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Filter Design for the Improvement of Robustness in Adaptive Control Systems

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Direct application of adaptive algorithm to the actual I/O is not a good strategy. The importance of using a kind of filter for estimation or control is strongly recommended. Simple pre-filtering method and pseudo-plant method is introduced. And, the properties of each methods are compared by analysis and/or simulations. A guideline for the choice of filters are proposed.

1. Introduction

Since Rohr's[1] pointing out about the undesirable phenomena of the basic MRAC with unmodelled dynamics and/or noise, there have been two kinds of meaningful changes in the research trends of adaptive control systems. One is the pessimistic view about the impracticality of adaptive control, which comes from its lackness of the robustness. The other is a more intensive study[2] to analyze and improve robustness of adaptive systems. In such areas as chemical reactors, distillation columns, heating systems, and PH controls, lots of the successful results with adaptive methods are reported[3]. Also applications of adaptive methods to various kinds of electrical machines are becoming interesting topics to many research groups[4]. However, the theoretical results on the robustness of adaptive control is not so clear, yet. In this paper, the role of filters in adaptive control systems is considered. Prefiltering and a special type filters -- pseudo-plant method[5] -- is described in section 2. The effect of filters in identification is described in section 3, while the effects of filters in cont- rol is described in section 4. Also some guidelines for the design of filters are suggested. Simulation results and conclusions are in section 5 and 6.

2. Simple prefiltering and pseudo-plant

Results of current applications of self-tuning algorithms to industrial processes[8,9] indicate the need for a prefilter for parameter estimation and an observer polynomial in the control law very strongly. Subsection 2.1 deals with simple prefiltering method and 2.2 deals with the pseudo-plant method.

2.1 Simple prefiltering

Assuming an input/output model and a some variant type estimator of RLS(recursive least square) estimator, the criterion to be minimized could be expressed as a some of weighted modelling error and weighted noise spectrum. To solve the minimization problem is not easy and the problem could be seperated one by one(ie. a weighted modelling error term and a weighted noise spectrum term). Solving the minimization of the weighted modelling error becomes an optimal input design problem which was considered extensively in 1970's[10,11]. Solving the minimization deigning a prefilter. The conditions of these problems are at variant with identification type(ie. open-loop or closed-loop identification.). However, rule-of-thumb guide lines for the prefilter were suggested in [12], and those are compared with the pseudo-plant method in this paper.

2.2 Pseudo-plant

A plant model with a user-chosen transfer function Pm is excited by the same input as the real plant. A combination of the filtered output of the real plant and the chosen plant model provides a pseudo-output $y\alpha(t)$ such that $y\alpha(t)$ equals the real output $y(t)$ when Pm equals the true plant transfer-function. The transfur-function between the control $u(t)$ and pseudo-output $y\alpha(t)$ is that of the pseudo-plant. This structure, as shown in Fig.1 provides a mechanism for generating controllers which are more robust in comparison with designs using the plant transfer-function alone. The idea of using a pseudo-plant was originated by Donati and Vallauri[6] and has been adopted by Koust et al.[7], Kim et al.[5].

A plant transfer-function with unstructured uncertainty can be expressed in multipicative form as thus

$$P(.) = Pm(.) \cdot (1 + \delta(.)) \qquad (1)$$

where (.) denotes the differential operator s in continuous systems: it can also denote the delay operator q^{-1} in discrete time. Here $\delta(.)$ is a rational function which represents the model uncertainty and is assumed to be stable (ie. the plant model captures all the unstable poles). if there is no need to distinguish between s and q^{-1}, (P, Pm, δ) will be used for simplicity of notation. The transfer-function of the pseudo-plant $P\alpha$ is

$$P\alpha = Pm + \alpha Fo Pm \delta = Pm(1 + \alpha Fo \delta), \qquad (2)$$

where Fo is usually a low-pass filter and α is a real between 0 and 1.
The norm of $\alpha Fo\delta$ can be made arbitrarily small compared with the norm of δ. Both the multipicative and additive uncertainties are reduced in the pseudo-plant. As the design of robust controllers is related to one of these uncertainties, and filters can be designed which have

good uncertainty attenuation properties, the pseudo plant method looks useful in deriving robust control algorithms, and hence improve the robustness of corresponding adaptive algorithms. As the pseudo-plant method includes a plant model in the loop, some relationship with internal model control (IMC, [13]) can be derived. Fig.2(a) shows the feedback control scheme using a pseudo-plant, while fig.2(b) shows that of IMC. In IMC, the closed-loop transfer-function from w and disturbance l to y becomes

$$y=\frac{CP}{1+CF_0(P-P_m)}w+\frac{1-CF_0P_m}{1+CF_0(P-P_m)}l \quad (3)$$

In the pseudo-plant method, the closed-loop transfer-function from w and l to y is

$$y=\frac{PC_p}{1+C_pP_m+\alpha C_pF_0(P-P_m)}w+\frac{1+C_p(1-\alpha F_0)P_m}{1+C_pP_m+\alpha C_pF_0(P-P_m)}l \quad (4)$$

Let us assume the perfect matching case (ie. P = Pm), = 1, and all-pass filter Fo = 1, then (3) is changed to (5).

$$y=CP\cdot w+(1-CP)\cdot l \quad (5)$$

while (4) is changed to (6),

$$y=\frac{C_pP}{1+C_pP}w+\frac{1}{1+C_pP}l \quad (6)$$

Equation (5) is the simplest case of IMC and (6) is the simplest case of Pseudo-plant control, so (5) and (6) clearly explain the conceptual defference between the two methods. Firstly, the transfer-function form set-point w to output y can be considered. Equation (5) says that IMC is an open-loop control scheme, whilst (6) says that the pseudo-plant method is an output-feedback scheme. The open-loop characteristics of IMC limit its direct use to open-loop stable plants. For open-loop unstable plants, the feedback-equivalent form of IMC should be used or the plant should be stabilized before IMC is applied[13]. However, as seen in (6), pseudo-plant feedback control under the assumption is exactly the same as output feedback control and it enables the use of the pseudo-plant method to open-loop unstable plants. Secondly, disturbance rejection can be considered. For IMC, the condition for disturbance rejection is CP = 1, and it means that the control gain should be carefully chosen to satisfy the relation of C = 1/P, which is generally impractical. In the case of pseudo-plant control, the condition for disturbance rejection is simply to increase the control gain and coinsides with general feedback control methods.

3. Identification with a reduced order model

Adaptive Control is in many cases based on the real time estimation of the plant or controller parameters. But parameter estimation is a function of a chosen model, input-output data, chosen algorithm and some implementation techniques. As any actual process can not be modelled completely by a finite order system, any chosen model has some unmodelled dynamics, and parameter estimation with a reduced order model is not only difficult but also quite often-happening problem. The concept of true parameter dose not exist in this problem. Only the concept of good parameter estimation in the user chosen band-width exist and has the practical importance. So in practical parameter estimation, some implementation techinques for a meaningful result is necessary. The following example shows the frequency-dependent phenomena in a reduced order identification.

Let us consider the Rohr's benchmark example[1].

$$P(s)=\frac{458}{(s+1)(s^2+30s+299)}$$

If we try to identify the plant with structure $Pm(s) = \hat{b}/(s+\hat{a})$ using sinusoidial signals of frequency w, $\hat{a}$ and $\hat{b}$ will be functions of frequency. Substituting s=jw and matching the plant with the model in real and imaginary parts in the form of $\hat{b}(w)/(jw+\hat{a}(w))$, where $\hat{a}(w)$ and $\hat{b}(w)$ are real functions of w, gives

$$\hat{a}(w)=\frac{229-31w^2}{259-w^2}$$

and

$$\hat{b}(w)=\frac{458}{259-w^2}$$

Fig.3 represents the parameter estimates as function of the test signal frequency. As $w\to\sqrt{259}$, the estimates tend to infinity

Considering estimation of the above system through a low-pass filter of 1/(s+1), the filtered plant P(s) is as follows.

$$Pf(s)=\frac{458}{(s+1)^2(s^2+30s+229)}$$

Identification of the above process, Pf(s), by a $\hat{b}/(s+\hat{a})$ type estimator gives the results as Fig.4. Fig. 5 is the results of using a second order filter $1/(s+1)^2$.

Let us show the same procedure for a suitable chosen pseudo-plant. As the first break-point frequency of the plant is 1 and the DC gain of the plant is 2, choice of Fo as $1/(s+1)$ and Pm $2/(s+1)$ may well be acceptable. Then the pseudo-plant becoms

$$P_\alpha(s)=\frac{2s^3+(62-2\alpha)s^2+(518-60\alpha)s+458}{s^4+32s^3+290s^2+488s+229}$$

The variations of $\hat{a}_\alpha(w)$ and $\hat{b}_\alpha(w)$ are shown in Fig.6. The pseudo-plant parameter estimates are less sensitive than the plant parameter estimates to the frequency of the test-signals. Furthermore, as α decreases, the pseudo-plant parameters become even less sensitive. As the pseudo plant contains a model to guide in the estimation, the variation in the estimates is bounded to be near to the chosen model. From (2), it is found that, as $\alpha\to 0$, the pseudo-plant approaches the chosen model and the identification of the pseudo-plant becomes a simple chosen model identification. So the choice of a suitable model $\hat{P}m$ is important: if the plant parameters vary $\hat{P}m$ could be updated, though implementation of this scheme is not dealt with here.

4. Closed-loop analysis

For ease of analysis, pole assignment control is adopted. The robustness improvement effect using the pseudo-plant method with Ortega's[14] algorithm is shown in Kim et al.[5]. Only the closed-loop analysis for the pseudo-plant method and the observer known as T-filtering is compared.

Let us assume P = B/A and Pm = $\hat{B}/\hat{A}$.

Fig.7 shows the pole-assignment loop and T is an observer polynomial.

If T = 1 and the model is perfect in Fig.7, the closed-loop pole is

Pc = RA + SB.

As T is introduced, the closed-loop pole equation becomes

$$R\hat{A} + S\hat{B} = PcT$$

The actual closed-loop becomes

$$y(t) = \frac{B}{Pc}\,\frac{1 + \hat{B}\Delta/Pc}{1 + \hat{B}\Delta S/PcT}\,w(t) + \frac{RA}{PcT}\,\frac{1}{1 + \hat{B}\Delta S/PcT}\,d(t) \quad (7)$$

Fig.8 shows the pole-assignment control of the pseudo-plant. In this loop, no T filter is introduced, but a kind of low-pass filter Fo is used in the construction of pseudo-plant.

The closed-loop becomes

$$y(t) = \frac{B}{Pc}\,\frac{1 + \Delta}{1 + Fo\Delta BS/Pc}\,w(t) + \frac{AR - BFoS}{Pc(1+Fo\Delta BS/Pc)}\,d(t) \quad (8)$$

As Δ is assumed to be stable but unknown dynamics term, the closed-loop behaviour of equation (7) or (8) is not so clear. However, considering the closed-loop relation from w(t) to y(t), the pole equations could be made the same if T = 1/Fo. But the zero equations are different in each cases.

As a guide-line for T-filter, it is said, "To ensure good robustness properties, choose $T = (1 - \beta q^{-1})^n$ such that the cut-off frequency β coincide with the dominant time constant of $\hat{A}$ and n is the degree of $\hat{A}$".

But we propose a physically meaningful method. As Pc is a pre-known value, if we know $\hat{B}$, S could be known. But in the process of delf-tuning control, $\hat{B}$ and $\hat{S}$ could not be pre-known precisly. However, some nominal model of the plant could be known and the chosen nominal model is assumed to model the process well. In this case, as the design object of Fo is to reduce the perturbation term FoΔBS, to choose Fo to minimize FoΔ could be a nice one. The term Δ is not exactly known but could be assumed not to have a big gain in the frequency band of the chosen model Pm, and the choice of Fo = Pm is recommended. (But the gain Fo should be tuned to 1). Another merit of the proposed method is insensitive to the frequency changes of the input signals. However, the pseudo-plant should contain a suitable model Pm in it. So to be applied for time-varying plants, an isolated identification loop for model selection should be used. This could be the concept of expert or fault detection technique, and dealing it in more detail is defered.

5. Simulation results

The same example as in section 3 is used. Sampling the system by 0.04 seconds, the discrete plant becomes as follows.

$$P(q^{-1}) = \frac{q^{-1}(0{\cdot}0036 + 0{\cdot}0107q^{-1} + 0{\cdot}0019q^{-2})}{1 - 2{\cdot}0549q^{-1} + 1{\cdot}3524q^{-2} + 0{\cdot}2894q^{-3}}$$

Fig.9 is the result of a first-order controller with the pseudo-plant method. Here $Pm = 0.0647q^{-1}/(1 - 0.9693q^{-1})$, which is the first-order estimation of the discrete plant by the test-signal

$u(t) = 1.9\sin(0.00001\pi t) + (0.51\sin(0.0005\pi t)$
$+ 0.23\sin(0.001\pi t)$

The magnitude of each component was chosen randomly, but the frequencies were chosen in the low-frequency band. Even the plant is third-order and the first-order controller stabilize the plant well. The choice of $Pm = q^{-1}0.0784/(1 - 0.9608q^{-1})$, which is the discrete version of $2/(s + 1)$, shows almost the same result. In this simulation, $\alpha = 1$ and $Fo = q^{-1}0.0392/(1 - 0.9608q^{-1})$, the discrete version of $1/(s + 1)$. Another choice for $Fo = q^{-1}0.0769/(1 - 0.9231q^{-1})$, the discrete version of $2/(s + 2)$, shows a similar result.

As an another example, the plant,

$$P(q^{-1}) = \frac{0.0376q^{-1} - 1.3148q^{-2} + 0.7184q^{-3} - 0.1129q^{-4}}{1 - 2.219q^{-1} + 1.7653q^{-2} - 0.5152q^{-3} + 0.0505q^{-4}}$$

is tested. We design Pm tobe a reduced-order model of the nominal model. That is,

$$Pm(q^{-1}) = \frac{0.2628q^{-1} - 0.2279q^{-2}}{1 - 1.7765q^{-1} + 0.7944q^{-2}}$$

We also design so that the filter Fo of the pseudo-plant has the same poles as Pm. That is,

$$Fo = \frac{0.054q^{-1}}{1 - 1.7765q^{-1} + 0.7944q^{-2}}$$

Fig.10 is the Bode Diagram of the plant, the model, and the filter. Fig.11 shows outputs of the plant. Using the filter Fo, the robustness is improved. So it is recommended that the filter Fo is to be similar to Pm (ie. they have same poles or similar shapes of Bode diagram.).

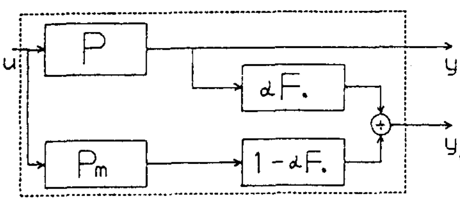

Fig.1 The pseudo-plant

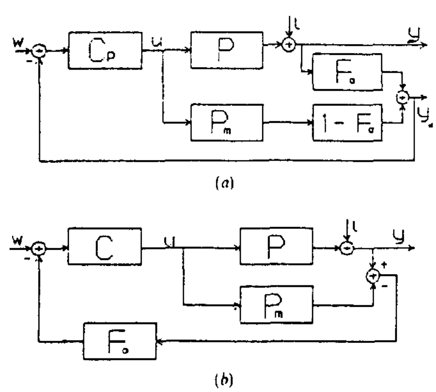

Fig.2 Feedback control of the following:
(a) the pseudo-plant
(b) internal model control

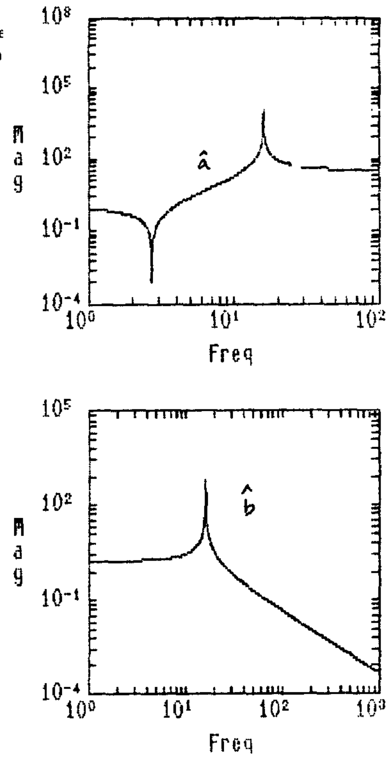

Fig.3 Variations of the plant parameter estimates

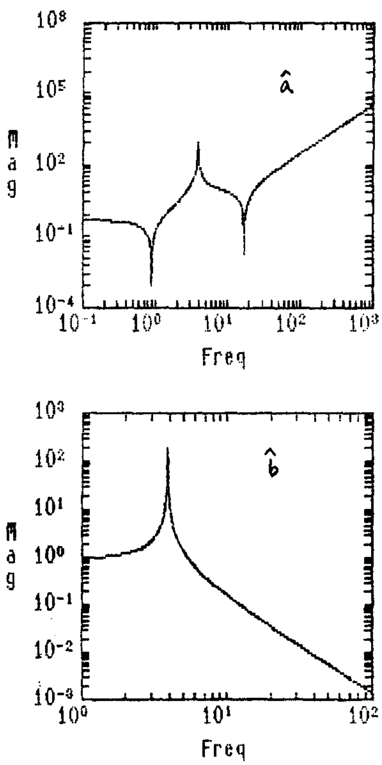

Fig.4 Variations of the plant parameter estimates

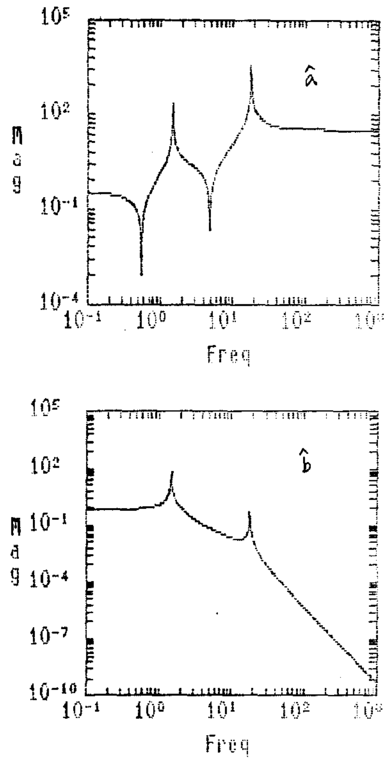

Fig.5 Variations of the plant parameter estimates

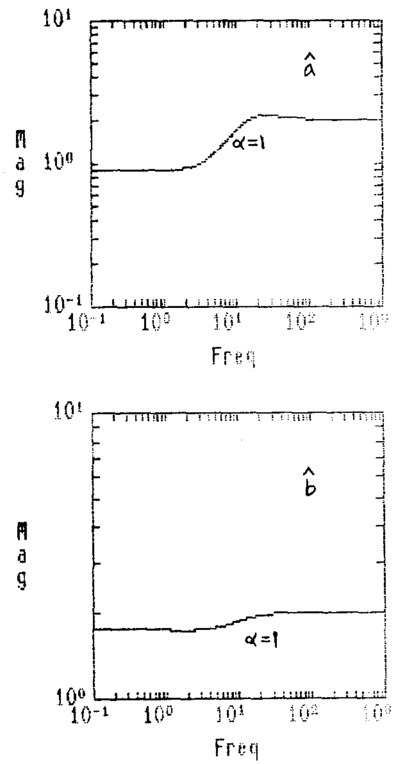

Fig.6 Variations of the pseudo-plant parameter estimates

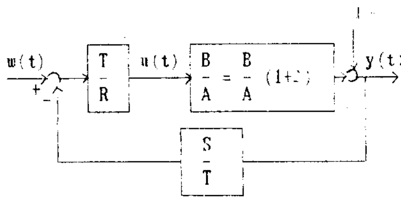

Fig. 7 The pole-assignment loop

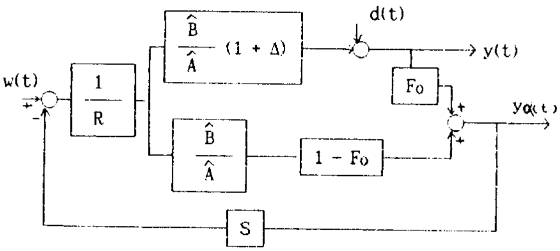

Fig. 8 The pole-assignment control
of the pseudo-plant

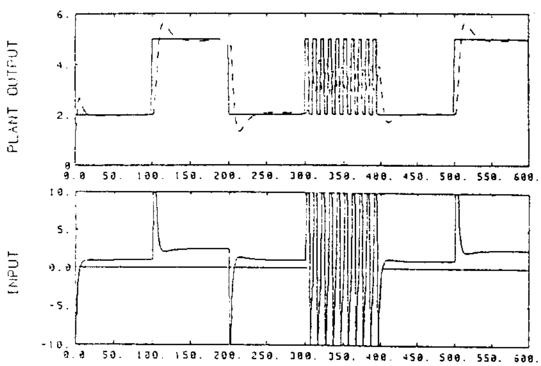

Fig. 9 Rohrs' example: T=0.04 s,
pseudo-plant method,
first-order adaptive control

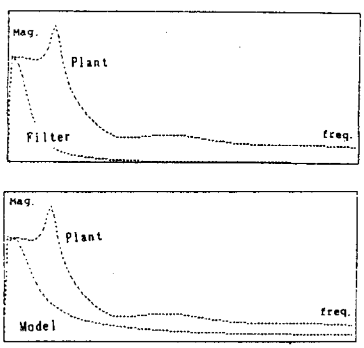

Fig. 10 Bode diagram of plant, model & filter

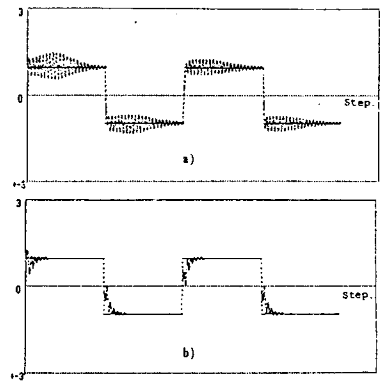

Fig. 11 Output of the pseudo-plant
(a) without filter (b) with filter

6. Conclusion

The concept of pseudo-plant is introduced and the comparative results of heuristic identification with reduced order model shows the effectiveness of the proposed method. A relation between the pseudo-plant method and the IMC is clarified. Also the closed-loop comparision of the proposed method and T-filtering is done. A guide-line for the pseudo-plant implementation is proposed and simulation shows robustness of the proposed method. The theoretical results can be found in [5].

7. References

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