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A Method of Collision-Free Trajectory Planning for Two Robot Arms

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In this paper we outline an approach for the collision-free trajectory planning of two robot arms which are modeled as connected line segments. A new approach to determine the collision between two robot arms and the boundary of the collision region in the coordination space is proposed. The coordination curve may then be chosen to avoid the collision region. For minimum time trajectory, time is assigned to this curve by dynamic time scaling under constraints such as maximum torque or maximum angular velocity of each actuator. A comparison of the proposed method and the graphical method of determining the collision region is also included. Finally, as an example, some simulation results for two SCARA type robots are presented.

1. Introduction

When a task requires two robots to move in common work space, there may be collision between the two robots with independently planned trajectories. Thus some motion coordination is required to determine the collision-free trajectory pair for the two robots in a cooperative sense.

Compared with the case of a single robot with stationary obstacles[1][2][3], the collision-free motion planning for multiple robot system has inherent difficulties because each robot acts as moving obstacle to others. Different from the method of adjusting the velocity of one robot along its prespecified path in two robot system[4], a method which determines the collision-free trajectory pair simultaneously was proposed in [5]. In that work, a graphical method was used in determining collision region, which is difficult to be applied for general case.

In this paper, we consolidate the concept of Coordination Space of [5], propose a new method of determining the collision region, and calculate time optimal trajectory pair for two robots using bang-bang type control law[7] and dynamic time-scaling method[8][9].

Problem is formulated in Section 2 and the method of determining collision region is proposed in Section 3. In Section 4, assuming that a collision-free coordination between two robots is given, minimum time trajectory pair is sought considering dynamic constraints on actuator torque and Cartesian velocities of the two robots.

2. Problem Formulation

To investigate the collision-free trajectory planning for two robots, the robot system under consideration is explained in Section 2-(1), and problem formulation is presented in Section 2-(2).

(1) Description of a two-robot system

In this work, we assume the robots as planar type arms. It has been reported that various types of these robots are applied usefully in industrial field, and those types of planar arms are listed in [5]. For the brevity of presentation, we simplify the robot as connected straight line segments. By referring Fig. 1 and [3], we describe the robot system as follows:

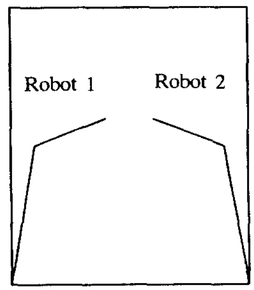


Fig. 1 Two robot system

The kinematics of robot r, r=1,2, is represented by

$$X^r = F^r(\theta^r), \quad r=1,2 \tag{1}$$

and the dynamics by

$$D^r(\theta^r)\ddot{\theta} + C^r(\theta^r,\dot{\theta}^r) = u^r, \quad r=1,2 \tag{2}$$

where X^r is the Cartesian coordinate of the end effector of robot r, $\theta^r,\dot{\theta}^r,\ddot{\theta}^r$ are the joint displacement , joint velocity, and joint acceleration, and $\mathbf{u}^r$ is the joint force/torque of robot r. And we assume that the joint force/torque and the joint velocity are limited by the following constraints.

$$v^r(\theta^r,\dot{\theta}^r)\le u^r \le w^r(\theta^r,\dot{\theta}^r) \quad r=1,2 \tag{3}$$

$$a^r(\theta^r)\le \dot{\theta}^r \le b^r(\theta^r) \quad r=1,2 \tag{4}$$

(2) Problem Formulation and a Solution Process

Using the description of robot system in previous Section , we can restate the problem as in [5]:

When a task is given that the two robots, described by equations (1)-(2), move from their corresponding initial points to final points along the designated paths, find a collision-free trajectory for the two robots under constraints on torques and velocities of actuators of two robots which are described by (3) and (4)

The complete general solution of this problem is known to be very difficult to derive, hence the decomposed solution step of [5] is accepted in this paper. The steps are outlined as follows :

1) We define a configuration space to describe the configuration of robots and to identify collision region.
2) We select a collision-free curve in that configuration space.
3) Along the chosen curve in the configuration space, we plan trajectories for the two robots simultaneously.

As mentioned before, we assume that the robots move prespecified paths, hence the positions of end effector can be described in terms of traveled distance along the paths as

$$X^r = G^r(s^r), \quad r=1,2 \tag{5}$$

where s^r denote the normalized traveled distance along the path of the robot r. By (1) and (5), we can determine the configuration of robot and the minimum distance between two robots with given s^1, s^2. The minimum distance between the two robots is represented as

$$d = H(s^1, s^2) \tag{6}$$

At this point we consider the 3D space composed by s^1, s^2 and d (Distance Run-length space, DR space, Fig. 2), then any combination of the configurations of the robots along their paths corresponds to a point of a plane or of one of several planes or of some volume according to the number of inverse kinematic solutions of the robots. If we consider the case in which unique configuration is determined from any s^1 and s^2 combination, all the values of (s^1, s^2, d) along the paths of robots form a plane (Distance Plane) in DR space. And the collision region is some part of Distance Plane which satisfies $d=0$. If one wants to give some safety margin (d_0) for the sake of safe trajectory planning even with uncertainties in robot geometries, he may simply determine the collision region as the parts of Distance Plane under the plane $d=d_0$(see Fig. 2). When the unique correspondence of configuration is ensured, it is enough to consider only $d=0$ plane, and that plane was called Coordination Space (CS) in [5].

3. Determination of Collision Region

As mentioned previously, we assume that the robot under consideration is composed of straight line segments. Before considering the collision of two robots, we introduce the method used in checking the intersection of two line segments[6]. We represent the points of Fig. 3 as:

$$P^r(x^r_p, y^r_p), \quad r=1,2 \tag{7-a}$$

$$Q^r(x^r_q, y^r_q), \quad r=1,2 \tag{7-b}$$

and introduce the following index quantities :

$$u = \frac{(x^2_p - x^2_q)(y^2_q - y^1_q) - (y^2_p - y^2_q)(x^2_q - x^1_q)}{DET} \tag{8-a}$$

$$v = \frac{(x^1_p - x^1_q)(y^2_q - y^1_q) - (y^1_p - y^1_q)(x^2_q - x^1_q)}{DET} \tag{8-b}$$

$$DET = (x^2_p - x^2_q)(y^1_p - y^1_t) - (y^2_p - y^2_q)(x^1_p - x^1_q) \tag{8-c}$$

$u(v)$ is normalized distance from $P^1(P^2)$ to the intersection point. As suggested in [6], if

$$0 \le u \le 1 \tag{9}$$

$$0 \le v \le 1 \tag{10}$$

there is an intersection between two line segments.

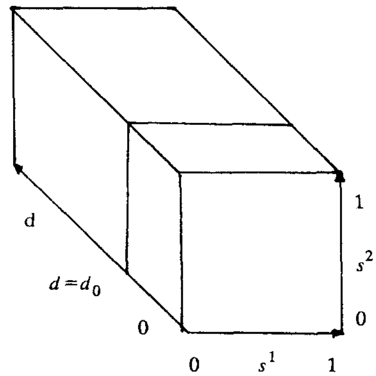


Fig. 2 Distance Run-Length Space

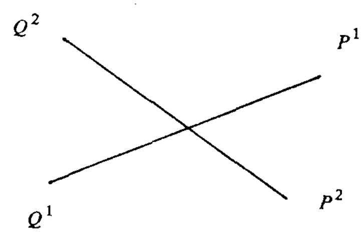
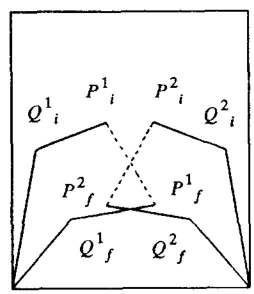


Fig. 3 Two line segments



Robot 1 Robot 2

Fig. 4 Two robots moving straight line paths

To apply this index quantities to the determination of collision region in CS, we consider the motion of two robots in Fig. 4, in which the end-effectors of the robots move along straight lines. Then any point in path is described as:

$$x^r_p = x^r_{p_i} + (x^r_{p_f} - x^r_{p_i})s^r \quad r=1,2 \tag{11-a}$$

$$y^r_p = y^r_{p_i} + (y^r_{p_f} - y^r_{p_i})s^r \quad r=1,2 \tag{11-b}$$

If we approximate the motions of points Q^1, Q^2 as straight line, we get

$$x^r_q = x^r_{q_i} + (x^r_{q_f} - x^r_{q_i})s^r \quad r=1,2 \tag{12-a}$$

$$y^r_q = y^r_{q_i} + (y^r_{q_f} - y^r_{q_i})s^r \quad r=1,2 \tag{12-b}$$

Assuming that collision occurs only between the end links of the robots, we propose the following guide line to determine the collision region.

(GL1). The collision region in CS is the region which satisfys the following conditions:

$$0\leq u(s^1,s^2)\leq 1 \quad (13\text{-a})$$

$$0\leq v(s^1,s^2)\leq 1 \quad (13\text{-b})$$

where u and v are the same to the equation (8) and now are the functions of s^1 and s^2.

More specifically, one can deduce that the boundaries of collision region are composed by some parts of curves such as

$$u(s^1,s^2)=1 \quad (14\text{-a})$$

$$v(s^1,s^2)=1 \quad (14\text{-b})$$

$$s^1=0 \quad (14\text{-c})$$

$$s^1=1 \quad (14\text{-d})$$

$$s^2=0 \quad (14\text{-e})$$

$$s^2=1 \quad (14\text{-f})$$

To get the configuration of the curve of (14-a,b), we substitute (10),(11) for (8), then (14-a) and (14-b) becomes respectly

$$a_1(s^2)^2+a_2s^2+a_3s^1s^2+a_4s^1+a_5=0 \quad (15)$$

$$b_1(s^1)^2+b_2s^2+b_3s^1s^2+b_4s^2+b_5=0 \quad (16)$$

After rearranging, we get from (15) and (16)

$$s^1=-a_1(s^2)^2-a_2s^2-\frac{a_5}{a_3s^2+a_4} \qquad a_3s^2+a_4\neq 0 \quad (17)$$

$$s^2=-b_1(s^1)^2-b_2s^1-\frac{b_5}{b_3s^1+b_4} \qquad b_3s^1+b_4\neq 0 \quad (18)$$

And (17) may be arranged to be

$$s^2=As^1+B+\frac{E}{Cs^1+D} \quad (19)$$

and an example is depicted in Fig. 5.

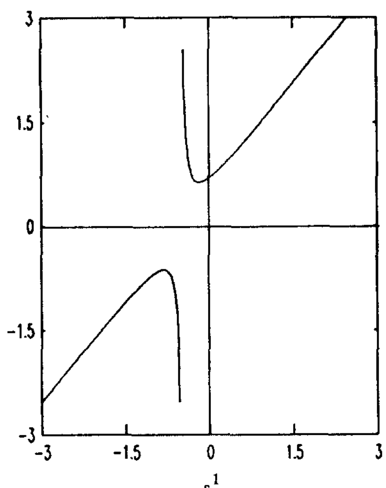


Fig. 5. The curve of

$$s^2 = s^1+0.5+\frac{0.1}{s^1+0.5}$$

Using (GL1), we calculate the collision region for the motion of Fig. 6-a as Fig. 6-b. In this case, each path of the robot is composed by 2 straight line segments, hence there are four combinations of the line segments pair. As a result, the whole CS is divided into four regions, and the collision region is determined respectly. If the paths of the robots are composed of many line segments, N for robot 1 and M for robot 2, we divide CS into NM region, and apply (GL1) to every sub-region.

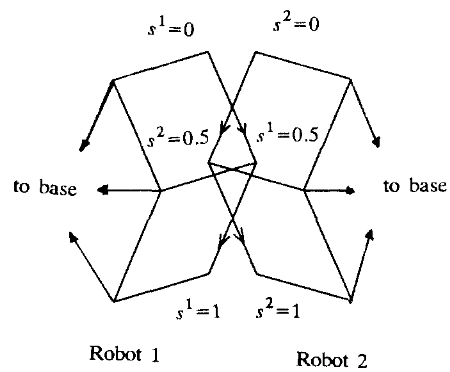


Fig. 6. (a) Robot motions with given straight line paths

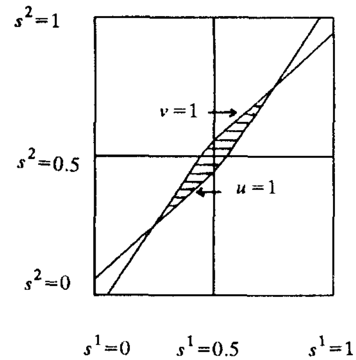


Fig. 6. (b) Collision region in CS for the paths of (a)

4.Minimum Time Trajectory Planning

In Section 3, we proposed the method of determining collision region in CS. Now assuming that the collision-free coordination curve(CFCC) in CS is given, we present the method of finding minimum time trajectory pair. To get the minimum time trajectory pair for two robots, we introduce the results of [7]. In [7], Chen and Desrochers showed that the structure of minimum time control law requires that at least one of the actuators is always in saturation while the others adjust their torques so that some constraints on the motion are not violated while enabling the manipulator to achieve its final destination. With this idea and the Dynamic Scaling methods[8][9], time optimal trajectory pair with prespecified paths is determined through dynamic programming technique. This technique is applied to the example of [5] under the constraints

$$-25\ Nm \le u^r{}_1 \le 25\ Nm, \quad -7\ Nm \le u^r{}_2 \le 7\ Nm$$

$$r=1,2 \tag{20}$$

and the velocity constraints

$$0 \le \dot{s}^1 \le 3\ m/sec \tag{21}$$

$$0 \le \dot{s}^2 \le 5\ m/sec \tag{22}$$

along the coordination curve $s^2=(s^1)^2$. The result is in Fig. 7 and one can see that at any time one of the joint of the two robots is in its saturation.

5. Discussion and Concluding Remarks

A collision-free trajectory planning for two robots is investigated in this paper by decomposing it into 3 stages as in [5].

Assuming that the paths of the robots are given, we analyzed the characteristics of CS, and derived the explicit form of the collision region in CS for the straight line motion of two robots.

With a given CFCC in CS, we apply the known result of torque condition[7] and dynamic scaling method via dynamic programming technique to get the minimum time trajectory pair.

In extending this algorithm to 3D case, some modification of the method of checking intersection between two line segments and some consideration for calculating the distance between two 3D line segments should be included in determining the collision region in CS.

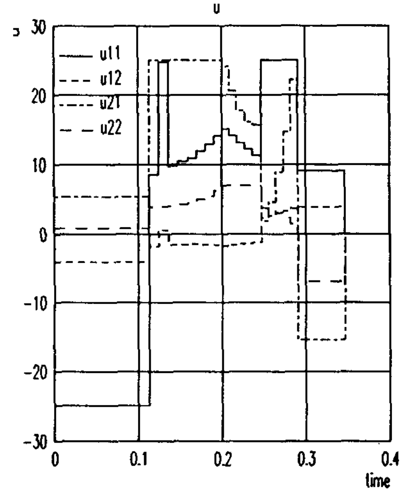


Fig. 7 (a) Minimum time joint torque of two robots along a collision-free coordination of $s^2=(s^1)^2$.

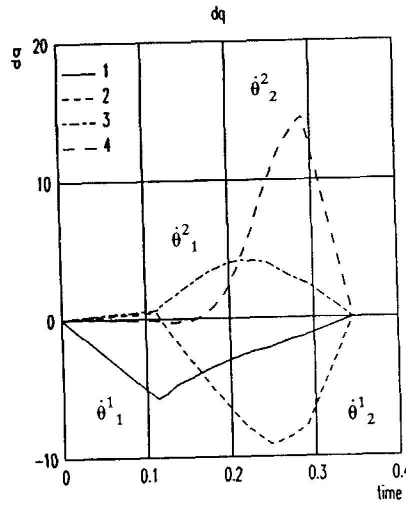


Fig. 7 (b) Minimum time joint velocity of two robots along a collision-free coordination of $s^2=(s^1)^2$.

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