\pm$ j0.4 } and a value of increment ΔJ_1 = 2.7238.

Next, we transform a pair of complex conjugate poles, and a valueof increment ΔJ_2 being less than 3. If q_2=0, then from Theorem 3 and Lemma 8 we can obtain q_1=0.2, q_3=0.1. Then utilizing q_1= 0.2, q_3=0.1 at Step 2 and Step 3, we can obtain the closed-loop matrix A_2. The closed-loop poles of the system $A_2$are located as follows:{ 0.1342, 0.25 $\pm$ j0.125 } and a value of increment ΔJ_2 = 0.41004.

6. Conclusion

In this paper, we have proposed a decision method of determining quadratic weightings of an LQ-problem to locate all poles of the closed-loop system in the specified region and to keep the value of the quadratic cost function less than the specified value.

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