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Buckling Strength Analysis of Box-Column Including the Coupling Effect Between Local and Global Buckling

by
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ABSTRACT

In this study, a formulation of the idealized plate element based upon the idealized structural unit method(ISUM) firstly proposed by Ueda et.al is made in an attempt to analyze the geometric nonlinear behaviour up to the buckling strength of thin-walled long structures like box-column structure including the coupling effect between local and global buckling. An application to the example box-column is also performed and it is found that the present method gives reliable results with consuming very short computing times and therefore is very useful for evaluation of the buckling strength of thin-walled long structures.

1. Introduction

Buckling strength of thin-walled long structures under axial compression such as box-column largely decreases compared with Euler buckling strength which is obtained assuming that the local member does not buckle if local buckling in the structural components takes place as the external load increases because the global bending rigidity of the whole structure is remarkably reduced after local buckling.

Therefore, in order to evaluate actual and reliable buckling strength of thin-walled structures, the coupling effect between the local and global buckling should be included.

Finite element method is a powerful approach to solve this problem but requires the enormous computing times for the analysis of large sized structures. As a countermeasure to this problem, Ueda et. al. proposed the idealized structural unit method(ISUM)[1,2] in the early 70s. This method is very useful and practical for evaluation of buckling and ultimate strength of large sized structures such as ship and offshore structures. In this method, some idealized elements for the objective structure to be analyzed should be developed in advance, like in the case of FEM.

In this study, ISUM is applied to estimate the buckling strength of box-column structure under axial compression considering the local and global geometric nonlinearity and then an idealized plate element for box-column composed of the plate elements shown in Fig.1 is developed in advance. After the accuracy and reliability of the developed element is checked comparing with the results by the other methods like FEM and experiments, it is then applied to estimation of the buckling strength of the example box-column.

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2. Modelling of the Example Box-Column

Box-column structure used in marine and onshore structures is composed of the plate members in the outer shell as shown in Fig.1 and in general, diaphram is also attatched preventing the transverse cross-section from the racking deformation. Therefore, the plate member is considered to be one of the elements in this study and then actual geometric nonlinear behaviour of this element is simplified.

2.1 Boundary Condition of the Element

In general, boundary condition of the plate element depends upon the restraints of the supporting members and the adjacent elements.

The boundaries of the idealized plate element developed in this study is assumed that :

1) Since the bending rigidity of the supporting members is large enough, local deflection at edges does not occur,
2) The restraint to the rotation at all edges is free,
3) The in-plane movement freely takes place but all edges remain straight even after the plate deflection occurs.

2.2 Loading Condition of the Element

If box-column structure is subjected to axial compression, axial compressive (or tensile) stress, shear stress and/or in-plane bending stress develop on the plate elements as shown in Fig.2. Since the magnitude of the in-plane bending stress is relatively small and then its influence to the geometric nonlinear behaviour of the element which is very small as compared with the whole structure is negligible, only average axial and shear stresses are considered in the local element level(see Fig.2) but the effect of bending stresses to the whole structural level is of course included.

3. Analytical Theory - Formulation of the Idealized Plate Element

3.1 Nodal Forces and Nodal Displacements

In actual plate elements, large deflection due to the local and global buckling of the structure takes place and therefore at least five degrees of freedom at each nodal point are necessary to express the geometric nonlinear behaviour of the element when using the finite element method.

In the idealized structural unit method, however, the geometric nonlinear behaviour of the element is carefully evaluated by analytical, numerical and/or experimental method and then based upon the results, the equivalent flat plate comprsing the effect of reduction of the in-plane stiffness after local buckling is formulated. Therefore, the idealized plate element with the equivalent in-plane stiffness is considered to be always flat even after local buckling and then only three degrees of freedom at each nodal point are necessary in this case :

$$\{R\} = \{R_1\ R_2\ R_3\ R_4\}^T,\ \{R_i\} = \{R_{xi}\ R_{yi}\ R_{zi}\}^T$$
$$\{U\} = \{U_1\ U_2\ U_3\ U_4\}^T,\ \{U_i\} = \{u_i\ \ v_i\ \ w_i\}^T \tag{1}$$

where, $\{R\}$ is nodal force vector and $\{U\}$ nodal displacement vector and R_x, R_y and R_z are axial forces in the x and y direction, respectively and u,v and w displacements in the x and y direction, respectively (see Fig.3).

3.2 Displacement Function

Displacement function corresponding to the nodal displacements defined in the above is adopted as :

$$u = a_1 + a_2 x + a_3 y + a_4 xy + \frac{b_4}{2}(b^2 - y^2)$$
$$v = b_1 + b_2 x + b_3 y + b_4 xy + \frac{a_4}{2}(a^2 - x^2)$$
$$w = c_1 + c_2 x + c_3 y + c_4 xy \qquad (2)$$

3.3 Relationship Between Displacements and Strains

Displacement-strain relation including the geometric nonlinear effect due to both the in-plane and out-of-plane large deformation is given as :

$$\epsilon_x = u_x - z \cdot w_{xx} + \frac{1}{2}(u_x^2 + v_x^2 + w_x^2)$$
$$\epsilon_y = v_y - z \cdot w_{yy} + \frac{1}{2}(u_y^2 + v_y^2 + w_y^2)$$
$$\gamma_{xy} = u_y + v_x - 2z \cdot w_{xy} + u_x u_y + v_x v_y + w_x w_y \qquad (3)$$

where ϵ_x, ϵ_y and γ_{xy} are the membrane strains.

In the incremental and matrix form, Eq.(3) may become :

$$\{\Delta\epsilon\} = [B_p]\{\Delta U\} + [C_p][G_p]\{\Delta U\} + \frac{1}{2}[\Delta C_p][G_p]\{\Delta U\} + [C_b][G_b]\{\Delta U\} + \frac{1}{2}[\Delta C_b][G_b]\{\Delta U\} - z \cdot [G_{bb}]\{\Delta U\} \qquad (4)$$

where, $\{\Delta\epsilon\} = \{\Delta\epsilon_x \;\; \Delta\epsilon_y \;\; \Delta\gamma_{xy}\}^T$,

$\{\Delta U\} = \{\Delta u \;\; \Delta v \;\; \Delta w\}^T$,

$\{u_x \;\; v_y \;\; u_y + v_x\}^T = [B_p]\{U\}$,

$\{u_x \;\; v_x \;\; u_y \;\; v_y\}^T = [G_p]\{U\}$,

$\{w_x \;\; w_y\}^T = [G_b]\{U\}$,

$\{w_{xx} \;\; w_{yy} \;\; 2w_{xy}\}^T = [G_{bb}]\{U\}$,

$$[C_p] = \begin{bmatrix} u_x & v_x & 0 & 0 \\ 0 & 0 & u_y & v_y \\ u_y & v_y & u_x & v_x \end{bmatrix}, \quad [C_b] = \begin{bmatrix} w_x & 0 \\ 0 & w_y \\ w_y & w_x \end{bmatrix}.$$

3.4 Relationship Between Stresses and Strains

In general, stress-strain relation in the incremental form is given :

$$\{\Delta\sigma\} = [D]\{\Delta\epsilon\} \qquad (5)$$

where,

[D] : stress-strain matrix.

3.4.1 Before Buckling

In the prebuckling range, stress-strain matrix is given as :

$$[D] = [D]^E = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad (6)$$

where, E is Young's modulus and ν is Poisson's ratio.

3.4.2 After Buckling

In the postbuckling range, the in-plane stress distribution of the plate element is not uniform, in which maximum stress is developed at the corner and minimum stress in the middle of the plate that is largely deflected and then is acted by the membrane stress (see Fig.4).

In this study, deflected plate is replaced by the equivalent flat plate which includes the effect due to reduction of the in-plane stiffness. Since boundaries of the plate element remain straight, axial strains at the edges are calculated as :

$$\epsilon_x = \frac{1}{E}\sigma_x)_{y=0,b} - \frac{\nu}{E}\sigma_y)_{y=0,b}$$
$$\epsilon_y = -\frac{\nu}{E}\sigma_x)_{x=0,a} + \frac{1}{E}\sigma_y)_{x=0,a} \qquad (7)$$

And also shear strain is defined by using the effective shearing modulus G_e [3,4]:

$$\gamma_{xy} = \frac{\tau}{G_e} \qquad (8)$$

Since the stress distribution σ_x, σ_y in Eq.(7) and the effective shearing modulus G_e in Eq.(8) can be evaluated by the analytical and/or numerical method [3-5], stress-strain relation in the postbuckling range is obtained in the incremental form :

$$\{\Delta\sigma\} = [D]^B\{\Delta\epsilon\} \qquad (9)$$

where, $[D]^B$: stress-strain matrix in the postbuckling range.

3.5 Derivation of the Tangent Stiffness Matrix

3.5.1 Total Lagrangian Formulation

The tangent stiffness equation of one element in the local coordinate is finally expressed by the following equation :

$$\{L\} + \{\Delta R\} = [K]^E \{\Delta U\} \qquad (10)$$

In the above equation, $\{L\}$ is the unbalance force vector which is due to the discrepancies between the internal and ecternal forces and should be made to become zero to secure the equilibrium condition in which the iteration procedure is introduced and $[K]^E$ is the tangent stiffness matrix of the element.

In general, $[K]^E$ can be subdivided into four terms as :

$$[K]^E = [K]_p + [K]_b + [K]_G + [K]_\sigma \qquad (11)$$

In the right hand side of Eq.(11), the first term $[K]_p$ represents the small, in-plane deformation and the second term $[K]_b$ the small, out-of-plane deformation. The third term $[K]_G$ is so called the initial deformation stiffness matrix which consists of three terms indicating the initial deformation effect associated with in-plane, out-of-plane and their interactions and the last term $[K]_\sigma$, so called the initial stress stiffness matrix indicates the large deformation effect due to the existence of the initial stress and is composed of two terms related to in-plane and out-of-plane large deformation, in which the term to their interations is not appeared.

In the calculation Eq.(11), $[D]^E$ in the prebuckling range and $[D]^B$ in the post-buckling range are used as the stress-strain matrix of the element.

3.5.2 Updated Lagrangian Formulation

The stiffness matrix $[K]^E$ in Eq.(11) was derived under the consideration the global coordinate of the objective plate to be analyzed is fixed with regard to the overall, space one, which results in the possibility of the use of the identical transformation matrix through every incremental loading steps.

On the other hand, using the concept of the updated Lagrangian formulation which the global coordinate may become to be updated at each deformed state, although the transformation matrix should be newly set up, the initial deformation stiffness matrix $[K]_G$ in Eq.(11) can be removed as:

$$[K]^E = [K]_p + [K]_b + [K]_\sigma \qquad (12)$$

3.6 Buckling condition

As the external load increases, if the stress components acted on the element satisfy the buckling condition, the plate may buckle. Therefore, in this study, the buckling interaction equation proposed in Ref.[6] which is formulated in terms of stress components is adopted :

$$\Gamma_B = \Gamma_B(\{\sigma\}) = 0 \qquad (13)$$

Buckling judgement is made by the magnitude (or sign) of Γ_B in Eq.(12): if $\Gamma_B < 0$, the plate is still in the pre-buckling range, if $\Gamma_B = 0$, the plate just buckles and if $\Gamma_B > 0$, the plate is in the postbuckling range.

3.7 Transformation Matrix

In general, the formulation of the exact transformation matrix for the rectangular plate is difficult to get it so that the approximate assuming that the element is in a plane including at least three nodal points is made in this study.

4. Numerical example

Based upon the above analytical theory a computer program was completed. The computer program applies the updated Lagrangian formulation for the stiffness matrix, sky line method for the solution of the stiffness equation and Newton-Rapson method for the convergence of the unbalance force.

An application to the example box-column structure as shown in Fig.1 is performed in an attempt to analyze the geometric nonlinear behaviour up to the buckling strength, including the coupling effect between the local and global buckling.

Fig.5.a shows the load-shortening curve and Fig.5.b the load-deflection curve for the example box-column structure, indicating that the global buckling strenth of the structure when not considering the local buckling is in good agreement with Euler buckling strength and when considering the local buckling effect, since the global bending rigidity considerably decreases after local buckling, the global buckling strength is remarkably reduced and the reduction amount for this example is about half of Euler buckling strength.

5. Conclusion

In this study, an idealized plate element based upon the idealized structural unit method (ISUM) is formulated in an attempt to analyze the geometric nonlinear behaviour up to the buckling strength of the thin-walled long structures like box-column structure which are composed of the plate members. An application to the example box-column structure is also made and it is found that the present method gives the reliable results with consuming very short computing times.

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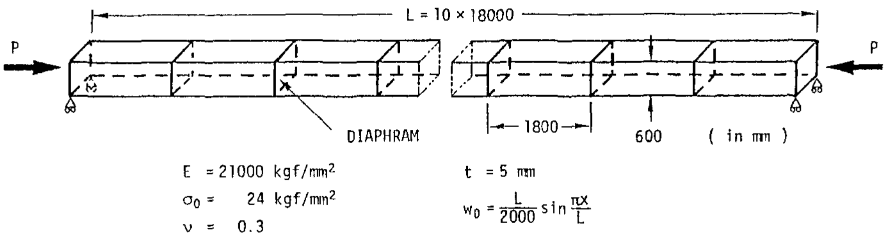

Fig.1 Example box-column structure and applied load

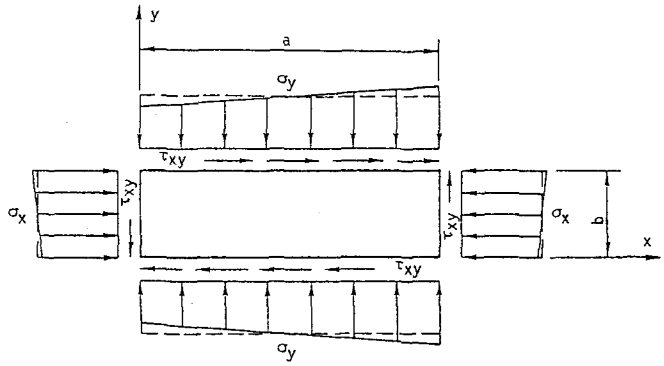

Fig.2 Stresses acting on the plate element

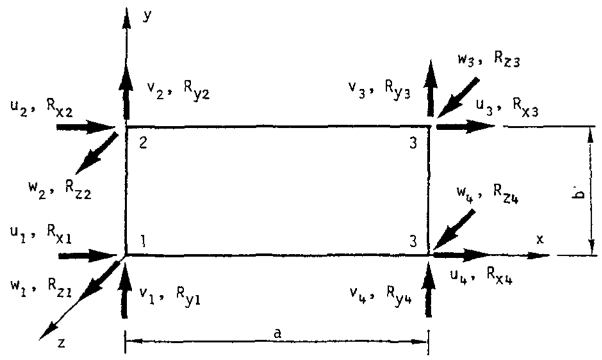

Fig.3 Nodal displacement and force of the idealized plate element

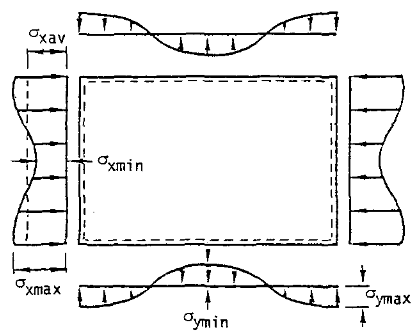

Fig.4 Stress distribution after buckling (edges remain straight)

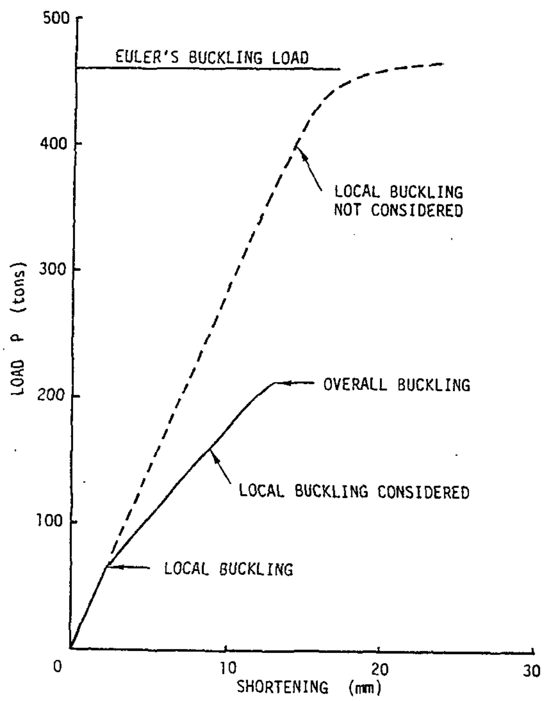

Fig.5.a Load-shortening curve for the example box-column structure

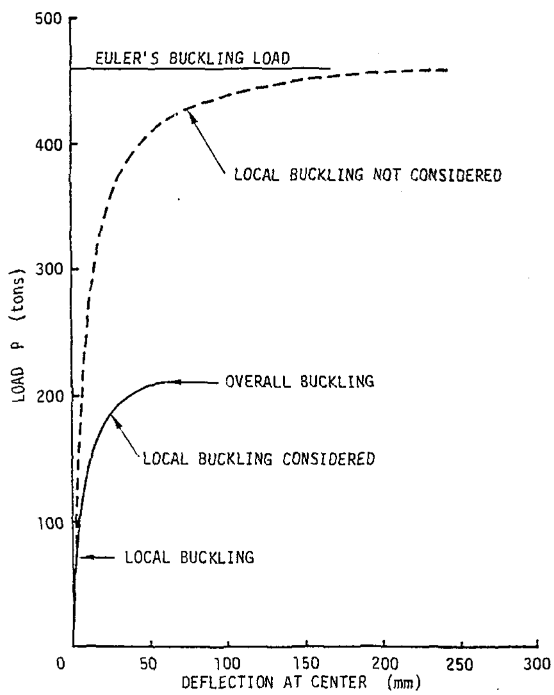

Fig.5.b Load-deflection curve for the example box-column structure