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AN ACTIVE VIBRATION CONTROL SYSTEM WITH STATE FEEDBACK

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Abstract: This paper considers an active vibration control system based on pole placement incorporating the internal model principle when the system is subjected to disturbances which are generated by a linear dynamical system. Experimental results are presented which show the effectiveness of the method when the system is excited by a sine wave disturbance and system parameters are known. An adaptive control design is also discussed.

1. Introduction

Isolation of a mechanical system from vibrations produced by the environment requires the use of a vibration control system.

In the past, passive vibration control systems which use masses, springs and dampers were considered widely by designers. However, passive systems have their own performance limitations and a designer has no choice but to accept a compromise solution when designing a passive vibration control system.

Recently the possibility of using active systems has been explored[1-8]. Active systems have several theoretical advantages. First, they can supply energy, whereas a passive system can only absorb it. A second advantage is that active systems can be adapted to various conditions. In cases where the performance limitations of passive systems cannot be accepted and the cost of active control systems is justifiable in view of their improved performance, active vibration control systems are powerful candidates.

Active vibration control methods are classified into two categories; one is a type supported at a fixed point[1-5] and the other is a type using an auxiliary mass[6-8]. The former has a simple structure and a high vibration control performance, but needs a fixed point to support an active controller. Meanwhile, the latter has a feature that the application field is wider than the former because fixed points are not required.

This paper investigates an active vibration control system based on pole placement with the internal model principle. The advantage is that it is effective for all kinds of deterministic disturbance which is generated by a linear dynamical system although disturbances are disregarded on design or a special type of disturbance is considered in other studies. It has been already shown that this controller achieved an excellent suppression result experimentally for the vibration control system of a type supported at a fixed point[5]. In this paper, the control technique mentioned above is applied to another type of active vibration control system; an auxiliary mass system. Experimental results are presented which indicate the effectiveness of the method when the system is excited by a sine wave disturbance.

2. Problem Formulation

Consider an active vibration control system shown in Fig.1. The system consists of a main vibratory mass-spring-damper system, an active control mass-spring-damper system, a servo controller and a digital computer. The system output is the position of the main mass and the input is the driving voltage to the servo amplifier. The main vibratory mass is disturbed by an exciting force through the spring of the main vibratory system. The mass of the active controller is actuated by the servo motor. The inertia force of the active control mass is the driving force to suppress the vibration of the main vibratory mass.

The vibration system of Fig.1 is modelled by the equation

$$M\ddot{y}+d_2\dot{y}+k_2y+k_1(y-z)+d_1(\dot{y}-\dot{z}) = k_3(w-y)-k_4(x-z+y) \qquad (2.1)$$

$$m\ddot{z}+k_1(z-y)+d_1(\dot{z}-\dot{y})=k_4(x-z+y) \qquad (2.2)$$

- M : mass of main vibratory system
- m : mass of active controller
- d_i : viscous damping coefficient
- k_i : spring constant
- y : displacement of main mass from the normal point on the fixed plane
- z : displacement of active

controller mass from the normal point on the fixed plane

x : displacement of control force driver from the normal point on the main mass

w : displacement of disturbance

The model of the servo controller can be given by

$$\dot{x}=sx+ru \tag{2.3}$$

u : voltage input to servo amplifier

This system is controllable and observable.

The equations (2.1), (2.2) and (2.3) are converted into a discrete-time system for the use of a digital computer.

$$A(q^{-1})y(t)=B(q^{-1})u(t)+C(q^{-1})w(t) \tag{2.4}$$

$$A(q^{-1})=1+a_1q^{-1}+...+a_nq^{-n} \qquad (n=5)$$

$$B(q^{-1})=b_1q^{-1}+...+b_mq^{-m} \qquad (m=5)$$

$$C(q^{-1})=c_1q^{-1}+...+c_\ell q^{-\ell} \qquad (\ell=5)$$

where q^{-1} is the unit delay operator and parameters a_i, b_i and c_i can be represented by the system parameters and a sampling period T.

It is assumed that w(t) is a deterministic disturbance generated by a linear finite-dimensional dynamical system of the form

$$D(q^{-1})w(t)=0 \tag{2.5}$$

where $D(q^{-1})$ is a polynomial of degree p having distinct roots on the unit circle. If the disturbance is the sum of sine wave signals or a periodic signal or a step signal, then it is describable by (2.5).

It is also assumed that the polynomials $A(q^{-1})D(q^{-1})$ and $B(q^{-1})$ are relatively prime and that the degrees n, m and p are known.

The objective is to make the output y(t) converge to zero asymptotically, that is, to achieve suppression of vibration of the main vibratory mass by measurement of the displacement y(t).

3. Control Theory

Pole placement control combined with the internal model principle is designed for the vibration suppression system.

In this section, coefficients of polynomials $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ are assumed to be known.

The control law is given as

$$S(q^{-1})D(q^{-1})u(t)=-R(q^{-1})y(t) \tag{3.1}$$

where the controller polynomials $S(q^{-1})$ and $R(q^{-1})$ are obtained by solving the Diophantine equation

$$A(q^{-1})D(q^{-1})S(q^{-1})+B(q^{-1})R(q^{-1}) =A^*(q^{-1}) \tag{3.2}$$

$$S(q^{-1})=1+s_1q^{-1}+...+s_{m-1}q^{-m+1}$$

$$R(q^{-1})=r_1q^{-1}+...+r_{n+p-1}q^{-n-p+1}$$

The polynomial $A^*(q^{-1})$ is arbitrarily selected to be stable with desired closed-loop poles and with the degree<n+m+p. Then the closed-loop system resulting from the feedback law (3.1) has the following property.

$$\lim_{t\to\infty} y(t)=0 \tag{3.3}$$

The proof is shown as follows: Since the polynomials $A(q^{-1})D(q^{-1})$ and $B(q^{-1})$ are relatively prime, the equation (3.2) always has the unique solution. From (3.2)

$$A^*(q^{-1})y(t) =A(q^{-1})D(q^{-1})S(q^{-1})y(t) +B(q^{-1})R(q^{-1})y(t) \tag{3.4}$$

It is relatively easy to show by (2.4) and (2.5) that

$$A(q^{-1})D(q^{-1})y(t)=B(q^{-1})D(q^{-1})u(t) \tag{3.5}$$

Substituting (3.5) into (3.4) and using (3.1) give

$$A^*(q^{-1})y(t) =B(q^{-1})D(q^{-1})S(q^{-1})u(t) +B(q^{-1})R(q^{-1})y(t) =0 \tag{3.6}$$

The objective (3.3) is attained since the polynomial $A^*(q^{-1})$ is given to be stable.

4. Experiment

In experiment, a disturbance was set to be a sine wave. Hence

$$D(q^{-1})=1-2\cos(\omega T)q^{-1}+q^{-2} \tag{4.1}$$

where ω is the angular velocity of a disturbance. Parameters of the experimental system and design parameters are shown in Table 1.

The experimental results are depicted in Figs.2-4. The control started at 3 sec. The output converges to near the origin. However, small deviation remains because of the measurement noise and a nonlinear friction. Figure 4 shows the power spectrum of the displacement of the main vibratory mass at the steady state. The spectrum of the maximum amplitude in the case of no control is 0 dB. The amplitude of the output is suppressed to about 4.5% at the fundamental frequency of the disturbance.

5. Adaptive Design

Plant parameters and the dynamics of a disturbance were assumed to be known in Section 3 and 4. However, they are usually unknown. In such a case, adaptive control is effective. An adaptive control design is discussed in this section.

It is assumed that coefficients of polynomials $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ are unknown.

For the purpose of parameter estimation, the nonminimal model of the plant (3.5) is written as

$$y(t)=\theta^T\phi(t) \tag{5.1}$$

where

$$\phi^T(t)=[y(t-1),\dots,y(t-n-p),\; u(t-1),\dots,u(t-m-p)]$$

$$\theta^T=[-\bar{a}_1,\dots,-\bar{a}_{n+p},\; \bar{b}_1,\dots,\bar{b}_{m+p}]$$

The elements of θ are the coefficients of the polynomials

$$A(q^{-1})D(q^{-1})=\bar{A}(q^{-1}) =1+\bar{a}_1q^{-1}+\dots+\bar{a}_{n+p}q^{-n-p} \tag{5.2}$$

$$B(q^{-1})D(q^{-1})=\bar{B}(q^{-1}) =\bar{b}_1q^{-1}+\dots+\bar{b}_{m+p}q^{-m-p} \tag{5.3}$$

The weighted least squares algorithm is implemented for identification of these parameters.

$$\hat{\theta}(t)=\hat{\theta}(t-1)-\frac{P(t-1)\phi(t)(\hat{y}(t)-y(t))}{\lambda+\phi^T(t)P(t-1)\phi(t)} \tag{5.4}$$

$$P(t)=\frac{1}{\lambda}[P(t-1)-\frac{P(t-1)\phi(t)\phi^T(t)P(t-1)}{\lambda+\phi^T(t)P(t-1)\phi(t)}] \tag{5.5}$$

$$\hat{y}(t)=\hat{\theta}^T(t-1)\phi(t) \tag{5.6}$$

where $P(0)=P^T(0)>0$, $0<\lambda\leq 1$ and

$$\hat{\theta}^T(t)=[-\hat{\bar{a}}_1(t),\dots,-\hat{\bar{a}}_{n+p}(t),\; \hat{\bar{b}}_1(t),\dots,\hat{\bar{b}}_{m+p}(t)]$$

is the estimated parameters vector of θ.

In order to calculate the estimated greatest common divisor polynomial $\hat{D}(q^{-1})$ of $D(q^{-1})$, the following polynomials are defined

$$\hat{\bar{A}}(t,q^{-1}) =1+\hat{\bar{a}}_1(t)q^{-1}+\dots+\hat{\bar{a}}_{n+p}(t)q^{-n-p} \tag{5.7}$$

$$\hat{\bar{B}}(t,q^{-1}) =\hat{\bar{b}}_1(t)+\dots+\hat{\bar{b}}_{m+p}(t)q^{-m-p} \tag{5.8}$$

and Euclidean algorithm is applied to $\hat{\bar{A}}(t,q^{-1})$ and $\hat{\bar{B}}(t,q^{-1})$[5]. Dividing $\hat{\bar{B}}(t,q^{-1})$ by $\hat{D}(t,q^{-1})$ yields the estimated polynomial $\hat{B}(t,q^{-1})$ of $B(q^{-1})$. The input is given as follows;

$$\hat{S}(t,q^{-1})\hat{D}(t,q^{-1})u(t) =-\hat{R}(t,q^{-1})y(t) \tag{5.9}$$

where $\hat{S}(t,q^{-1})$ and $\hat{R}(t,q^{-1})$ are the solution of the Diophantine equation

$$\hat{\bar{A}}(t,q^{-1})\hat{S}(t,q^{-1})+\hat{B}(t,q^{-1})\hat{R}(t,q^{-1}) =A^*(q^{-1}) \tag{5.10}$$

$$\hat{S}(t,q^{-1})=1+\hat{s}_1(t)q^{-1}+\dots+\hat{s}_{m-1}(t)q^{-m+1}$$

$$\hat{R}(t,q^{-1})=\hat{r}_1(t)q^{-1}+\dots+\hat{r}_{n+p-1}(t)q^{-n-p+1}$$

As an illustration of the approach presented above, a numerical simulation was carried out. Design parameters are shown in Table 2. The control began at 5.0 sec. The displacement of the main vibratory system is demonstrated in Fig.5. Figure 6 indicates the input voltage to the servo amplifier. From Fig.5, the desired performance is obtained.

This adaptive algorithm will be applied to the experimental system in the future.

6. Conclusions

This paper deals with an active vibration control problem using an auxiliary mass-damper-spring system. Pole placement control combined with the internal model principle is proposed. Experimental results show that this control algorithm provides a desired performance in the case when the system parameters and the angular velocity of a sine wave disturbance are known. Furthermore, an adaptive design of this controller is discussed and the effectiveness is confirmed by simulation.

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Table 1 Experimental Condition

main mass active controller mass	$M = 5.6$ kg $m = 1.2$ kg
viscous damping coefficient	$d_1 = 10.0$ Ns/m $d_2 = 10.0$ Ns/m
spring constant	$k_1 = 166$ N/m $k_2 = 1300$ N/m $k_3 = 1400$ N/m $k_4 = 1800$ N/m
servo controller	$s = -1.8$, $r = 0.015$
disturbance	amplitude = 0.0007 m $\omega = 2\pi$ rad/s
sampling period	$T = 0.1$ s
desired closed-loop polynomial	$A^* = 1-0.8q^{-1}$

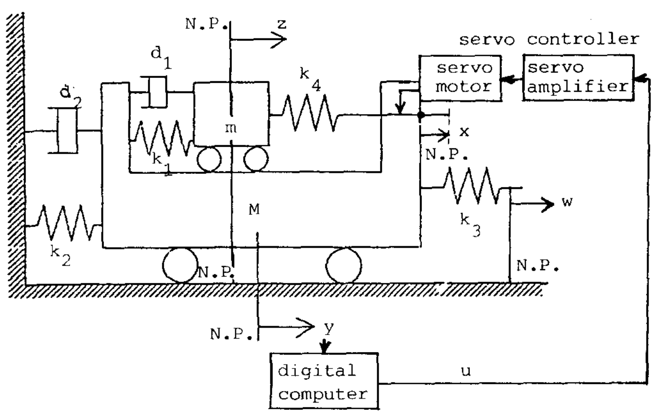

Fig.1 Active vibration control system

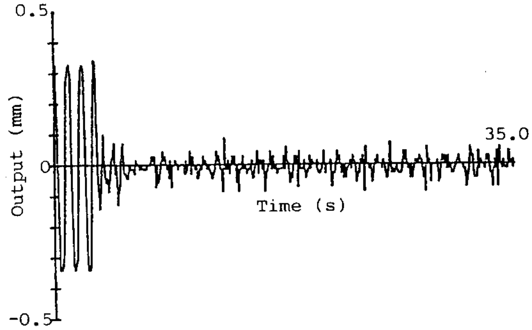

Fig.2 Displacement of the main mass

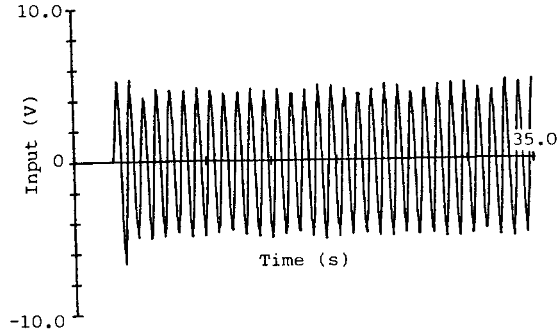

Fig.3 Input voltage to the amplifier

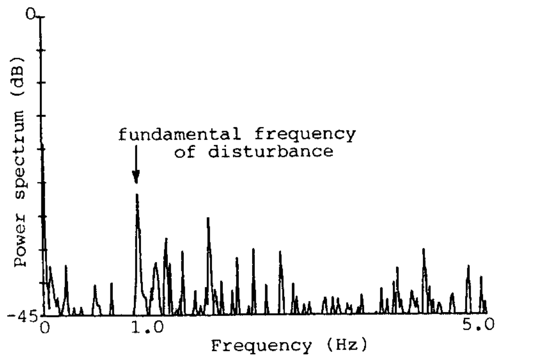

Fig.4 Power spectrum of the displacement of the main mass

Table 2 Simulation Condition

sampling period	$T = 0.1$ s
desired closed-loop polynomial	$A^* = 1-0.7q^{-1}$
parameter identification law	$\lambda = 0.5$ $P(0) = 10^6 I$ $\hat{\theta}(0) = 0.9\theta$

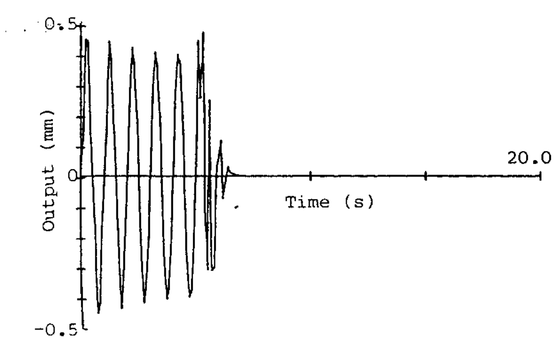

Fig.5 Displacement of the main mass

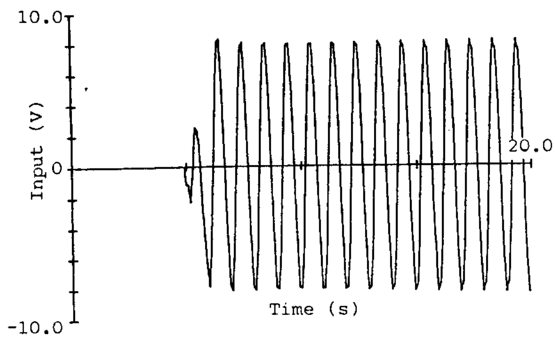

Fig.6 Input voltage to the amplifier