

# A Practical Identification Method for Robot System Dynamic Parameters

Sungkwun Kim

Robotics & Automation R/D Div., Information System Business  
Samsung Electronics  
Gumi, Kyung Buk, Korea

A practical method of identifying the inertial parameters, viscous friction and Coulomb friction of a robot is presented. The parameters in the dynamic equations of a robot are obtained from the measurements of the command voltage and the joint position of the robot. First, a dynamic model of the integrated motor and manipulator is derived. An off line parameter identification procedure is developed and applied to the University of Minnesota Direct Drive Robot. To evaluate the accuracy of the parameters the dynamic tracking of robot was tested. The trajectory errors were significantly reduced when the identified dynamic parameters were used.

## 1. Introduction

The model-based control schemes such as the computed torque and resolved acceleration [Luh, Walker and Paul, 1980a,b] controller has been shown to compensate for manipulator non-linearities of high speed robots. These control schemes depend on the accuracy of the dynamic parameters. Therefore, precise parameter identification is essential for accurate robot control.

Earlier work in identification of robot dynamics concentrated on estimating the mass of payload. Paul[1981] presented two techniques with valid for the manipulators at rest. The first method used the joint torques/forces, and the second method used a wrist torque/force sensor. Coiffet[1983] extended this technique to also estimate the center of mass of the payload. By using special test torques and moving only one joint at a time, the moment of inertia of the payload can also be estimated. Atkeson, An and Hollerbach[1986] have proposed an approach which uses a wrist torque/force sensor to estimate the inertial parameters of a manipulator. Their approach has also been extended to identify the inertial parameters of all the links of a robot [Atkeson, C. G., An, C. H. and Hollerbach, J. M., 1986]. Khosla and Kanade [1985] developed an algorithm to estimate the inertial parameters of a robot from the measurements of inputs (actuating torques/forces) and outputs (joint positions, velocities and accelerations). One major problem associated with their method is that joint acceleration has to be obtained by numerical differentiation of position of velocity signals. This introduce noise and affects the accuracy of the estimated parameters. Also there is no way to find the friction force.

A new method to identify the dynamic parameters of a robot is presented from the measurements of its inputs(command voltage) and only joint positions. A mathematical model is introduced from the integrated system of links and actuators. The solution of the model is obtained. The identification method developed is implemented on the University of Minnesota Direct Drive Arm [Kazerooni and Kim, 1987, 1988, Kim, 1988].

## 2. Mathematical model for robot dynamic parameter identification

A permanent magnet AC synchronous motor(DC brushless motor) is considered. The electrical equation of a brushless DC motor is given by

$$V_t(t) - E_a(t) + R_a i_a(t) + L_a \frac{d i_a(t)}{dt} \quad (1)$$

where  $V_t$  is the applied voltage to the armature terminals of a motor,  $i_a$  is the armature current,  $E_a$  is the induced back emf,  $R_a$  and  $L_a$  are the armature winding resistance and inductance, respectively

The electromagnetic torque,  $T_{em}$ , and back emf,  $E_a$ , produced by the motor are expressed as

$$T_{em}(t) = K_t i_a(t) \quad (2)$$

$$E_a(t) = K_e \omega(t) \quad (3)$$

where  $K_t$  and  $K_e$  are the motor constant and back emf constant, respectively.  $\omega$  is the angular velocity of the shaft

The above equations can be combined into

$$V_i(t) = K_e \omega(t) + R_a \frac{T_{em}(t)}{K_t} + \frac{L_a}{K_t} \frac{dT_{em}(t)}{dt} \quad (4)$$

In general, the armature inductance,  $L_a$ , in a brushless torque motor is low enough so the amplifiers can be considered as current sources. The resistance of the motor armature is the dominant source of impedance. This allows simplification of equation (4) as follows

$$V_i(t) = K_e \omega(t) + R_a \frac{T_{em}(t)}{K_t} \quad (5)$$

The arbitrary load requires a load torque, which the motor must provide.

$$T_{em}(t) = T_{load}(t) \quad (6)$$

where the load torque,  $T_{load}(t)$ , is the sum of the joint torque,  $\tau(t)$ , and friction torque  $T_F(t)$ .

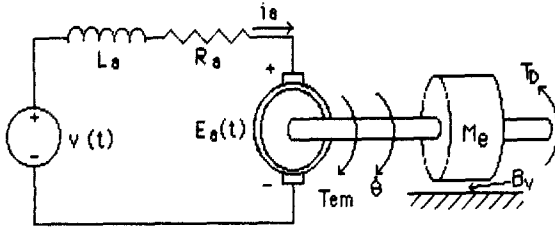


Figure 1 Integrated system for a motor and its load

The load torque,  $T_{load}(t)$ , can be considered as the sum of inertial torque,  $T_I$ , friction torque,  $T_F$ , and explicit load torque,  $T_L$ :

$$T_{load}(t) = T_I(t) + T_F(t) + T_L(t) \quad (7)$$

For integrating the motor dynamics with the manipulator dynamics and friction, a convenient approach is to view each joint motor as a subsystem with these systems interconnected by disturbance torques. The inertial torque required by the combination of the motor and load inertia at joint  $i$  can be expressed as

$$T_{Ii}(t) = (M_{mi} + M_{ij}[\theta(t)])\ddot{\theta}_i(t) - M_{ei}[\theta(t)]\dot{\theta}_i(t) \quad (8)$$

where  $M_{mi}$  denotes the combined moment of inertia of motor drive shaft and rotor assembly, and  $M_{ij}[\theta(t)]$  is the

effective inertia of joint  $i$ , which is the  $i$ -th diagonal element of matrix  $M[\theta(t)]$ , and note that  $M_{ei}[\theta(t)] = M_{mi} + M_{ij}[\theta(t)]$ .

Friction torque is given by

$$T_{Fi}(t) = B_{vi} \dot{\theta}_i(t) + T_{nfi}(t) \quad (9)$$

where  $B_{vi}$  is the combined viscous friction coefficient of the motor shaft and joint  $i$ , and  $T_{nfi}$  is nonlinear friction torques for joint  $i$  (Coulomb and stiction).

The explicit load torque,  $T_L$ , can be expressed as

$$T_{Li}(t) = \sum_{\substack{j=1 \\ j \neq i}}^n M_{ij}[\theta(t)] \ddot{\theta}_j(t) + C_i[\theta(t), \dot{\theta}(t)] \quad (10)$$

where  $M_{ij}[\theta(t)]$  is  $(i, j)$ -th element of matrix  $M[\theta(t)]$  and  $C_i[\theta(t), \dot{\theta}(t)]$  denotes the  $i$ -th element of vector of the centrifugal, Coriolis, and gravity forces.

Substituting equations (8) through (10) into (7) yields

$$T_{loadi}(t) = M_{ei}[\theta(t)]\ddot{\theta}_i(t) + B_{vi} \dot{\theta}_i(t) + T_{Di}(t) \quad (11)$$

where  $T_{Di}(t) = T_{Li}(t) + T_{nfi}(t)$

Since the load torque consists of inertial, friction terms, and disturbance load, the integrated dynamic equation of a manipulator from equations (5) and (11) is given by

$$V_{ti}(t) = K_{ei} \dot{\theta}_i(t) + (M_{ei}[\theta(t)] \ddot{\theta}_i(t) + B_{vi} \dot{\theta}_i(t) + T_{Di}(t)) R_{ai} / K_{ti} \quad (12)$$

Eliminating the subscript "i", equation (12) for an individual joint can be simplified as

$$\ddot{\theta}(t) + \frac{1}{M_e[\theta(t)]} (B_v + \frac{K_e K_t}{R_a}) \dot{\theta}(t) = \frac{1}{M_e[\theta(t)]} (V_i(t) \frac{K_t}{R_a} - T_D(t)) \quad (13)$$

where  $M_e$  is the combined moment of the inertia of the motor rotor, shaft and links, and  $B_v$  is the viscous friction which is proportional to the angular velocity.  $T_D$  is the disturbance load including Coulomb friction, off-diagonal terms of inertial force, centrifugal, Coriolis, and gravity forces.

To simplify, equation (13) can be rewritten in the form

$$\ddot{\theta} + A \dot{\theta} = U \quad (14)$$

$$\text{where } A = \frac{1}{M_e} (B_v + \frac{K_e K_t}{R_a}) = \frac{B}{M_e} \quad (15)$$

$$U = \frac{1}{M_e} \left( V_t \frac{K_t}{R_a} - T_D \right) \quad (16)$$

For a given step voltage, all the terms in the equations (15) and (16) are constants. The torque constant,  $K_t$ , and armature resistance,  $R_a$ , are either known from the manufacturers specification or can be measured, but the combined moment of inertia,  $M_e$ , damping,  $B$ , and disturbance,  $T_D$ , are unknown. Note that since stiction is only experienced, it is not included in the dynamic model.

Equation (14), which is nonhomogeneous, can be solved by the general method. The corresponding solution of differential equation (14) is

$$\theta(t) = E_1 + E_2 e^{-At} + (U/A)t \quad (17)$$

where  $E_1$  and  $E_2$  are constants.

The initial conditions of equation (17) are

$$\theta(0) = 0, \dot{\theta}(0) = 0$$

By substituting the initial conditions into this equation, the constants are given by

$$E_1 = -U/A^2, E_2 = U/A^2$$

Substitution into equation (17) yields

$$\theta(t) = \frac{U}{A^2} (e^{-At} + At - 1) \quad (18)$$

Substituting Taylor series for the exponent,  $e^{-At}$ , into equation (18) gives

$$\theta(t) = U \left( \frac{t^2}{2!} - \frac{A t^3}{3!} + \frac{A^2 t^4}{4!} - \frac{A^3 t^5}{5!} + \frac{A^4 t^6}{6!} \dots \right) \quad (19)$$

Constants  $A$  and  $U$  are obtained by Least Square method applied to equation (19) using many data,  $\theta(t)$  and  $t$ , for a given step input voltage. The combined moment of inertia,  $M_e$ , and damping,  $B$ , relative to the viscous friction and the back emf are determined by equations (15) and (16). Coulomb friction can be obtained using a data set from a different input voltage. From equations (15), (16) and (18), we note the following properties: 1) From measuring a step input voltage and output (only angular position) as a function of time,  $A$  and  $U$  can be obtained.  $A$  and  $U$  are constants for a given step input voltage, consisting of an input voltage, the combined term of viscous friction and back emf, Coulomb friction, armature winding resistance

and the combined inertial moment of the motor and the robot links. 2) The combined moment of inertia,  $M_e$ , the combined term of viscous friction and back emf,  $B$ , and the Coulomb friction,  $T_D$ , are directly determined from  $A$  and  $U$ . It is not necessary to separate the viscous friction from the combined term because the viscous friction and the back emf both act as damping term.

### 3. Dynamic Properties of a Manipulator

In the absence of friction or other disturbance, the dynamics of an n-link rigid manipulator can be written as

$$M(\theta)\ddot{\theta} + C(\theta)(\dot{\theta}^2) + G(\theta)(\dot{\theta}) + G(\theta) = \tau \quad (20)$$

In this form of the dynamic equations, the complexity of the computation is seen to be in the form of computing various parameters which are a function of inertial parameters and the manipulator position,  $\theta$ . The closed form dynamic model of a six degrees of freedom robot is in general very complex, but the closed form dynamic equation can be obtained using a symbolic computation program [Mathlab group, 1983]. To investigate the forms of each term in the dynamic equation, the closed form dynamic equations of the University of Minnesota Direct Drive Manipulator were derived in [Kim, 1988].

The internal properties of the dynamic equations are as follows: Centrifugal force at joint depends on the square of the other joint velocities because diagonal terms in the centrifugal coefficient matrix are zero. Coriolis force is proportional to the product of two different joint velocities. The gravity term depends on only  $\theta$ . Since the inertial parameters of all the links are constant, the inertial parameters can be directly applied to the dynamic equations for a control algorithm.

For parameter identification, when only one joint moves, the Coriolis and the centrifugal terms disappear in the dynamic equation. The dynamic equation can then be written by

$$\tau = M(\theta)\ddot{\theta} + G(\theta) \quad (21)$$

If the robot is statically balanced, only inertial term appears in the equation

$$\tau = M(\theta)\ddot{\theta} \quad (22)$$

To identify the dynamic parameters, only the diagonal terms in the inertial matrix are needed. An example of these inertial terms of the University of Minnesota Direct Drive Manipulator is given in [kim, 1988]

## 4. Identification Procedure for an N Degree of Freedom Robot

The identification procedure for a manipulator is presented in this Section. To simplify the derivation of the dynamic equation for the  $N$  links of the manipulator, the parameter identification problem is started from link  $N$  (the tip) and proceeds sequentially to link 0 (the base). The inertial parameters of each link are individually identified. The inertial parameters identified for link  $i$  become known parameters in the dynamic equation of the link  $i-1$ . The parameter identification procedure is as follows.

1) All the joints are locked at their desired positions except the joint to be tested.

2) Apply a step input voltage to the robot and measure the position of the joint as a function of time. To estimate the Coulomb friction, the measurements of the position must be conducted for two different step inputs because there are three unknowns in two equations (The position of a joint can be measured from the encode or resolver mounted at the shaft of the joint).

3) Calculate  $A$  and  $U$  using the experimental data (time and position). Determine the combined inertia,  $M_e$ , and the combined damping term,  $B$  using equations (15) and (17).

4) Compare the combined inertia  $M_e$  obtained experimentally with the inertial term from the closed form dynamic equation. (Note that only the inertial term in the dynamic equation is used for the parameter identification, because there are no effects of centrifugal and Coriolis force by locking all the joints except the joint tested).

## 5. Experimental results

### Hardware

Experiments for identifying dynamic parameters were performed on the University of Minnesota Direct Drive Robot (as shown in figure 2), which has three degrees of freedom. An IBM AT microcomputer, hosting a 4-node parallel processor, is used as the main controller of this robot. Each node is an independent 32-bit processor with local memory and communication links to the other nodes in the system. A high speed AD/DA converter is used to read the velocity signals and to send analog command signals to the servo controller unit. The servo controller unit produces three phase, Pulse Width Modulated (PWM), sinusoidal currents for the power amplifier. A PWM power amplifier, which provides up to 47 Amperes of drive current from a 325 volt power supply, is used to power the motors. The

peak torque of motor 1 is 118 Nm, while the peak torques of motors 2 and 3 are 78 and 58 Nm respectively.



Figure 2. The University of Minnesota Direct Drive Robot

### Identification Results

A step command signal to a joint results in a constant torque output. Joint position was measured using a resolver. The position of the joint was sampled at 2.4 ms intervals. Each data point consists of the command voltage, position and time. A step input was applied to one motor (actuating one joint). The other joints were locked at desired positions using a fixed position control algorithm. Real time and joint position information was stored in the direct memory of the computer while the link was tested. A file storing the time and the position data was created after moving the link. Inertial parameters were obtained using the identification procedure outlined in previous Section. As an example, Figure 4 shows the position vs. time for a given step command voltage. The computed trajectories from the identified dynamic parameters agree well with the experimental trajectory curves. The results of the identification for the robot dynamic parameters are summarized in Table 1. Because the first link has only one degree of freedom (about the Z - axis), only the  $I_{z1}$  term in the dynamic equation appears. Some parameters in the Table 1 can only be identified in linear combinations. The 6 inertial parameters of joint 3 must be lumped together because joint 3 is operated by a four bar mechanism.

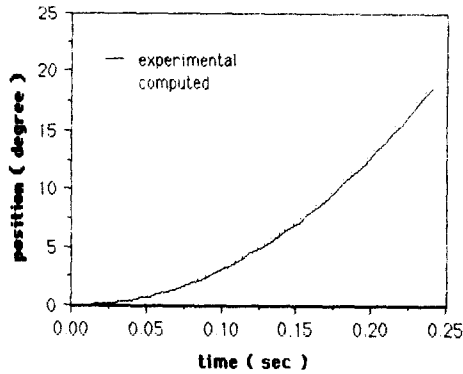


Figure 4 Position vs. time for a step input for motor 2

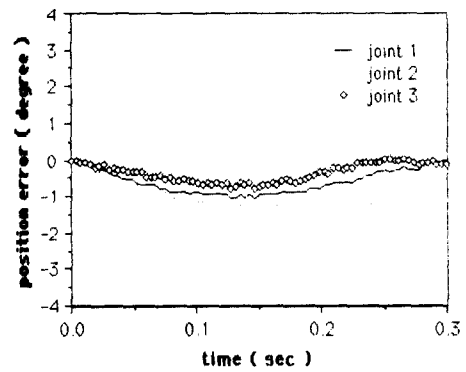
Table 1 Inertial parameters for the University of Minnesota Direct Drive Manipulator

value	Computed value	Parameters	identified	
Inertial (kg-m <sup>2</sup> )		$I_{z1}$	0.19719	0.1752
		$I_{x2}$	0.02592	0.04125
		$m_2 \bar{x}_2^2 + I_{e2} + I_{y2}$	1.68135	1.06955
		$m_2 \bar{x}_2^2 + I_{e2} + I_{z2}$	1.78135	1.16955
		$I_{e1} + I_{e5}$	0.28922	0.298045
		$I_{e1} + I_{e6}$	0.27759	0.296158
		$I_{e3}$	0.20	0.2498
		$I_{e4}$	0.14543	0.1319
Damping in joint (Nm-sec/rad)	motor 1	0.036	-	
	motor 2	0.078	-	
	motor 3	0.02	-	
Coulomb in joint (Nm)	motor 1	2.52	-	
	motor 2	1.0	-	
	motor 3	0.27	-	

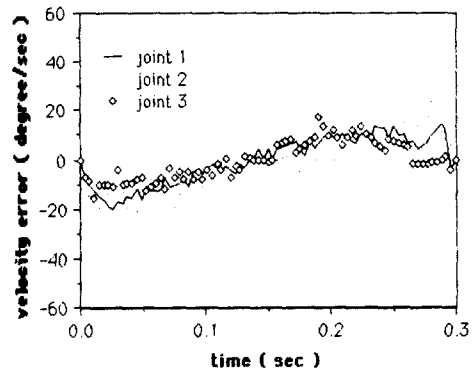
## 6. Feedforward Control Experiments

To verify the accuracy of the experimental dynamic parameters, feedforward control without feedback compensation is applied to the robot. The integrated dynamic model and the identified parameters are used for the control law. The dynamic model does not include the gravity terms because the University of Minnesota Manipulator is statically balanced. The reference trajectory in the experiment is generated by a cubic polynomial. The experimental trajectory is compared with the desired trajectory. The robot control program, written in C language, yields a 250 Hz sampling frequency. Each joint was commanded to move 30 degrees in 0.3 seconds from a predetermined origin. The maximum velocity and acceleration for each joint are 150 degree/sec and 2000 degree/sec<sup>2</sup>, respectively.

The trajectory and velocity errors for each joint are depicted in Figures 5 and 6. Figure 5 shows the trajectory and velocity errors when each joint was commanded to move 30 degrees in 0.3 seconds from a predetermined origin. The maximum tracking errors are -1.06°, -1.35°, and 0.78° for joint 1, 2 and 3, respectively. Figure 6 shows the trajectory and velocity errors when all joints were commanded to simultaneously move 30 degrees in 0.3 seconds from a predetermined origin. The peak trajectory errors are 1.38°, 3.4° and 0.85° for joint 1, 2 and 3 respectively. The results show that the trajectory and velocity errors are increased when all joints were simultaneously moved, because the complex nonlinear dynamic characteristics and unmodeled dynamics exist in the system. Since the modeled system is never the same as the actual system, a closed loop feedback control method is required to compensate for small error.

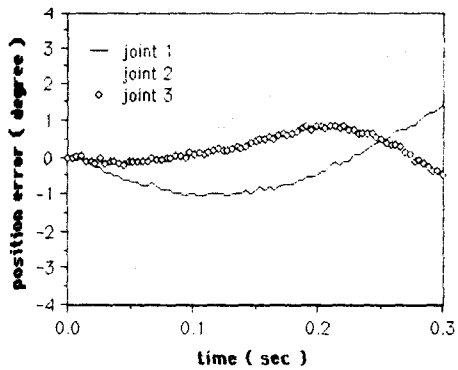


(a)

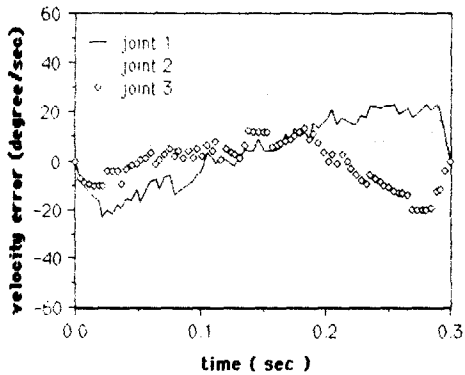


(b)

Figure 5 Trajectory and velocity error curves in each joint with single joint motion. (a) position error. (b) velocity error



(a)



(b)

Figure 6 Trajectory and velocity error curves in each joint with full robot motion (a) position error, (b) velocity error

## 7. Summary

The identification of the inertial parameters was obtained by a integrated robot dynamic system identification method. Damping friction and Coulomb friction were also identified. Most inertial parameters were directly identified by comparing the inertial terms in the closed form dynamic equation. Some dynamic parameters were identified in linear combinations, as in other methods (Atkeson, An, and Hollerbach, 1986; Khosla and Kanade, 1985). The accuracy of the parameters identified was experimentally proven by examining the dynamic tracking accuracy along a specified trajectory. The advantages of this method may be summarized as follows :

1) Dynamic parameters such as inertial parameters, viscous friction and Coulomb friction are identified

2) This method only needs to measure the joint position as a function of time for a given step input. A torque measuring device is not required to identify the parameters

3) The diagonal terms in the inertial matrix of a dynamic equation are used

4) This method can be extended to estimate a load at the end point of a robot.

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