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A STUDY ON INITIAL CONVERGENCE PROPERTIES OF THE KALMAN FILTERING ALGORITHM

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Abstract

In this paper we present initial convergence properties of the Kalman filtering algorithm. we put an arbitrary small positive correlation matrix as an initial condition in the recursive algorithm. This arbitrary small initial condition perturbs the Kalman filtering algorithm and may lead to initial instability. We derive a condition which insures the stable operation of the Kalman filtering algorithm from the stochastic Lyapunov difference equation.

1. Introduction

Recently many adaptive filtering techniques are being realized by the advant of advances of digital technology in the field of adaptive signal processing such as adaptive echo cancellation, adaptive channel equalization, adaptive line enhancement and adaptive speech processing. Among them the Kalman filtering algorithm is the fastest converging optimal algorithm. Gauss used the Kalman filtering algorithm for the first time in the early nineteenth century. Since then the algorithm has been widely used in various fields.

In order to initiate the Kalman filtering algorithm we assign an initial condition which implies that the correlation matrix should be strictly positive-definite. Otherwise, the algorithm becomes singular in the initialization stage. During the initiation stage we do not have sufficient information to determine the filter coefficients. The arbitrary small positive matrix as an initial condition can be interpreted as some additive finite variance noise signal to the Kalman filtering algorithm. This initial condition perturbs the Kalman filtering algorithm and sometimes leads to its initial instability. In order to insure the stable operation of the algorithm in the initial stage, we need a condition for robustness of the Kalman filtering algorithm. Our aim is to study golbal convergence and initial convergence behaviors of the Kalman filtering algorithm.

The Kalman filtering algorithm can solve the deterministic Wiener-Hopf equation in the optimal manner. The Kalman filtering algorithm has many variations which are more numerically efficient in updating algorithms at every iteration such as the fast Kalman algorithm, the fast transversal filter algorithm and the recursive least-squares lattice algorithm. These algorithms are utilizing a Toeplitz-like autocorrelation matrix property and very efficient in computing the inverse of the autocorrelation matrix in the recursive form. Their computational complexity is of the order of 0(M), where M is the number of the adaptive filter taps. But their convergence properties are the same since they are all the Kalman filtering algorithms.

2. Basic Results of the Kalman Filtering Algorithm

Now we review the Kalman filtering algorithm. Suppose that $d(i)$ is the desired response at time i, then the linear regression model will be given as

$$d(i) = \sum_{k=1}^{M} w_{ok}^{*}\, u(i-k+1) + e_0(i) \tag{2.1}$$

where $u(i)$, $u(i-1)$, $\cdots$, $u(i-M+1)$ are prewindowed wide-sense stationary input data, i.e. $u(i) = 0$, $i < 0$. The w_{ok} are unknown parameters and $e_0(i)$ represents the white measurement error with zero mean and variance σ^2. The asterisk denotes the complex conjugate. In the Kalman filtering estimation problem, we choose the tap weights $w_1, w_2, \cdots, w_M$ which minimize the sum of error squares, for $N \geq M$

$$E(w_1, \cdots, w_M) = \sum_{i=1}^{N} |\varepsilon(i)|^2, \tag{2.2}$$

where

$$\varepsilon(i) = d(i) - \sum_{k=1}^{M} w_k^{*}\, u(i-k+1) \tag{2.3}$$

The optimal solution $\underline{\hat{w}}^T = (\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_M)$ satisfies the following Wiener-Hopf equation:

$$U^H U \underline{\hat{w}} = U^H \underline{v} \tag{2.4}$$

where

$$U^H = \begin{bmatrix} u(1) & u(2) & \cdots & u(N) \\ 0 & u(1) & \cdots & u(N-1) \\ \cdot & 0 & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & u(N-M+1) \end{bmatrix} \tag{2.5}$$

and $\underline{v}^T = (d(1), d(2), \cdots, d(N))$, where H denotes the Hermitian. If the $M \times M$ matrix $U^H U$ is nonsingular, then the least-squares estimate $\underline{\hat{w}}$ becomes

$$\underline{\hat{w}} = (U^H U)^{-1} U^H \underline{v} \tag{2.6}$$

This algorithm is very inefficient in computation. We now derive the Klaman filtering algorithm in the recursive form.

Let

$$\Phi(n) = U^H(n) U(n) = \sum_{i=1}^{n} \underline{u}(i) \underline{u}^H(i) \tag{2.7}$$

$$\underline{p}(n) = U(n) \underline{v}(n) = \sum_{i=1}^{n} \underline{u}(i) d^{*}(i), \tag{2.8}$$

where $\underline{u}^T(i) = [u(i), u(i-1), \cdots, u(i-M+1)]$.

then

$$\Phi(n) = \Phi(n-1) + \underline{u}(n) \underline{u}^H(n) \tag{2.9}$$

$$\underline{p}(n) = \underline{p}(n-1) + \underline{u}(n) d^{*}(n). \tag{2.10}$$

Since $\underline{p}(n) = \Phi(n) \underline{\hat{w}}(n)$, we have the recursive algorithm from the above equations:

$$\underline{\hat{w}}(n) = \underline{\hat{w}}(n-1) + \Phi^{-1}(n)\underline{u}(n)[d(n) - \underline{\hat{w}}^H(n-1)\underline{u}(n)]^*. \quad (2.11)$$

Now we define

$$\underline{k}(n) = \Phi^{-1}(n)\underline{u}(n) \quad (2.12)$$

$$\alpha(n) = d(n) - \underline{\hat{w}}^H(n-1)\underline{u}(n), \quad (2.13)$$

where $\underline{k}(n)$ is the Kalman gain and $\alpha(n)$ is the a priori estimation error.

$\Phi^{-1}(n)$ can be obtained from the matrix inversion lemma:

$$\Phi^{-1}(n) = \Phi^{-1}(n-1) - \frac{\Phi^{-1}(n-1)\underline{u}(n)\underline{u}^H(n)\Phi^{-1}(n-1)}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)} \quad (2.14)$$

Consequently we have the Kalman filtering algorithm:

$$\underline{\hat{w}}(n) = \underline{\hat{w}}(n-1) + \Phi^{-1}(n)\underline{u}(n)\alpha^*(n) \quad (2.15)$$

$$\alpha(n) = d(n) - \underline{\hat{w}}^H(n-1)\underline{u}(n) \quad (2.16)$$

$$\Phi^{-1}(n) = \Phi^{-1}(n-1) - \frac{\Phi^{-1}(n-1)\underline{u}(n)\underline{u}^H(n)\Phi^{-1}(n-1)}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)} \quad (2.17)$$

$$\Phi(0) = \delta I \quad (2.18)$$

$$\underline{\hat{w}}(0) = 0, \quad (2.19)$$

where δ is a small positive number.

3. Initial Convergence Analysis of the Kalman Filtering Algorithm

Next we study initial convergence properties of the Kalman filtering algorithm. We define the a posteriori estimation error such as

$$e(n) = d(n) - \underline{\hat{w}}^H(n)\underline{u}(n) \quad (3.1)$$

then we have the following lemma:

Lemma 1. The a posteriori estimation error $e(n)$ has the following relationship with the a priori estimation error $\alpha(n)$:

$$e(n) = \alpha(n)[1 - \underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n)] = \frac{\alpha(n)}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)}. \quad (3.2)$$

Proof Since $e(n) = d(n) - \underline{\hat{w}}^H(n)\underline{u}(n)$,

$$e(n) = d(n) - \underline{\hat{w}}^H(n-1)\underline{u}(n) - \alpha(n)\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n) \quad (3.3)$$

$$= \alpha(n)[1 - \underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n)]. \quad (3.4)$$

From the equation (2.17), we can have,

$$\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n) = \underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n) - \frac{|\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)|^2}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)} \quad (3.5)$$

$$= \frac{\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)} \quad (3.6)$$

therefore,

$$1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n) = \frac{1}{1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)}. \quad (3.7)$$

This completes the proof.

Now we let the tap weight error $\underline{\theta} = \underline{\hat{w}}(n) - \underline{w}_0$ and define the stochastic Lyapunov function as follows:

$$V(n) = \underline{\theta}^H(n)\Phi(n)\underline{\theta}(n). \quad (3.8)$$

Then we have the following result:

Lemma 2. The stochastic Lyapunov function $V(n)$ satisfies the following difference equations:

$$V(n) - V(n-1) = |e_0(n)|^2 - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2 \quad (3.9)$$

$$= |e_0(n)|^2 - (1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))\ |e(n)|^2 \quad (3.10)$$

Proof Now $V(n) = \underline{\theta}^H(n)\Phi(n)\underline{\theta}(n)$, (3.11)

$$V(n) = \underline{\theta}^H(n)\Phi(n)\underline{\theta}(n) = (\underline{\hat{w}}(n) - \underline{w}_0)^H\ \Phi(n)(\underline{\hat{w}}(n) - \underline{w}_0) = \underline{\hat{w}}^H(n)\Phi(n)\underline{\hat{w}}(n) - \underline{w}_0^H\Phi(n)\underline{\hat{w}}(n) - \underline{\hat{w}}^H(n)\Phi(n)\underline{w}_0 + \underline{w}_0^H\Phi(n)\underline{w}_0 \quad (3.12)$$

On the other hand, since

$$V(n-1) = \underline{\hat{w}}^H(n-1)\Phi(n-1)\underline{\hat{w}}(n-1) - \underline{w}_0^H\Phi(n-1)\underline{\hat{w}}(n-1) - \underline{\hat{w}}^H(n-1)\Phi(n-1)\underline{w}_0 + \underline{w}_0^H\Phi(n-1)\underline{w}_0, \quad (3.13)$$

$$V(n) - V(n-1) = \underline{\hat{w}}^H(n)\Phi(n)\underline{\hat{w}}(n) - \underline{\hat{w}}^H(n-1)\Phi(n-1)\underline{\hat{w}}(n-1) - \underline{w}_0^H(\Phi(n)\underline{\hat{w}}(n) - \Phi(n-1)\underline{\hat{w}}(n-1)) - (\Phi(n)\underline{\hat{w}}(n) - \Phi(n-1)\underline{\hat{w}}(n-1))^H\underline{w}_0 + \underline{w}_0^H(\Phi(n) - \Phi(n-1))\underline{w}_0. \quad (3.14)$$

Now we find

$$\Phi(n)\underline{\hat{w}}(n) = \Phi(n)(\underline{\hat{w}}(n-1) + \Phi^{-1}(n)\underline{u}(n)\alpha^*(n)) = \Phi(n)\underline{\hat{w}}(n-1) + \underline{u}(n)\alpha^*(n). \quad (3.15)$$

Also

$$\underline{w}_0^H(\Phi(n)\underline{\hat{w}}(n) - \Phi(n-1)\underline{\hat{w}}(n-1)) = \underline{w}_0^H(\Phi(n)\underline{\hat{w}}(n-1) + \underline{u}(n)\alpha^*(n) - \Phi(n-1)\underline{\hat{w}}(n-1)) = \underline{w}_0^H\underline{u}(n)\underline{u}^H(n)\underline{\hat{w}}(n-1) + \underline{w}_0^H\underline{u}(n)\alpha^*(n). \quad (3.16)$$

And

$$(\Phi(n)\underline{\hat{w}}(n) - \Phi(n-1)\underline{\hat{w}}(n-1))^H \underline{w}_0 = \underline{\hat{w}}(n-1)\underline{u}(n)\underline{u}^H(n)\underline{w}_0 + \alpha(n)\underline{u}^H\underline{w}_0 \quad (3.17)$$

From the equations (2.15) and (2.9), we have

$$\begin{aligned}
&\underline{\hat{w}}^H(n)\Phi(n)\underline{\hat{w}}(n) \\
&= [\underline{\hat{w}}(n-1)+\Phi^{-1}(n)\underline{u}(n)\alpha^*(n)]^H\Phi(n) \\
&\quad [\underline{\hat{w}}(n-1)+\Phi^{-1}(n)\underline{u}(n)\alpha^*(n)] \\
&= \underline{\hat{w}}^H(n-1)\Phi(n)\underline{\hat{w}}(n-1) \\
&\quad + \underline{u}^H(n)\underline{\hat{w}}(n-1)\alpha(n) \\
&\quad + \underline{\hat{w}}^H(n-1)\underline{u}(n)\alpha^*(n) \\
&\quad + \underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n)\,|\alpha(n)|^2 \\
&= \underline{\hat{w}}^H(n-1)\Phi(n-1)\underline{\hat{w}}(n-1) \\
&\quad + \underline{w}^H(n-1)\underline{u}(n)\underline{u}^H(n)\underline{\hat{w}}(n-1) \\
&\quad + \underline{u}^H(n)\underline{\hat{w}}(n-1)\alpha(n) \\
&\quad + \underline{\hat{w}}^H(n-1)\underline{u}(n)\alpha^*(n) \\
&\quad + \underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n)\,|\alpha(n)|^2 \\
&= \underline{\hat{w}}^H(n-1)\Phi(n-1)\underline{\hat{w}}(n-1)+|d(n)|^2 \\
&\quad - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2. \qquad (3.18)
\end{aligned}$$

Therefore

$$\begin{aligned}
&V(n) - V(n-1) \\
&= |d(n)|^2 - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2 \\
&\quad - \underline{w}_0^H\underline{u}(n)\underline{u}^H(n)\underline{\hat{w}}(n-1) \\
&\quad - \underline{w}_0^H\underline{u}(n)\alpha^*(n) \\
&\quad - \underline{\hat{w}}(n-1)\underline{u}(n)\underline{u}^H(n)\underline{w}_0 \\
&\quad - \alpha(n)\underline{u}^H(n)\underline{w}_0 + \underline{w}_0^H\underline{u}(n)\underline{u}^H(n)\underline{w}_0 \\
&= |d(n)|^2 - \underline{w}_0^H\underline{u}(n)d^*(n) \\
&\quad - d(n)\underline{u}^H(n)\underline{w}_0 + |\underline{w}_0^H\underline{u}(n)|^2 \\
&\quad - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2 \\
&= |d(n)-\underline{w}_0^H\underline{u}(n)|^2 \\
&\quad - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2 \\
&= |e_0(n)|^2 - (1-\underline{u}^H(n)\Phi^{-1}(n)\underline{u}(n))\ |\alpha(n)|^2
\end{aligned}$$

or

$$= |e_0(n)|^2 - (1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))\,|e(n)|^2. \qquad (3.19)$$

This completes the proof.

Remark If we have the deterministic system model, then the measurement error $e_0(n) = 0$, $n=1, 2, 3, \cdots$. In this case we can have the convergence analysis in Goodwin and Sin [1].

For the stochastic system model, if we take the expectation operator on both sides of the equation in Lemma 2, then we have

$$\begin{aligned}
&E[V(n)] - E[V(n-1)] \\
&= \sigma^2 \\
&\quad - E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))\ |e(n)|^2] \quad (3.20)
\end{aligned}$$

Now $1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n) \geq 1$, we have

$$E[V(n)] - E[V(n-1)] \leq \sigma^2 - E[|e(n)|^2] \qquad (3.21)$$

Since $\underline{\theta}^H(n)\underline{u}(n)$ is independent of $e_0(n)$,

$$E[|e(n)|^2] = E[|\underline{u}^H(n)\underline{\theta}(n)|^2] + \sigma^2 \qquad (3.22)$$

Therefore

$$\begin{aligned}
&E[V(n)] - E[V(n-1)] \\
&\quad \leq -E[|\underline{u}^H(n)\underline{\theta}(n)|^2] \qquad (3.23)
\end{aligned}$$

for $n=1, 2, 3, \cdots$.

It is easy to show that $V(n)$ is a supermartingale sequence [2]. And we conclude that

$$\lim_{n\to\infty} V(n) = V(\infty) \qquad (3.24)$$

almost surely.

Now we summarize the above results as follows:

Theorem 1. The supermartingale sequence $V(n)$ is convergent to $V(\infty)$ in probability one.

Remark $V(\infty)$ is not necessarily zero.

$$\text{Now} \quad 0 \leq E[V(n)] \leq E[V(0)] < \infty \qquad (3.25)$$

and

$$\begin{aligned}
E[V(n)] &= E[\underline{\theta}^H(n)\Phi(n)\underline{\theta}(n)] \\
&= tr\ E[\underline{\theta}^H(n)\Phi(n)\underline{\theta}(n)] \\
&= tr\ E[\Phi(n)\underline{\theta}(n)\underline{\theta}^H(n)] \\
&\geq \lambda_{\min}E[\Phi(n)]\ tr\ E[\underline{\theta}(n)\underline{\theta}^H(n)] \\
&= n\ \lambda_{\min}(E[\underline{u}(i)\underline{u}^H(i)])\cdot\|\underline{\theta}(n)\|_E^2, \qquad (3.26)
\end{aligned}$$

where

$$\|\underline{\theta}(n)\|_E^2 = tr\ E[\underline{\theta}(n)\underline{\theta}^H(n)]. \qquad (3.27)$$

Since $\lambda_{\min}(E[\underline{u}(i)\underline{u}(i)]) = 0, \quad 1 \leq i < M$

$$\begin{aligned}
&E[\underline{u}(i)\underline{u}^H(i)] = R, \quad i \geq M \\
&\lambda_{\min}(R) > 0, \qquad (3.28)
\end{aligned}$$

we have

$$\|\underline{\theta}(n)\|_E^2 \leq \frac{1}{(n-M+1)\ \lambda_{\min}(R)}\ E[V(0)], \quad n \geq M. \qquad (3.29)$$

Summarizing the above results, we have:

Theorem 2. The Kalman filtering algorithm for the stochastic system model is strongly convergent with probability one and the upper bound for the norm of the tap weight estimation error $\underline{\theta}(n) = \underline{\hat{w}}(n)-\underline{w}_0$ is given as follows:

$$\|\underline{\theta}(n)\|_E \leq \frac{\sqrt{\delta}}{\sqrt{(n-M+1)\ \lambda_{\min}(R)}}\ \|\underline{w}_0\|, \quad n \geq M. \qquad (3.30)$$

From the equation (3.9), we have

$$E[V(n)] = E[V(0)] + \sum_{k=1}^{n}\{\sigma^2 - E[(1+\underline{u}^H(k)\Phi^{-1}(k-1)\underline{u}(k))^{-1}|\alpha(k)|^2]\} \le E[V(0)] < \infty. \tag{3.31}$$

Hence

$$\lim_{n\to\infty} E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))^{-1}|\alpha(n)|^2] = \sigma^2 \tag{3.32}$$

and it is very easy to show that [3]

$$\lim_{n\to\infty} E[\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)] = 0. \tag{3.33}$$

We thus have the following result:

Theorem 3. For the a priori and a posteriori estimation errors, we have

$$\lim_{n\to\infty} E[|\alpha(n)|^2] = \lim_{n\to\infty} E[|e(n)|^2] = \sigma^2 \tag{3.34}$$

In the initiation stage ($1 \le n \le M-1$), we put an initial condition $\Phi(0)=\delta I$, where δ is a small number and I is the identity matrix. This insures the inverse operation of $\Phi(n)$, $n\ge 1$. On the other hand, this condition may cause initial instability due to uncertainty of the initial condition. If δ is small, then from the equation (3.20) we expect that the term $1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)$ gets large and drives the algorithm to converge during the initiation stage but it takes time to reach the minimum value. For a large value of δ, the term $1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)$ is near to 1 and $|e_0(n)|^2 > |e(n)|^2$, for $0\le n\le M$ because of initial uncertainty. Now we conclude that from the equation (3.10), for a very large valued of δ, $V(n)-V(n-1) > 0$, $1\le n\le M$. The stochastic Lyapunov function is increasing. This phenomena happen only in the initial stage since for $n\ge M$, the Kalman algorithm lies in the attraction region of Theorem 2 provided that we do not have roundoff error effects.

In order to prevent this initial divergence, we need to restrict the value δ in some bound. Hence we should have from the equation (3.20),

$$E[V(n)] - E[V(n-1)] = \sigma^2 - E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))|e(n)|^2] \le 0$$

Therefore, we have

$$\sigma^2 \le E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))|e(n)|^2]. \tag{3.35}$$

Since we can have approximately

$$E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))|e(n)|^2] = E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)u(n)]E[|e(n)|^2] \tag{3.36}$$

and we can assume $\Phi(n-1) = \delta I$, for a large δ, and for $1\le n\le M$, we have

$$[1+\delta^{-1}tr(R)]E[|e(n)|^2] \ge \sigma^2,$$

for $1\le n\le M$. (3.37)

Therefore

$$\delta \le \frac{tr(R)\ \sigma^2_{e,\min}}{\sigma^2 - \sigma^2_{e,\min}},$$

where

$$\sigma^2_{e,\min} = \min_{1\le n\le M} E[|e(n)|^2]. \tag{3.38}$$

Next we consider the following equation:

$$E[V(n)] - E[V(n-1)] = \sigma^2 - E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))^{-1}|\alpha(n)|^2]. \tag{3.39}$$

For the very small value of δ the term $1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n)$ gets very large and the right side of the equation (3.39) becomes positive. Hence the Kalman filtering algorithm diverges. Using the same arguments before, we can have

$$\sigma^2 \le E[(1+\underline{u}^H(n)\Phi^{-1}(n-1)\underline{u}(n))^{-1}|\alpha(n)|^2] \tag{3.40}$$

and

$$[1+\delta^{-1}\ tr(R)]^{-1}E|\alpha(n)|^2 \ge \sigma^2, \tag{3.41}$$

for $1\le n\le M$.

Therefore we have

$$\delta \ge \frac{tr(R)\ \sigma^2}{\sigma^2_{\alpha,\min} - \sigma^2},$$

where

$$\delta^2_{\alpha,\min} = \min_{1\le n\le M} E[|\alpha(n)|^2]. \tag{3.42}$$

We can summarize the above result as follows :

Theorem 4. In order to avoid initial divergence in the Kalman filtering algorithm, the arbitrary small initial condition $\Phi(0)=\delta I$ should be constrained as follows :

$$\frac{tr(R)\ \sigma^2}{\sigma_{\alpha,\min}-\sigma^2} \le \delta \le \frac{tr(R)\ \sigma^2_{e,\min}}{\sigma^2-\sigma^2_{e,\min}}$$

4. Conclusion

Initial convergence behaviors as well as golval convergence properties are dealt with. In the initialization stage we put an arbitrary initial condition in the Kalman filtering algorithm. The condition perturbs the algorithm and causes initial instability. We analyzed this behaviors by using the stochastic Lyapunov function difference equation. And we provided a condition to prevent this divergence.

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