# Robust Control of the Directly Drived Robot via Model Feedback Control System

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Robot manipulators are highly coupled nonlinear systems and their motions are influenced by uncertain dynamics. In this paper a design methodology which is called model feedback control system or plant model control scheme is presented for the purpose of reducing the influence of the uncertain dynamics. This control system is applied to the trajectly control of the directly drived robot. Theoretically and experimentally performances resulting from use of this control scheme show that the influences of the uncertain dynamics are reduced obviously.

## 1.Introduction

Robot manipulators are constructed with DC servo motors and other various mechanical elements. And then their motions are influenced by uncertain dynamics . The unknown dynamics and parameter variations while in work seriously disturb the performance of the control system. Then in the control system design of the robot manipulators robustness is thought to be most important [1,2]. Recently in order to improve the robustness many control design methodologies are proposed [3-7] . For example, model reference adaptive control system(MRACS) [4,5] and sliding mode control based on variable structure system(VSS) theory are developed [6,7] . But there are many unsolved problem for real implementation. Generally a huge calculation is required in both of these control systems. Especially in the case of MRACS strong restriction to the nonlinearity of robot dynamics is required and in the case of VSS discontinuous control law results in chattering of machinary system.

The aim of this study is to incorporate robustness considerarions into the classical control design methodology and to improve the robot control design methodology. Then in this study model feedback control system(MFCS) will be newly proposed and analyzed with paying attention to the robustness. It will be theoretically shown that the MFCS has high insensitivity and robustness for the single-input single-output(SI-SO) system. Still more it will be shown that this design methodology can be easily extended to the multi-input multi-output(MI-MO) system with maintaining the robustness. In the case of MI-MO system the classical Nyquist array methods and diagonal dominance conditions for uncertain matrix will be incorporated to accomodate the MI-MO MFCS for system uncertainties. The new theoretical framework shows that the synthesis of the MI-MO MFCS can be attributed to that of the SI-SO MFCS.

This MFCS will be applied to the trajectory control of the directly drived robot(DD robot) with 3-degrees of freedom . Experimental results show that the MFCS is effective control scheme under the existance of dynamical uncertainty and parameter variations, that is, the influences of uncertainties and parameter variations are reduced by the MFCS.

## 2. Single Input Single Output Sytem of MFCS

In this section, the properties of the model feedback control system shown in Fig.1 will be investigated. We consider an additive type of uncertainty in the transfer function description of the plant

$$G_p(s)=G_o(s)+\Delta G(s)$$

where $G_o(s)$ is the transfer function of the nominal approximating the true plant behaviour and $G_p(s)$ is the uncertain transfer function of the true plant. Generally it is not necessary to distinguish between the uncertainties occurring at the input and the output of the plant if the additive form of uncertainty is used[9]. $G_c(s)$ is compensator transfer function. f(s) is a transfer function inserted into the inner loop for the purpose of reducing the gain of the transfer function relation between r(s) and u(s) and is able to be arbitrarily chosen.

Main idea of this control scheme is illustrsted in Fig.2. In this figure there are three equivalent systems. If H(s) satisfies following condition, then input-output property of the system illustrated in Fig.2(a) is equivalent to that of the system illustrated in Fig.2(b).

$$G_p(s)/(1+H(s)G_p(s))=G_o(s) \tag{1}$$

From this condition H(s) is given by

$$H(s)=G_o^{-1}(s)-G_p^{-1}(s) \tag{2}$$

Combinig equation (2) with $u(s)=G_p^{-1}(s)y(s)$ gives

$$u(s)= r(s)+u(s)-G_o^{-1}(s)y(s) \tag{3}$$

This input-output property is illustrated in Fig.2(C).In this system the gain of the transfer function between r(s) and u(s) is infinity. Then f(s) must be inserted into this loop in order to reduce the gain. Usually f(s) is arbitrarily selected under the condition thatthe transfer function between r(s) and u(s) is stable. Similar control system had been first stated by Reswick (1956)[3]. His techniques imposed a term of $k(s)G_o(s)$ on f(s). Where k(s) was arbitrarily chosen. And inverse form $G_o^{-1}(s)$ was not used in the inner loop. Usually inverse form $G_o^{-1}(s)$ gives rise to difficulty of physical realization. Nevertheless inverse form is preferred to the form of $k(s)\,G_o(s)$ because it can be easily extended to multi-input multi-output system as stated later. And in our case $G_o(s)$ is not a transfer function of real system but model transfer function of the plant. Then we can choose $G_o(s)$ a proper transfer function which can be realized.

In the model feedback control system illustrated in Fig.1, $G_r(s)$, closed loop transfer function, $G_d(s)$, transfer function relationship between disturbance d(s) and output y(s), S(s), sensitivity function for a parameter variation, are given by the following equations.

$$G_r(s)=\frac{G_p(s)F(s)G_c(s)}{1+G_p(s)F(s)(G_c(s)+G_o^{-1}(s))} \tag{4}$$

$$G_d(s)=G_p(s)/(1+G_p(s)F(s)(G_c(s)+G_o^{-1}(s))) \tag{5}$$

$$S(s)=1/(1+G_p(s)F(s)(G_c(s)+G_o^{-1}(s))) \tag{6}$$

where $F(s)=1/(1-f(s))$.

For the case when f(s)=1, then the transfer function $G_r(s)=1$, $G_d(s)=0$ and the sensitivity function S(s)=0. This means that the property of tracking to the command input and the property of disturbance reduction and improvement of the sensitivity are all equivalent. On the other hand if the measurement noise n(s) is added to the output y(s), then the transfer function relationship between n(s) and y(s) is given by

$$G_n(s)=\frac{G_p(s)F(s)(G_c(s)+G_o^{-1}(s))}{1+G_p(s)F(s)(G_c(s)+G_o^{-1}(s))} \tag{7}$$

and the relationship between $G_n(s)$ and S(s) is given by

$$G_n(s)+S(s)=1 \tag{8}$$

This means that the trade-off between $G_n(s)$ and the sensitivity function exists. Usually in the low frequency area sensitivity is improved and in the high frequency area influence of the measurement noise is reduced.

For the robust stability of the MFCS, following propositions are hold. Before propositions are stated, we give two assumptions.

(a) If $\Delta G(s)=0$ then closed loop system is stable.
(b) Zeros of $G_o(s)$ are all in the open left half plane.

Under these assumptions following propositions are easily derived.

(Proposition 1)
If
(i) $Z_+\left[G_o(s)\right]=Z_+\left[G_o(s)+\Delta G(s)\right]$
(ii) $|1+H(j\omega)|>|\Delta G(j\omega)G_o^{-1}(j\omega)H(j\omega)|\quad \omega\in R$
then closed loop system is robust stable.
where $Z_+\left[P(s)\right]$ is a set of unstable poles of P(s) and $H(s)=F(s)(1+G_o(s)G_c(s))$

Furthermore conditions(i) and (ii) can be replaced as follows.

(Proposition 2)
If
(i) Zero of $(G_o(s)+\Delta G(s))$ are in the open left half plane.
(ii) $|1+G_o(j\omega)G_c(j\omega)|$
$$>|F^{-1}(j\omega)(1+\Delta G(j\omega)G_o^{-1}(j\omega))^{-1}|\quad \omega\in R$$
then closed loop system is robust stable.

3. Multi-input Multi-output system of MFCS

The MFCS illustrated in Fig.1 is easily extended to the MI-MO system. But discussions of the properties of the MI-MO MFCS are slightly different from the case of the SI-SO system. In the case of MI-MO sytem the classical Nyquist array methods and diagonal dominance conditions for uncertain matrix are incorporated to accomodate the MI-MO MFCS for system uncertainties. The new theoretical framework shows that the synthesis of the MI-MO MFCS can be attributed to that of the SI-SO MFCS.

The multivariable MFCS is illustrated also in Fig.1. The command input vector to the system is denoted by r(s), the disturbance vector by d(s) and the measurement noise vector by n(s). Plant transfer function matrix is denoted by $n\times n$ rational matrix $G_p(s)$. Compensator $G_c(s)$, f(s) and model transfer function matrix $G_o(s)$ are all $n\times n$ diagonal matrix whose elements are rational function of s. These are denoted by

$$G_c(s)=\mathrm{diag}\left[g_1^{\,c}(s),\dots,\ g_n^{\,c}(s)\right]$$
$$G_o(s)=\mathrm{diag}\left[g_1^{\,o}(s),\dots,\ g_n^{\,o}(s)\right]$$
$$f(s)=\mathrm{diag}\left[f_1(s),\dots,\ f_n(s)\right]$$

Then the transfer function matrix relationship between the output vector y(s) and r(s) is given by

$$W(s)=G_p(s)\left[I-f(s)+(G_c(s)+G_o^{-1}(s))G_p(s)\right]^{-1}\times G_c(s) \tag{9}$$

Letting $[I-f(s)]^{-1}=(1/k(s))I$, namely $f(s)=\mathrm{diag}\left[1-k(s),\ 1-k(s),\dots,\ 1-k(s)\right]$ then closed loop transfer function matrix W(s) is rewritten by

$$W(s)=\left[I+G_c^{-1}(s)(k(s)G_p^{-1}(s)+G_o^{-1}(s))\right]^{-1} \tag{10}$$

In the same way as SI-SO system an additive type of uncertainty is considered.

$$G_p(s)=G_o(s)+E(s)$$

$G_o(s)$ is defined above. E(s) is the model-plant mismatch or the error component. Then

$$W(s)=\left[I+G(s)+k(s)\Delta G(s)\right]^{-1}G(s) \tag{11}$$

where $\Delta G(s) = \left[I + E(s) G_o^{-1}(s)\right]^{-1}$, $G(s)=G_o(s)G_c(s)$, namely

$G(s) = \mathrm{diag}\left[g_1(s), \ldots, g_n(s)\right]$,

$g_i(s)=g_i^o(s)g_i^c(s)$

In this transfer function matrix equation, $k(s)\Delta G(s)$ can be considered as parameter variations similar to the SI-SO MFCS. Then discussions about robust stability can be done with new theoretical framwork. In order to do this the following definitions are needed.

$$\underline{\sigma}(I+G(j\omega)) = \min_{i,\omega} |1+g_i(j\omega)| \quad (12)$$

$$\overline{\sigma}(\Delta G(j\omega)) = \max_{i,\omega} \sum_{k=1}^{n} |\Delta g_{ik}(j\omega)| \quad (13)$$

where $\overline{\sigma}(.)$ is a matrix norm induced by vector norm.
By these definitions dominance conditions for uncertain matrix can be given as follows.

(Proposition 3)
Let G(s), $\Delta G(s)$ be complex matrix defined above.
If
$$|k(j\omega)| < \underline{\sigma}(I+G(j\omega))/\overline{\sigma}(\Delta G(j\omega)) \quad (14)$$
then
Complex matrix $I+G(s)+k(s)\Delta G(s)$ is diagonally row dominant.

The above proposition and following Lemma allow one to draw conditions about the robust stability of uncertain multivariable system.

(Lemma) Stability of Diagonal Dominance Matrix, Rosenbrock(1974) [8]
Let G(s) be open loop transfer function matrix.
Let I+G(s) be return difference matrix and also diagonal dominance.
Then
the close loop system is stable
if and only if
$$\sum_{i=1} N_{-1+jo}(g_{ii}(s)) = -P_o$$
Where $p_o$ is the number of open loop poles in the closed right half plane and $N_{-1+jo}(g(s))$ is the number which Nyquist diagram of g(s) encircles the (-1+jo) point clockwise.

Characteristic equation of the MFCS is given by
$$\det\left[I+G(s)+k(s)\Delta G(s)\right]=0$$
where G(s), $\Delta G(s)$ are defined in equ.(11). Stability of the closed loop system is determined by the location of zeros of characteristic equation.
But if $I+G(s)+k(s)\Delta G(s)$ is diagonal dominant, then stability of the MFCS can be checked by following proposition.

(Proposition 4)
Let G(s), $\Delta G(s)$ be complex matrix defined above.
If
(i) $|k(j\omega)| < \underline{\sigma}(I+G(j\omega))/\overline{\sigma}(\Delta G(j\omega))$

(ii) $\sum_i N_{-1+jo}(g_{ii}(s)) = -p_o$

then the closed loop system is stable

As shown Prop.3 and Prop.4 the MFCS can be easily extended to the multivariable system and it can be made to be robustly diagonal dominant [9].

Diagonal dominance makes an important role in the multivariable control system design. It had been reported first by Rosenbrock that Nyquist stability criterion and diagonal dominance can be incorporated to cover the case when the parameters of the system are subject to uncertainty. But no one has yet considered how to realize the diagonal dominance. On the other hand the MFCS can intrinsically realize the diagonal dominance.

4. Application to the robot manipulators

In this chapter the robust trajectory control system will be considered. Motions of the robot manipulators are governed by the highly coupled nonlinear system which is mixed with mechanics and electro-dynamics. Fig.3 shows the schematic model of the 3-degrees of freedom robot. For the case of Fig.3 the equations of motion are given by

$$\left[J_o+M(\theta)\right]\ddot{\theta} + h(\theta,\dot{\theta}) + g(\theta) + K\dot{\theta} + F\,\mathrm{sgn}\,\dot{\theta} = Dv \quad (15)$$

where $\theta=[\theta_1, \theta_2, \theta_3]^t$ denotes the vector of generalized coordinates and $v=[v_1, v_2, v_3]^t$ denotes the vector of input voltages supplied to the DC servo moters, $M(\theta)$ is an inertia matrix and $h(\theta, \dot{\theta})$ is the vector of Coriolis, centrifugal forces, $g(\theta)$ is the vector of gravitational forces. F sgn$\dot{\theta}$ denotes the vector of coulomb frictions and $J_o$ is a inertia matrix of DC servo moter, K denotes the mixed constants of back EMF and linear frictions. D is a constant matrix which represents the relationship between the voltage and torque.
Each of these matrix is given by

$$J_o=\mathrm{diag}\left[J_{11}, J_{22}, J_{33}\right]$$
$$h(\theta, \dot{\theta}) = \left[h_1, h_2, h_3\right]^t$$
$$g(\theta) = \left[g_1, g_2, g_3\right]^t, \quad K=\mathrm{diag}\left[K_1, K_2, K_3\right]$$
$$F=\mathrm{diag}\left[F_1, F_2, F_3\right], \quad D=\mathrm{diag}\left[D_1, D_2, D_3\right]$$
$$M(\theta) = \left[M_{ij}(\theta)\right], \quad (i,j=1,2,3)$$

Components of these matrix $M(\theta)$, $h(\theta,\dot{\theta})$, $g(\theta)$, are composed by the nonlinear functions of angular velocities and angular variables [12, 13]. In these equations gravitational forces and K,D can be identified easily as compared with $J_o+M(\theta)$ and other nonlinear terms.
Then gravitational forces can be cancelled by

$$v=D^{-1}(\tau+g(\theta)) \quad (16)$$

Substituting (16) into (15) gives

$$\left[J_o+M(\theta)\right]\ddot{\theta} + h(\theta, \dot{\theta}) + K\dot{\theta} + F\,\mathrm{sgn}\,\dot{\theta} = \tau \quad (17)$$

In order to design a complete trajectory control system, it is necessary to get a exact dynamic model of robot manipulator written by equ.(17). But unfortunately it is imposible because equ.(15),(17) are depending on the properties of servo moters and dynamic coupling effects between the joints and varying effective inertias of the links. Even if the physical prameter, masses and length or inertias, are given, exact identification of robotic systems is extremly difficult.

Some parameter identification schemes for the robot manipulators have been proposed [10,11]. But these identification shemes require precise experiments. For the simplicity after the cancellation of gravity loading term coupled nonlinear terms are frequently neglected. For the case of abbreviation influences of these neglected terms may disturb the system performances. On the other hand even if any detailed identification can be done, model-plant mismatch is inevitable. Therefore whenever the robot maniplators are controlled based on the mathematical model of the robot, it is necessary to take into considerations the robustness.

In order to do this, the MFCS is applied to the control of robot manipulators. Fig.4 shows a robust tracking sheme for the trajectory control. In this control scheme robot dynamics is completely unknown. But we can assume that the known terms are already cancelled by the feed-forward control. Then it may be expressed such as equ.(17). For the purpose of this paper it may be assumed that the mathematical model of the robot manipulator is given by

$$K\dot{\theta} = u \tag{18}$$

Then the inertia force and Coriolis and centrifugal forces are all neglected. These forces can be considered to be the disturbances and the parameter variations. Infiuences of these uncertainties may cause tracking errors or violate the stability. It can be shown that these harmful influences are reduced by the MFCS.

In order to demonstrate this, the MFCS is applied to the trajectory control of robot manipulators. Photo.1 shows the robot manipulator which is used at the experiments. This robot is newly constructed for the experiment. The gravitational force is approximately cancelled by the counter balance.

Joint angles are detected by variables resistance and these informations are sent to the mini-computer in order to calculate command signals. This sampling interval is fixed to 16ms. The schematic model of this robot is illustrated in Fig.3. Then the equations of motion is given by equ.(15).

Parameter identification of this robot is very difficult as stated above. Experimentally equations of motion are given by

$$\begin{bmatrix}3.0&0&0\\0&2.5&0\\0&0&0\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\\ddot{\theta}_2\\\ddot{\theta}_3\end{bmatrix}+\begin{bmatrix}15.5&0&0\\0&15.5&0\\0&0&20.0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\\dot{\theta}_3\end{bmatrix}+\begin{bmatrix}-0.03\sin\theta_2 \;\; -0.35\sin\theta_3\\-0.15\sin\theta_3\end{bmatrix}+\begin{bmatrix}0.2\;\mathrm{sgn}\theta_1\\0.08\;\mathrm{sgn}\theta_2\\0.35\;\mathrm{sgn}\theta_3\end{bmatrix}=\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \tag{19}$$

Input-output properties of coupled nonlinear terms can not be detected. Nevertheless intrinsically nonlinear terms exist, but it is very difficult to distinguish its input-output property with others. In equ.(19) model dynamics is chosen as

$$\begin{bmatrix}15.5&0&0\\0&15.0&0\\0&0&20.0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\\dot{\theta}_3\end{bmatrix}=\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \tag{20}$$

Using this model dynamics the trajectory control system is designed. This control scheme is illustrated in Fig.4. In this sheme model transfer function, $G_o(s)$, is given by equ.(20), that is

$$G_o(s)=\begin{bmatrix}1/15.5s&0&0\\0&1/15.0s&0\\0&0&1/20.0s\end{bmatrix}$$

and f(s) is given by

$$f(s)=\mathrm{diag}\left[T/s+T \quad T/s+T \quad T/s+T\right] \tag{21}$$

where T is a positive constant which is arbitrarily assigned. As illustrated in Fig.3 desired trajectory is a circle in a vertical plane. End effector coordinates of the robot manipulator must be satisfied

$$X= a+r\sin(\omega t)$$
$$Y= b$$
$$Z= c+r\cos(\omega t)$$

where $[X,Y,Z]^t$ is end effector coordinates, $[a,b,c]^t$ is the center coordinates of the circle, and r is a radius of the circle,  is an angular velocity of the trajectory.

For the case of the linear system written by equ.(20) control u(t) is given by

$$u(t)=\left[J(\theta)\,K^{-1}\right]^{-1}\left[\alpha\phi \;-Q(t)\right] \tag{22}$$

where $\alpha$ is a positive constant which is arbitrarily assigned [12, 13].

$$J(\theta)=\begin{bmatrix}-p(\theta)\sin\theta_1 & l_2\cos\theta_1\,Z(\theta) & l_3\cos\theta_1\cos(\theta_2+\theta_3)\\ p(\theta)\cos\theta_1 & l_2\sin\theta_1\,Z(\theta) & l_3\sin\theta_1\cos(\theta_2+\theta_3)\\ 0 & -l_2\sin\theta_2-l_3\sin(\theta_2+\theta_3) & -l_3\sin(\theta_2+\theta_3)\end{bmatrix}$$

$$p(\theta)=l_2\sin\theta_2+l_3\sin(\theta_2+\theta_3)$$

$$K=\mathrm{diag}\left[15.5,\ 15.0,\ 20.0\right]$$

$$\phi(\theta,t)=\begin{bmatrix}X(\theta)-a-r\sin(\omega t)\\Y(\theta)-b\\Z(\theta)-c-r\cos(\omega t)\end{bmatrix}$$

$$X(\theta)=p(\theta)\cos\theta_1$$
$$Y(\theta)=p(\theta)\sin\theta_1$$
$$Z(\theta)=l_2\cos\theta_2+l_3\cos(\theta_2+\theta_3)$$

$$Q(t)=\begin{bmatrix}-r\,\omega\cos(\omega t)\\0\\r\,\omega\sin(\omega t)\end{bmatrix}$$

In this case if real system is given by equ.(20), then the tracking control law written by equ.(22) can make the end-effector coordinates track the reference trajectory with sufficient accuracy. But equ.(20) denotes the model dynamics of real system and it differs from the real equations of motion such as equ.(15). Then for the robust tracking of the reference trajectory the influences of uncertainties must be reduced. In order to do this the MFCS is used in an inner loop system.

For the case of the robust tracking control input voltages sent to the actuators of the robot manipular are given by

$$v = u - K\dot{\theta} + T\int(u - K\dot{\theta})dt \qquad (23)$$

where u and K,T are given by equ.(22).
In this control law if v=u then the tracking control is performed without the MFCS.

Fig.5 (a) shows the performance of this control system without the MFCS. In this case feedback gain $\alpha$ is assigned to -0.8 and angular velocity of the desired trajectory is assigned to 0.2(rad/sec) and a radius of the circle is assigned to 5cm. Thus the trace of this desired trajectory is a circle with diameter 10cm and is moving on the circle with angular velocity 0.2 rad/sec. On the other hand $l_1$, $l_2$, $l_3$ is 78.0cm, 30.0cm, 20.0cm respectively. In Fig.5(a) bald-faced curve is just drawn by the pen equipped on the end-effector of the robot manipulator itself and fine stroked line is a trace of the desired trajectory.
Parformance of this control system is estimated by

$$I_p = \sum_{k=0}^{N}\left[(x(\omega kT_s)-a-r\ \sin\ (\omega kT_s))^2 + (Z(\omega kT_s)-C-r\ \cos(\omega kT_s))^2\right]$$

where $T_s$ is sampling time and $N=2\pi/\omega T_s$.
Setting $T_s$=0.016(sec) and $\omega$=0.2 give

$$I_p = 0.1456296$$

Fig.5(b) shows the performance in the case when the MFCS is applied. Conditions of this experiment are same to those of experiment illustrated in Fig.5 (a). That is $\alpha$=-0.8, $\omega$=0.2 rad/sec, etc. But in this case the MFCS is used in an inner loop. Then control input is given by equ.(23). In this case T is assigned to 2.0 and other paramenters are same as the case of Fig.5(a). Value of the performance index $I_p$ is evaluated as

$$I_p = 0.4825857 \times 10^{-2}$$

In the case of Fig.5(c) T is assigned to 3.0 and other parameters are same as the case of Fig.5(a).In this experiments value of the performance index $I_p$ is evaluated as

$$I_p = 0.2852761 \times 10^{-2}$$

Through these experiments the feedback gain $\alpha$ is fixed to -0.8 and sampling time $T_s$ is fixed to 16ms. For the case when $\alpha$ is less than -0.8, the motion of the closed loop system shows a tendency of unstable. Therefore in order to improve the performance of the tracking $\alpha$ can not be assigned less than -0.8 also in the case of Fig.5(a). All of these experiments are controlled by the mini-computer system (Toshiba DS 600/40).

Results of these experiments show that the MFCS can reduce the influences of the uncertainties and performance of the case of T=3.0 is better than that of T=2.0. Theoretically the more the modulus of T/(s+T) is close to 1,the more the performances are improved.But actually input voltages may be satulated or measurement noise may be amplified too much. Then modulus of T/(s+T) can not be assigned too close to 1. Usually the dicision of f(s) is depending on the uncertainties and the nonlinearities.f(s) may be decided experimentally by try and error. Then the dicision scheme of f(s) is remaining as future problem.

## 5. Conclusions

A design technique for uncertain multivariable system has been presented. The proposed method is called the Model Feedback Control System which is similar to the system developed by Reswick. In this paper properties of the MFCS have been discussed. Robust stability theorems have been developed and in the case of multivariable uncertain systems the classical diagonal dominance conditions have been incorporated into the MFCS.

The proposed design methodology has been applied to the tracking control of the robot manipulator. Experimentally it was shown that the proposed technique can be applicable to the control system design of multi-degree of freedom robot and the robustness of the MFCS is attractive because several performances could be improved without violating the system stability.

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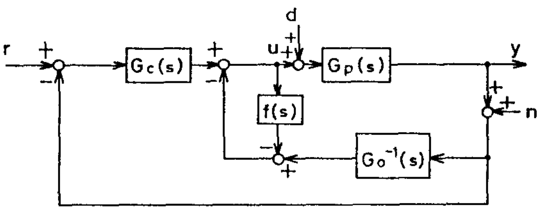


Fig.1 Model feedback control system

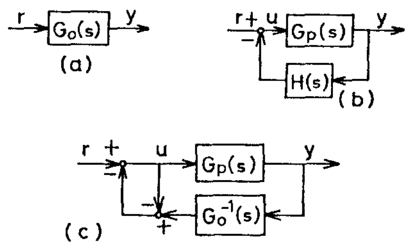


Fig.2 Equivalent systems

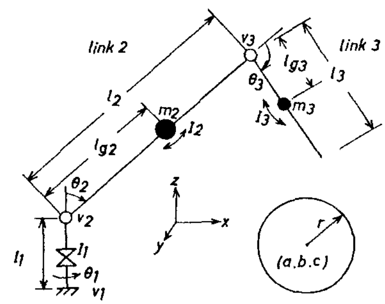


Fig.3 Schematic model of 3-degrees of freedom robot

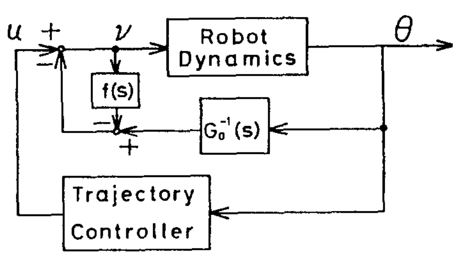


Fig.4 Control system of robust tracking of desired trajectory

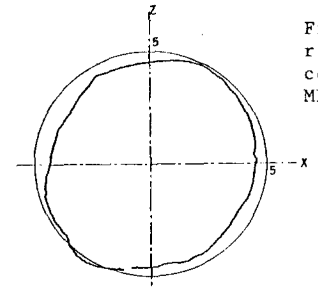

Fig.5(a) Experimental result of tracking control without the MFCS

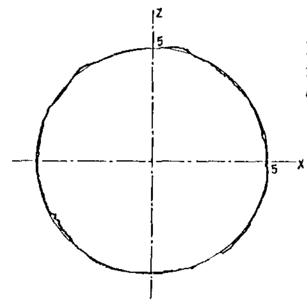

Fig.5(b) Experimental result of tracking control with the MFCS (T=2.0)

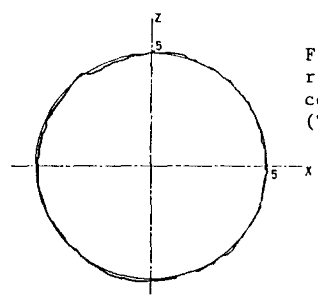

Fig.5(c) Experimental result of tracking control with the MFCS (T=3.0)

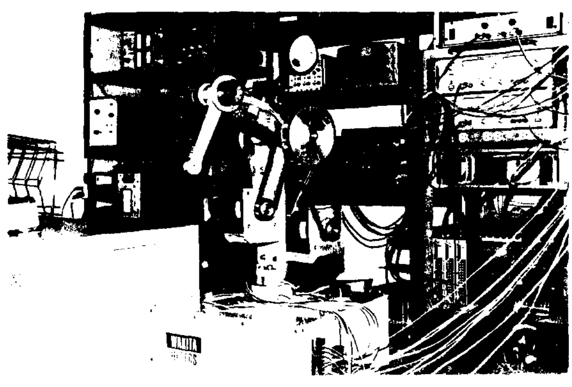

Photo.1 3-degrees of freedom robot