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A Transient Stability Analysis Algorithm Using Decoupled Network Solution

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Abstract

This paper presents a new algorithm using power flow solution which is given by the polar form Newton-Raphson method in a transient stability analysis. The computation time to solve network equations can be much saved by a decoupled power flow method. In addition, the time is much saved in performing a approximate stability analysis by linearizing the differential equations and using a voltage and angle sensitivity matrix given in network equations.

1. Introduction

A conventional approach to a transient stability analysis is to solve in turn system network equations and nonlinear differential equations representing the dynamics of a power system. The nonlinear differential equations of a power system are solved by a numerical integration method such as the Runge-Kutta method or the Euler method,etc. And system network equations represented as voltage-current equations or power flow equations are analyzed by the Gauss iterative method or the Newton-Raphson method. In a general method of stability analysis, a generator is treated as fictitious slack node followed by impedance or equivalent current source connected to shunt admittance in parallel. A load is generally represented as constant impedance or constant current or constant power.

In this paper, the loads are modeled as a function of terminal voltage and frequency. And the generators are modeled as voltage dependent sources of real and reactive power connected to the terminal buses. In a viewpoint of time , the analysis method suggested in this paper is faster than other approaches since a decomposition method can be applied by using polar coordinates instead of rectangular ones to solve network equations. Also, by combining a voltage and angle sensitivity matrix obtained in network equations and linearized state equations , the time is much saved in performing approximate stability analysis.

2. The representation of synchronous generators

Each generator is described by the following nonlinear differential equations.

$$\dot{W}i = \frac{\pi f}{Hi} (Pmi - Pei) \quad \text{-----(1)}$$

$$\dot{\delta} i = Wi - 2\pi f \quad \text{-----(2)}$$

$$Tdoi'\dot{E}qi' = Efd - Eqi' - (xd - xdi')Idi \quad \text{-----(3)}$$

for i=1,...,M

where
- M : the number of generator
- Pmi : mechanical power input
- Pei : electrical power input
- Wi : angular speed
- Hi : inertia constant
- δ i : rotor angle relative to a reference axis
- Tdoi': d - axis transient open circuit time constant
- Eqi' : q - axis voltage back of transient reactance
- Efd : field voltage acting along q-axis
- xdi : d - axis synchronous reactance
- xdi' : d - axis transient reactance
- Idi : d - axis current

The variables of each generator are related as shown in figure 1.

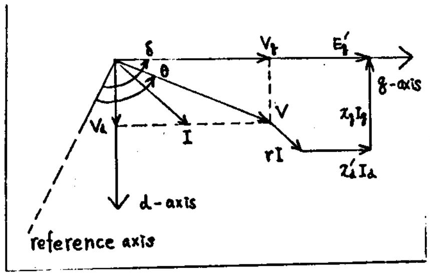


figure1. Phasor diagram of synchronous machine with sailency

Equations (1),(2),(3) are symbolically expressed as follows:

$$\dot{X} = f(X, V, \Theta) \quad \text{--------(4)}$$

where
- X : state vector of all generators
- V : voltage vector of all buses
- Θ : voltage angle vector of all buses

3. The formulation of network equations

The network of a power system consists of N buses. At the given time the following real and reactive power equations should be satisfied at all buses.

$$P_{gi} - P_{li} = P_{ni} \quad \text{---------(5)}$$

$$Q_{gi} - Q_{li} = Q_{ni} \quad \text{---------(6)}$$

for i=1,...N

where
- P_{gi}, Q_{gi} : real and reactive generator powers injected at bus i
- P_{li}, Q_{li} : real and reactive powers of load at bus i
- P_{ni}, Q_{ni} : real and reactive powers injection to a network at bus i

P_{gi}, Q_{gi} are given by (7),(8).
Here V_i and Θ_i mean voltage and voltage angle at bus i.

$$P_{gi} = \frac{V_i E_{qi}' \sin(\delta_i - \Theta_i)}{x_{di}'} + \frac{V_i^2 \sin 2(\delta_i - \Theta_i)(x_{di}' - x_{qi})}{2 x_{di}' x_q} \quad \text{---(7)}$$

$$Q_{gi} = \frac{V_i E_{qi}' \cos(\delta_i - \Theta_i)}{x_{di}'} - \frac{V_i^2 (x_{qi} \cos^2(\delta_i - \Theta_i) + x_{di}' \sin^2(\delta_i - \Theta_i))}{x_{di}' x_q} \quad \text{---(8)}$$

if sailency $x_{di}' = x_{qi}$

And P_{ni}, Q_{ni} are given as follows:

$$P_{ni} = V_i \sum_{j=1}^{N} (G_{ij} \cos(\Theta_i - \Theta_j) + B_{ij} \sin(\Theta_i - \Theta_j)) V_j \quad \text{----(9)}$$

$$Q_{ni} = V_i \sum_{j=1}^{N} (G_{ij} \sin(\Theta_i - \Theta_j) - B_{ij} \cos(\Theta_i - \Theta_j)) V_j \quad \text{----(10)}$$

where
- G_{ij}, B_{ij} : conductance and suceptance components of network admittance between i-th and j-th bus.

As P_{li} and Q_{li} are dependent on the terminal voltage and frequency, we are modeled P_{li} and Q_{li} as following equations (11),(12).

$$P_{li} = P_{loi} \left(\frac{V_i}{V_{oi}} \right)^{K_{pi}} (1 + \beta_{pi} \Delta f_i) \quad \text{---(11)}$$

$$Q_{li} = Q_{loi} \left(\frac{V_i}{V_{oi}} \right)^{K_{qi}} (1 + \beta_{qi} \Delta f_i) \quad \text{---(12)}$$

$$\Delta f_i = \frac{\Theta_i(t) - \Theta_i(t - \Delta t)}{\Delta t}$$

where
- P_{loi}:post-fault real power of load at bus i
- Q_{loi}:post-fault reactive power of load at bus i
- V_{loi}:post-fault voltage at bus i
- Δf_i :frequency deviation at bus i
- Δt :calculation step in stability analysis
- K_{pi} :effect constant of voltage for real load power
- K_{qi} :effect constant of voltage for reactive load power
- β_{pi} :effect constant of frequency for real load power
- β_{qi} :effect constant of frequency for reactive load power

For example, if the load of i-th bus is considered as constant impedance then the constants become like these. $K_{pi}=K_{qi}=2$, $\beta_{pi}=\beta_{qi}=0$.

4. Network solution scheme

The network solution scheme is given by the following procedure.

Solving the nonlinear differential equations representing generators at time t , we can obtain the state values of next step($t+\Delta t$). And the state values of next step are substitued for P_{gi}, Q_{gi}.

The next step voltages and angles are obtained by solving the network equations maintaining the state values of next step.

The network equations to be solved should be linearized at the values of votages and angles at time t. Then the equations should be solved iteratively and voltages and angles updated until network equations are satisfied.

The formulations are given by the following equations:

$$P_{gi}^{(k)} - P_{li}^{(k)} - P_{ni}^{(k)} = \sum_{j=1}^{N} \left(\frac{\partial P_{ni}}{\partial V_j} - \frac{\partial P_{gi}}{\partial V_j} + \frac{\partial P_{li}}{\partial V_j}\right) \Delta V_j^{(k)} + \sum_{j=1}^{N} \left(\frac{\partial P_{ni}}{\partial \theta_j} - \frac{\partial P_{gi}}{\partial \theta_j} + \frac{\partial P_{li}}{\partial \theta_j}\right) \Delta \theta_j^{(k)}$$

$$Q_{gi}^{(k)} - Q_{li}^{(k)} - Q_{ni}^{(k)} = \sum_{j=1}^{N} \left(\frac{\partial Q_{ni}}{\partial V_j} - \frac{\partial Q_{gi}}{\partial V_j} + \frac{\partial Q_{li}}{\partial V_j}\right) \Delta V_j^{(k)} + \sum_{j=1}^{N} \left(\frac{\partial Q_{ni}}{\partial \theta_j} - \frac{\partial Q_{gi}}{\partial \theta_j} + \frac{\partial Q_{li}}{\partial \theta_j}\right) \Delta \theta_j^{(k)}$$

and

$$V_i^{(k+1)} = V_i^{(k)} + \Delta V_i^{(k)}$$
$$\theta_i^{(k+1)} = \theta_i^{(k)} + \Delta \theta_i^{(k)}$$

-----------(13)

Generator power injections and loads are dependent on voltage and angle , and therefore their corresponding partial derivatives in (13) are no longer zero and expressed in (14).

$$\frac{\partial P_{gi}}{\partial V_i} = \frac{E_{qi} \sin(\delta_i - \theta_i)}{x_{di}'} + \frac{V_i \sin 2(\delta_i - \theta_i)(x_d' - x_{qi})}{x_{di}' x_{qi}}$$

$$\frac{\partial P_{gi}}{\partial \theta_i} = \frac{-E_{qi} V_i \cos(\delta_i - \theta_i)}{x_{di}'} - \frac{V_i^2 \cos 2(\delta_i - \theta_i)(x_{di}' - x_{qi})}{x_{di}' x_{qi}}$$

$$\frac{\partial Q_{gi}}{\partial V_i} = \frac{E_{qi} \cos(\delta_i - \theta_i)}{x_{di}'} - \frac{2V_i (x_{di}' \sin^2(\delta_i - \theta_i) + x_{qi} \cos^2(\delta_i - \theta_i))}{x_{di}' x_{qi}}$$

$$\frac{\partial Q_{gi}}{\partial \theta_i} = \frac{E_{qi} V_i \sin(\delta_i - \theta_i)}{x_{di}'} - \frac{V_i^2 \sin 2(\delta_i - \theta_i)(x_{di}' - x_{qi})}{x_{di}' x_{qi}}$$

$$\frac{\partial P_{li}}{\partial V_i} = K_{pi} P_{loi} \frac{V_i^{K_{pi}-1}}{V_{oi}^{K_{pi}}}$$

$$\frac{\partial P_{li}}{\partial \theta_i} = P_{loi} \left(\frac{V_i}{V_{oi}}\right)^{K_{pi}} \frac{\beta_p}{t}$$

$$\frac{\partial Q_{li}}{\partial V_i} = K_{qi} Q_{loi} \frac{V_i^{K_{qi}-1}}{V_{oi}^{K_{qi}}}$$

$$\frac{\partial Q_{li}}{\partial \theta_i} = P_{loi} \left(\frac{V_i}{V_{oi}}\right)^{K_{qi}} \frac{\beta_q}{t}$$

----------------------(14)

$$\text{if } i \neq j, \quad \frac{\partial P_{gi}}{\partial V_j} = \frac{\partial P_{gi}}{\partial \theta_j} = \frac{\partial Q_{gi}}{\partial V_j} = \frac{\partial Q_{gi}}{\partial \theta_j} = 0$$

$$\text{if } i \neq j, \quad \frac{\partial P_{li}}{\partial V_j} = \frac{\partial P_{li}}{\partial \theta_j} = \frac{\partial Q_{li}}{\partial V_j} = \frac{\partial Q_{li}}{\partial \theta_j} = 0$$

-------------------(15)

Define Pi,Qi as follows.

$$P_i^{(k)} = P_{ni}^{(k)} - P_{gi}^{(k)} + P_{li}^{(k)}$$
$$Q_i^{(k)} = P_{ni}^{(k)} - Q_{gi}^{(k)} + Q_{li}^{(k)}$$

-------------------------(16)

Here k denotes an iteration index.

To reduce network inversion times, let Jacobian matrix be constant. But if the condition of power system changes rapidly (i.e. just after fault or fault clear) ,the Jacobian matrix should be changed at each iteration.

Voltages and angles are updated until the absolute values of (16) are within a tolerance.

A matrix equation is shown in (17). A symbolic form of the matrix equation is given by (18).

$$\begin{bmatrix} -P_1 \\ \vdots \\ -P_N \\ -Q_1 \\ \vdots \\ -Q_N \end{bmatrix}^{(k)} = \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \cdots & \frac{\partial P_1}{\partial \theta_N} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_N} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_N}{\partial \theta_1} & \cdots & \frac{\partial P_N}{\partial \theta_N} & \frac{\partial P_N}{\partial V_1} & \cdots & \frac{\partial P_N}{\partial V_N} \\ \frac{\partial Q_1}{\partial \theta_1} & \cdots & \frac{\partial Q_1}{\partial \theta_N} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_N} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_N}{\partial \theta_1} & \cdots & \frac{\partial Q_N}{\partial \theta_N} & \frac{\partial Q_N}{\partial V_1} & \cdots & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \vdots \\ \Delta \theta_N \\ \Delta V_1 \\ \vdots \\ \Delta V_N \end{bmatrix}^{(k)}$$

--------------(17)

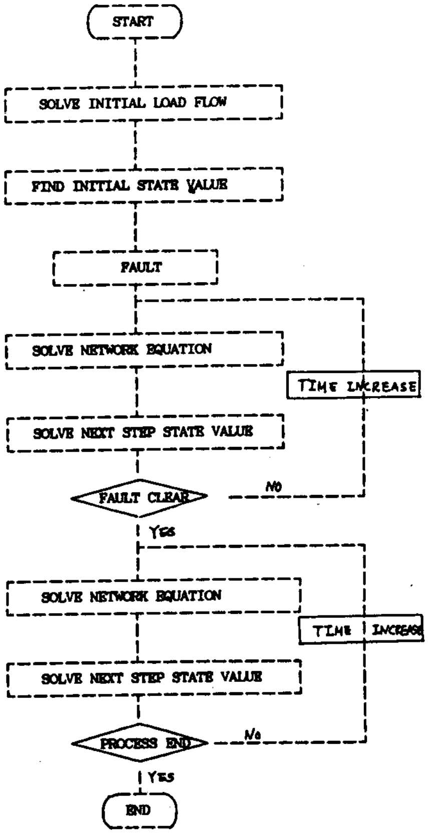
$$\begin{bmatrix} P^{(k)} \\ Q^{(k)} \end{bmatrix} = \begin{bmatrix} J1' & J2' \\ J3' & J4' \end{bmatrix} \begin{bmatrix} \Delta \theta^{(k)} \\ \Delta V^{(k)} \end{bmatrix}$$

----------------(18)

This Jacobian matrix is an equal form of the steady state Jacobian matrix except for the diagonal elements of J1',J2',J3',J4'. As the value of J2' elements is much smaller than that of J1', the matrix J2' can be assumed to be zero matrix . By the same reason ,J3' can be ignored. Consequently, the Jacobian matrix is decoupled as follows:

$$[P^{(k)}] = [J1'][\Delta \theta^{(k)}]$$
$$[Q^{(k)}] = [J4'][\Delta V^{(k)}]$$

The algorithm is described as follows:



5. The analysis of linearized equations

The network equations and the nonlinear differential equations of generators at a given time t are linearized respectively, and they are combined together.
First, linearizing differential equations as follows:

$$X = F(X, V, \theta)$$

where

$$X = \begin{bmatrix} X1 \\ \vdots \\ X2 \end{bmatrix} \quad V = \begin{bmatrix} V1 \\ \vdots \\ V2 \end{bmatrix} \quad \theta = \begin{bmatrix} \theta 1 \\ \vdots \\ \theta 2 \end{bmatrix}$$

$$\Delta\dot{X} = A\Delta X + [B : C] \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} + g \qquad (19)$$

Secondly, linearizing network equations as shown below:

$$Pg(X, V, \theta) - P(V, \theta) = Pn(V, \theta)$$
$$Qg(X, V, \theta) - Q(V, \theta) = Qn(V, \theta)$$

$$S\Delta X = [J'] \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$$

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = [J']^{-1} S\Delta X \qquad (20)$$

where
$[J']^{-1}S$: voltage angle sensitivity matrix

From eqs. (19),(20)

$$\Delta\dot{X} = A'\Delta X + g$$

where

$$A' = (A + [B : C][J']^{-1} S)$$

the values of states are obtained as a transition matrix by changing only g each step, and those of voltage and angle are obtained simply by eq. (20).

6. Case study

The transient stability studies of a power system consisting of 2 generators, 5 buses and 7 lines are made as an example. The 3-phase fault was placed on generator 1 ,and the fault was cleared in 0.1 second. The conventional algorithm was compared with the Newton-Raphson method using the polar coordinates suggested in this paper. In the conventional algorithm, the network equations are solved by the Gauss-Seidal iterative method.

In Figure 3, the conventional algorithm was used, and in Figure 4, the suggested one. In Figure 5, the linearized method was applied.

The results of this simulation show that it takes less time to apply the suggested algorithm to solve network equations.

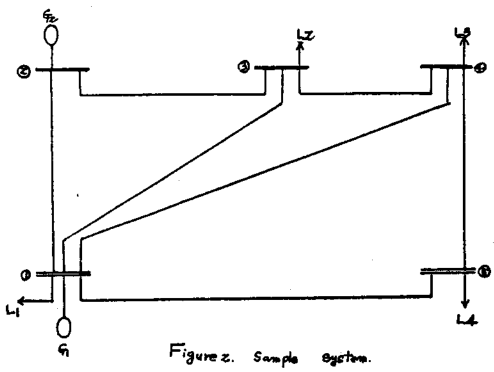


Figure 2. Sample system.

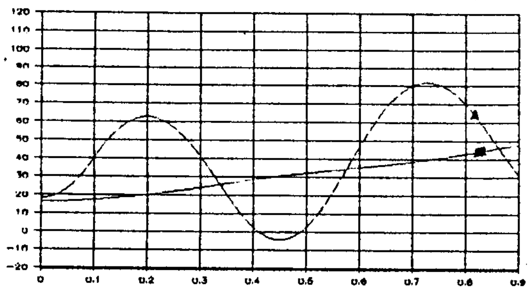


figure 3. conventional analysis.
▶ : angle of generator 1.
■ : angle of generator 2.

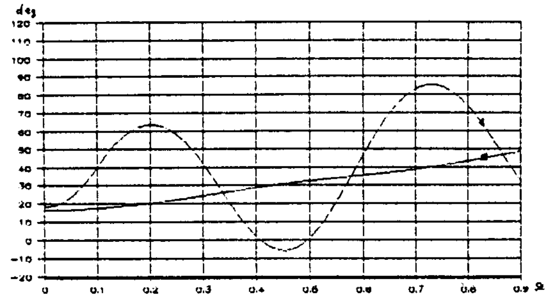


figure 4. suggested analysis.
▶ : angle of generator 1.
■ : angle of generator 2.

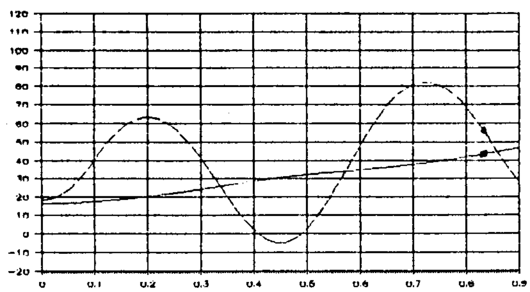


figure 5. linearized analysis.
▶ : angle of generator 1.
■ : angle of generator 2.

7. Conclusion

The main features of this paper are summarized as follows:

a) The direct inclusion of the formulas for generators and loads powers in a network equations is possible.

b) The decomposition is possible by using the polar coordinates in the Newton-Raphson method.

c) Due to the suggested linearized method, much time is saved in analyzing approxi- te transient stability.

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