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APPLICATION OF INVERSE DYNAMICS FOR HYBRID TRANSLATIONAL POSITION/FORCE CONTROL OF A FLEXIBLE ROBOT ARM

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ABSTRACT: A new simple method for controlling compliant motions of a flexible robot arm is presented. The method aims at controlling translational tip motion, force and moment by directly computing the base motion or torque. A numerical inversion of Laplace transform is used to obtain the results in the time domain. The results show the effectiveness of the method for the hybrid translational position/force control of a flexible robot arm.

INTRODUCTION

Development of light weight and higher performance robots for both commercial and space-based applications requires the research of a flexible robot manipulator. A lot of papers concerning the positioning or vibration control of a flexible robot arm have been published for several years[1-6]. However most of these papers concerned with the rotational motion of the flexible arm and few dealt with translational motion of the flexible arm. Also, inverse problems are important to robot control and programming, since they allow one to find the appropriate inputs necessary for producing the desired outputs[7-8].

For some applications, such as assembly of the mechanical parts or performing a machining process, a robot arm is required to interact compliantly with an environment. This sometimes necessitates the force control at the tip of robot manipulator. Research of rigid manipulator in this area has been active for over a decade as witnessed by the works of Whitney[9], Salisbury[10], Raibert and Craig[11], Roberts, Paul, and Hillberry[12]. However, few research has been done on the hybrid position/force control of a flexible arm.

With this background in mind, the method proposed here aims at controlling tip motion by directly computing the base motion or torque, which is necessarily applied at the base of the link to achieve the desired trajectory, moment, and force at the tip of the arm. A numerical inversion of Laplace transform is used to obtain the results in the time domain. The problem of inverting the Laplace transform can often be solved analytically by applying a partial fraction expansion or an integration along some contour in the complex plane. Sometimes, this method becomes too difficult or impossible to have the analytical results. In that case numerical methods will be necessary. The procedure used here is the approximation of the exponential function in the Bromwich integral. A simple and efficient numerical algorithm is applied to a particular one-link flexible arm. The results show the effectiveness of the method for the hybrid position/force control of flexible robot arms.

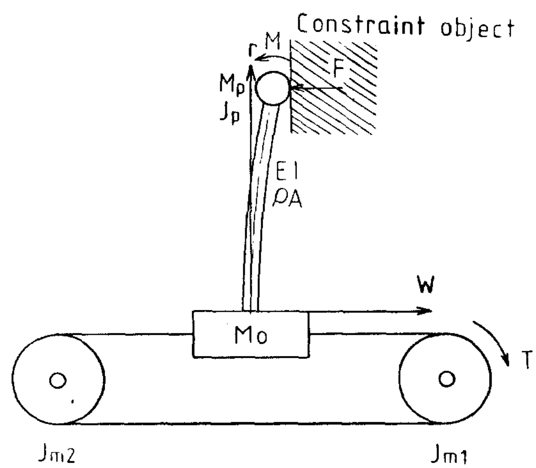

Figure 1 Flexible manipulator model.

FORMULATION AND ANALYSIS

Figure 1 shows a uniform arm of length l with a payload mass at the tip of the arm. The base of the arm is moved translationally by a DC motor. The arm is subjected to a force and moment at the tip. Here we define the flexural displacement of the arm $W(r,t)$ as $W(r,t)=W_s(r,t)-W_b$. In the case, the equation of motion of the arm is given by

$$E\left(1+c\frac{\partial}{\partial t}\right)I\frac{\partial^4 W}{\partial r^4}+\rho A\left(\frac{\partial^2 W}{\partial t^2}+\frac{\partial^2 W_b}{\partial t^2}\right)=0 \quad (1)$$

where ρ is the mass density, W is the flexural displacement, A is the cross-sectional area, I is the moment of inertia of arm cross-section, t is the time, E is the Young's modulus, W_b is translational displacement and c is the internal damping coefficient. Boundary conditions of the arm and the equilibrium equation of drive system are given by

$$W(0,t)=0,\ \partial W(0,t)/\partial r=0, \quad (2,3)$$

$$J_p\frac{\partial^2}{\partial t^2}\left(\frac{\partial W(1,t)}{\partial r}\right)=-E\left(1+c\frac{\partial}{\partial t}\right)I\frac{\partial^2 W(1,t)}{\partial r^2}+M, \quad (4)$$

$$M_p\frac{\partial^2}{\partial t^2}\{W_b+W(1,t)\}=E\left(1+c\frac{\partial}{\partial t}\right)I\frac{\partial^3 W(1,t)}{\partial r^3}+F, \quad (5)$$

$$M_0\frac{\partial^2}{\partial t^2}\{W_b+W(0,t)\}+c_s\frac{\partial}{\partial t}(W_b+W(0,t))$$

$$=-E\left(1+c\frac{\partial}{\partial t}\right)I\frac{\partial^3 W(0,t)}{\partial r^3}+\frac{K_\tau}{r_1 R}u(t)$$

$$-\left(\frac{J_{m1}}{r_1^2}+\frac{J_{m2}}{r_2^2}\right)\frac{\partial^2}{\partial t^2}\{W_b+W(0,t)\}$$

$$-\left(\frac{\varepsilon_1}{r_1^2}+\frac{\varepsilon_2}{r_2^2}-\frac{K_\tau K_b}{Rr_1^2}\right)\frac{\partial}{\partial t}(W_b+W(0,t)), \quad (6)$$

where M_p is the payload mass, M_0 is the mass of arm base, c_s is the damping coefficient between the base and rack, J_{m1} and J_{m2} are the moment of inertia of the pinion and shaft, ε_1 and ε_2 are the viscous friction coefficient, r_1 and r_2 are the radius of pinion, J_p is the payload mass moment of inertia, $u(t)$ is the armature input voltage, R is the resistance of the armature, F is the desired end point force, M is the desired end point moment, and K_τ and K_b are constants related to the motor torque and the back electromotive force, respectively. Equation (1) and (6) can be solved by applying the method of the Laplace transform with respect to t, defined by

$$L[W(r,t),W_b(t),u(t),F(t),M(t)]$$

$$=\int_0^\infty [W(r,t),W_b(t),u(t),F(t),M(t)]\exp(-st)dt$$

$$=[W(r,s),W_b(s),u(s),F(s),M(s)] \quad (7)$$

$$L^{-1}[W(r,s),W_b(s),u(s),F(s),M(s)]$$

$$=(2\pi i)^{-1}\int_{\zeta-\infty}^{\zeta+\infty}[W(r,s),W_b(s),u(s),F(s),M(s)]\exp(st)ds$$

$$=[W(r,t),W_b(t),u(t),F(t),M(t)]. \quad (8)$$

Transforming equations (1)-(6) gives

$$E(1+cs)Id^4W/dr^4+\rho As^2(W+W_b)=0, \quad (9)$$

$$W(0)=0,\ dW(0)/dr=0, \quad (10,11)$$

$$J_p s^2(dW(1)/dr)+E(1+cs)Id^2W(1)/dr^2=M, \quad (12)$$

$$M_p s^2\{W(1)+W_b\}-E(1+cs)Id^3W(1)/dr^3=F, \quad (13)$$

$$M_0 s^2\{W_b+W(0)\}+c_s s\{W_b+W(0)\}$$

$$=-E(1+cs)Id^3W(0)/dr^3+(K_\tau/Rr_1)u(s)$$

$$-(J_{m1}/r_1^2+J_{m2}/r_2^2)s^2\{W_b+W(0)\}$$

$$-(\varepsilon_1/r_1^2+\varepsilon_2/r_2^2-K_\tau K_b/Rr_1^2)s\{W_b+W(0)\}. \quad (14)$$

Hereinafter, the following dimensional and non-dimensional quantities are introduced:

$\beta^4=-\alpha^2/(1+\eta_c\alpha)=-\rho As^2 l^4/EI(1+cs)$, $\bar{W}=W/l$,

$\bar{W}_b=W_b/l$, $\bar{r}=r/l$, $\bar{r}_1=r_1/l$, $\bar{r}_2=r_2/l$, $\bar{J}_p=J_p/\rho Al^3$,

$\bar{M}_p=M_p/\rho Al$, $\bar{c}_s=c_s(\rho Al^4/EI)^{-1/2}$,

$\bar{J}=(M_0 l^2+J_{m1}/r_1^2+J_{m2}/r_2^2)/\rho Al^3$, $\alpha^2=\rho Al^4s^2/EI$,

$\bar{F}=Fl^2/EI$, $\bar{M}=Ml/EI$,

$\eta_c=c(\rho Al^4/EI)^{-1/2}$, $K_1=K_\tau l/EIRr_1$,

$\bar{\varepsilon}=c_s+(\varepsilon_1/r_1^2+\varepsilon_2/r_2^2-K_\tau K_b/R)/(\rho AL^2EI)^{1/2}$. (15)

A general solution to equation (1) is

$$\bar{W}(\bar{r})=A\cos\beta\bar{r}+B\sin\beta\bar{r}+C\cosh\beta\bar{r}+D\sinh\beta\bar{r}-\bar{W}_b \quad (16)$$

A, B, C and D are unknown constants determined from the boundary conditions. Substitution of eq.(16) into equations (10)-(14) leads to

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\begin{vmatrix} A \\ B \\ D \end{vmatrix}=\begin{vmatrix} M \\ F \\ K_1u(s) \end{vmatrix}$$

where

$$a_{11}=\bar{J}_p\alpha^2\beta(\cos\beta-\cosh\beta)$$

$$-(1+\eta_c\alpha)\beta^2(\sin\beta+\sinh\beta),$$

$$a_{12}=-\bar{J}_p\alpha^2\beta\sin\beta-(1+\eta_c\alpha)\beta^2\cos\beta,$$

$a_{13}=\bar{J}_p \alpha^2 \beta \sinh\beta+(1+\eta_c)\beta^2\cosh\beta$,

$a_{21}=\bar{M}_p \alpha^2(\sin\beta-\sinh\beta)$

$+(1+\eta_c\alpha)\beta^3(\cos\beta+\cosh\beta)$,

$a_{22}=\bar{M}_p \alpha^2\cos\beta-(1+\eta_c\alpha)\beta^3\sin\beta$,

$a_{23}=\bar{M}_p \alpha^2\cosh\beta-(1+\eta_c\alpha)\beta^3\sinh\beta$,

$a_{31}=-2(1+\eta_c\alpha)\beta^3$,

$a_{32}=\bar{J}\alpha^2+\bar{\varepsilon}\alpha$,

$a_{33}=\bar{J}\alpha^2+\bar{\varepsilon}\alpha$.

Here we put $a_{31}=0$, $a_{32}=0$, $a_{33}=1$ in eq.(17) when input is applied as a rotating angle. From eq.(16) one has

$$\bar{W}(\bar{r},s)=[\Delta_A(\sin\beta\bar{r}-\sinh\beta\bar{r})+\Delta_B(\cos\beta\bar{r}-1)+\Delta_D(\cosh\beta\bar{r}-1)]/\Delta \quad (18)$$

where

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix},\quad \Delta_A=\begin{vmatrix} \bar{M} & a_{12} & a_{13}\\ \bar{F} & a_{22} & a_{23}\\ K_1u(s) & a_{32} & a_{33}\end{vmatrix},$$

$$\Delta_B=\begin{vmatrix} a_{11} & \bar{M} & a_{13}\\ a_{21} & \bar{F} & a_{23}\\ a_{31} & K_1u(s) & a_{33}\end{vmatrix},\quad \Delta_D=\begin{vmatrix} a_{11} & a_{12} & \bar{M}\\ a_{21} & a_{22} & \bar{F}\\ a_{31} & a_{32} & K_1u(s)\end{vmatrix}.$$

Therefore we can get u(s) in the form

$$u(s)=[\Delta W(1)-\{\bar{M}(a_{22}a_{33}-a_{23}a_{31})+\bar{F}(a_{32}a_{31}-a_{12}a_{33})\}\times(\sin\beta-\sinh\beta)-\{\bar{M}(a_{23}a_{31}-a_{21}a_{33})+\bar{F}(a_{11}a_{33}-a_{13}a_{31})\}\times(\cos\beta-1)-\{\bar{M}(a_{21}a_{32}-a_{22}a_{31})+\bar{F}(a_{12}a_{31}-a_{32}a_{11})\}\times(\cosh\beta-1)]/K_1\{(a_{12}a_{23}-a_{31}a_{22})\times(\sin\beta-\sinh\beta)+(a_{21}a_{13}-a_{23}a_{11})\times(\cos\beta-1)+(a_{11}a_{22}-a_{21}a_{12})\times(\cosh\beta-1)\}. \quad (19)$$

THE CALCULATION OF INVERSION OF LAPLACE TRANSFORM

There have been two fairly general numerical approaches to the problem of inverting the Laplace transform. One is to expand the unknown function as a series in a complete set of orthogonal functions. The other is expressed the Laplace transform as a Fourier cosine transform whose inverse is expressible as a Fourier cosine series. The procedure we used here is the approximation of the exponential function in the Bromwich integral. The essential point of this method consists in the approximation of the exponential function exp(s) by

$$E_{ec}(s,a)=\exp(a)/2\cosh(a-s)$$

$$=(e^a/2)\sum_{n=-\infty}^{\infty} j(-1)^n/[s-a-j(n-0.5)\pi]$$

$$=e^s-e^{-2a}e^{3s}+e^{-4a}e^{5s}-\ \cdots\quad (20)$$

We define a function for t>0

$$f_{ec}(t,a)=(2\pi j)^{-1}\int_{\zeta-i\infty}^{\zeta+i\infty} F(s)E_{ec}(st,a)ds \quad (21)$$

On substituting (20) into (21), we have

$$f_{ec}(t,a)=f(t)-e^{-2a}f(3t)+e^{-4a}f(5t)-\ \cdots \quad (22)$$

and

$$f_{ec}(t,a)=(e^a/t)(F_1+F_2+F_3+\ \cdots) \quad (23)$$

where

$$F_n=(-1)^n I_m F\{[a+j(n-0.5)\pi]/t\} \quad (24)$$

Equation (22) shows that the function $f_{ec}(t,a)$ gives a good approximation when a>>1 and will be used in error estimation. On the other hand, (23) can be used to compute the numerical value of the inverse Laplace transform effectively. In practice, we must truncate the infinite series in (23) to some finite terms. Simple truncation, however, results in a relatively large error and is not realistic. In this respect, an effective method using Euler transformation has been developed. We transform (23) as follows

$$f_{ec}(t,a)=(e^a/t)\left(\sum_{n=1}^{k-1} F_n+\sum_{n=0}^{\infty} D^nF_k/2^{n+1}\right) \quad (25)$$

In practice, (25) is truncated to some finite terms, so that it is more convenient to use the expression

$$\sum_{n=0}^{m} D^nF_k/2^{n+1}=(1/2^{m+1})\sum_{n=0}^{m} A_{mn}F_{k+n} \quad (26)$$

where the A_{mn} are defined recursively by

$$A_{mm}=1,\ A_{mn-1}=A_{mn}+\binom{m+1}{n} \quad (27)$$

Thus we calculate the (l,m)th approximation by

$$f_{ec}^{lm}(t,a)=(e^a/t)\left(\sum_{n=1}^{l-1} F_n+2^{-m-1}\sum_{n=0}^{m} A_{mn}F_{l+n}\right) \quad (28)$$

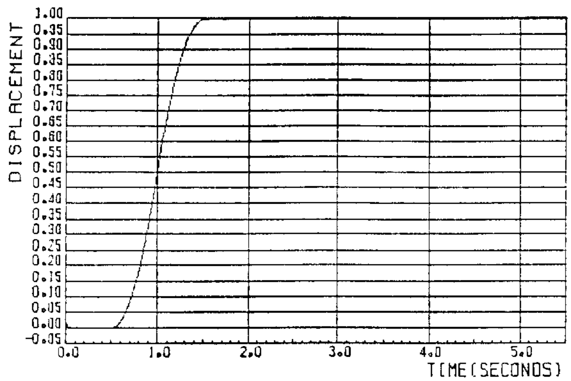

Fig. 2 Desired tip trajectory

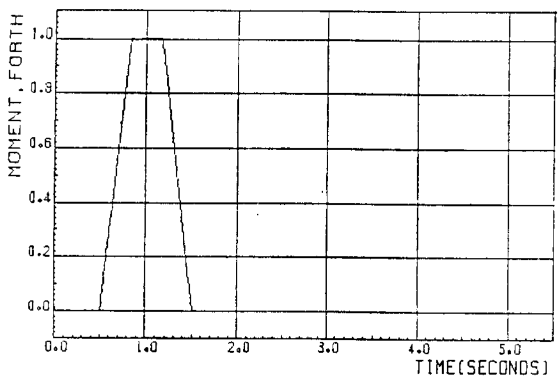

Fig. 3 Desired tip force and moment

SIMULATION RESULTS AND DISCUSSION

Here we consider the desired tip trajectory $\bar{W}_s(1,s)=e^{-0.5s}(1-e^{-0.5s})^2/0.25s^3$ as shown in Figure 2. Figure 3 shows the desired tip moment and force. This desired tip trajectory is based on bang-bang torque, which is required for time optimal control in the case of a single link rigid robot arm. The results in the time domain are obtained through the application of the numerical inversion of Laplace transform mentioned above. Figure 4 is the calculated input torque when the arm has the payload mass. Figure 5 is the calculated base motion when the arm has the payload mass. Figure 6 shows the calculated input torque when $\bar{F}$ and $\bar{M}$ are zero. Figure 7 is the calculated torque for $\bar{W}_s(1,s)=0.0$. Figure 8 is the result for the calculated input torque with no payload mass. Figure 9 shows the calculated input torque when $\bar{F}$ and $\bar{M}$ are applied at t=1.0(seconds). Figure 10 is the result when $\bar{F}$ and $\bar{M}$ are subjected at t=1.5(seconds). Compared with Figure 4 and 5 the base motion input is more smooth than torque input. As a matter of fact the base motion input is more easy to control than the torque input. The control input consists of the translational motion factor and the moment and force factor. Thus the positive torque component is greater than the negative one. A delay of starting time of moment and force shifts the positive torque component toward the later region of t.

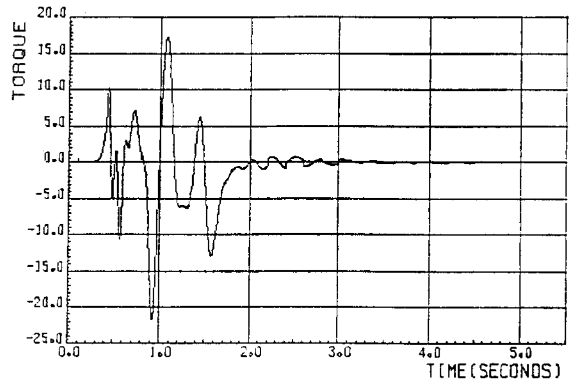

Fig. 4 Calculated input torque

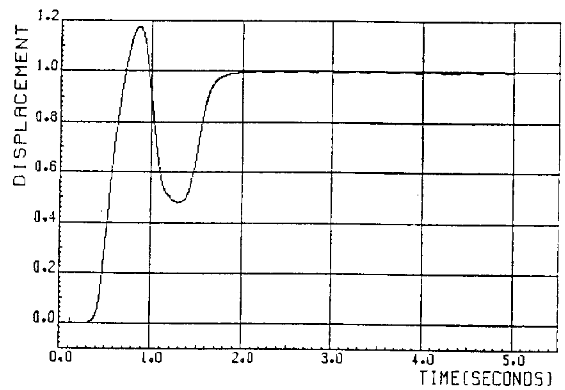

Fig. 5 Calculated input base displacement

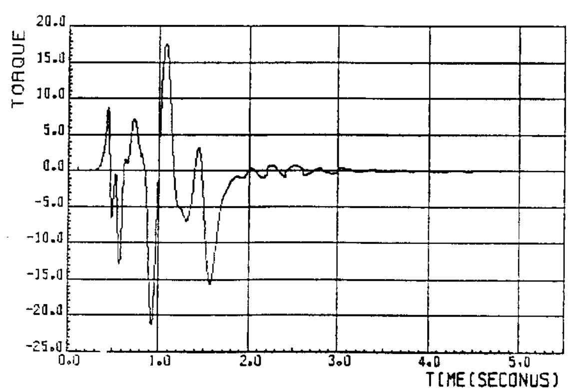

Fig. 6 Calculated input torque
($\bar{F}$,$\bar{M}$=0)

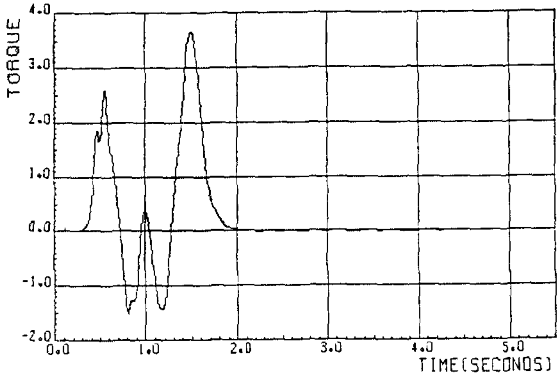

Fig. 7 Calculated input torque

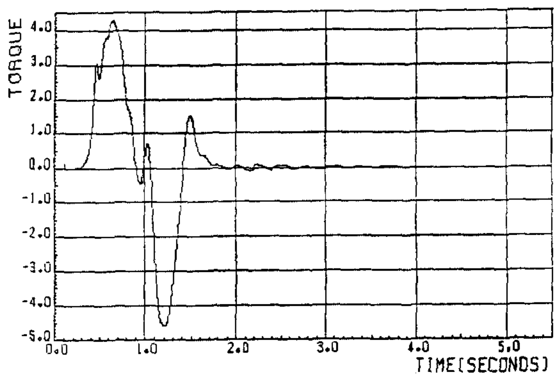

Fig. 8 Calculated input torque

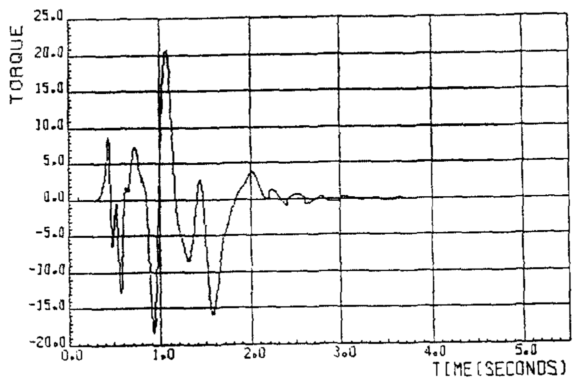

Fig. 9 Calculated input torque

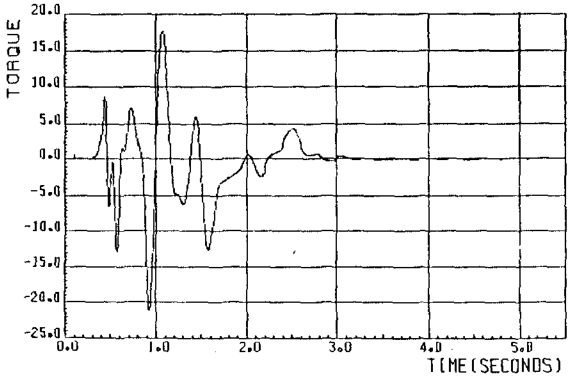

Fig. 10 Calculated input torque

CONCLUSIONS

A new method has been developed for the hybrid position/force control of a flexible robot arm. It is based on the numerical inversion of Laplace transform. Results obtained are summarized as follows.

(1) The advantage of this method is that we need not consider the mode numbers and the natural frequencies of the system.

(2) Comparing the input torque with the base motion, the base motion has no higher modes waves. Therefore, the base motion input is more easy to control than the torque input.

(3) The proposed procedure has the effectiveness for the hybrid position/force control of flexible robot arms.

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