TRAJECTORY CONTROL OF A FLEXIBLE ROBOT ARM USING INVERSE DYNAMICS

Minoru Sasaki*, Eiji Yamagata** and Hikaru Inooka***
* Department of Mechanical Engineering
Miyagi National College of Technology
Medeshima Natori, Miyagi, 981-12, Japan
**Terumo Corp., Fujinomiya Shizuoka, 418, Japan
***Department of Mechanical Engineering
Faculty of Engineering
Tohoku University,
Sendai, 980, Japan

Abstract: In recent years there has been much interest in using light-weight, higher performance arms for both commercial and space-based applications, leading to the research of flexible robot manipulator. This paper is concerned with the trajectory control of a flexible arm using inverse dynamics. Inverse problems are important to robot control and programming, since they allow one to find the appropriate inputs necessary for producing the desired outputs. The input is obtained by the numerical inversion of Laplace transformation in the time domain. And we attempt the trajectory control experiment of a flexible arm using this calculated input. In this article we compare the numerical results with experimental results and can find good agreement. The results make clear that this technique has the good potential for the control of tip trajectory of flexible robot arms.

1.Introduction

In recent years, there has been technological interest in the design and control of light-weight robots and several papers on it have been published during past years. There are many researches concerning the control of a flexible arm. They are used some special sensing devices as a sensor for the vibration; for example, optical sensor, strain gage, CCD camera and accelerometer[1-6].

But there are few papers about an inverse dynamics problem for the trajectory control of flexible arms. Zheng-Dong Ma, et al. [7] computed actuator torques required for a flexible arm to track a given trajectory by using virtual rigid link coordinates. Eduardo Bayo[8] presented a structural finite element technique based on Bernoulli-Euler beam theory which will permit the finding of the torques that are necessary to apply at one end of a flexible link to produce a desired motion at the other end.

But the method proposed here aims at controlling tip motion by directly computing the base angle necessary to apply at one end of the link to achieve the desired trajectory at the other end. A laplace transformation method is used to obtain the results in the time domain. The problem of inverting the Laplace transform of determining time function from its Laplace transform can often be solved analytically by applying a partial fraction expansion or integration along some contour in the complex plane. When this proves to be too difficult or impossible due to the complexity of the formula, numerical methods may be necessary. We used here the procedure for using the approximation of the exponential function in the Bromwich integral[9-10]. An efficient and simple numerical algorithm is then proposed for the solution of equations. Finally, the proposed technique is applied to a particular flexible one-link robot arm with payload mass. The arm is driven by a DC motor about an axis through the arm's fixed end. And we attempt the trajectory control experiment of a flexible arm. This experiment is the use of the signal produced by the computer simulation; the computer reproduces the stored signal in real time which can be a reference input to the system.

In this article we compare the analytical results with experimental results and can find good agreement. The results make clear that this technique has the good potential for the control of tip trajectory of flexible robot arms.

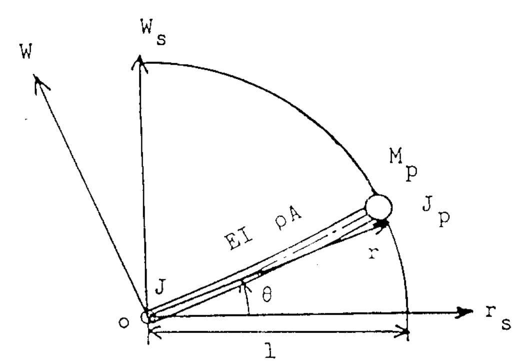

Figure 1 A flexible arm model.

2. Formulation and Analysis

Figure 1 shows a uniform arm of length l with a payload mass at the tip of the arm. The other end of the arm is clamped on a vertical shaft, which is driven by a DC motor. Let us consider small motions about the equilibrium state and low angular velocity. This assumption implies retaining only linear terms in the equations of motion. Here we define the flexural displacement of the arm W(r,t) as $W(r,t)=Ws(r,t)-r\theta$. In that case, the equation of motion of the arm is given by

$$E(1+c\frac{\partial}{\partial t})I\frac{\partial^4 W}{\partial r^4}+\rho A(r\frac{\partial^2\theta}{\partial t^2}+\frac{\partial^2 W}{\partial t^2})=0 \qquad (1)$$

where ρ is the mass density, W is the flexural displacement, A is the cross-sectional area, I is the moment of inertia of arm cross-section, t is the time, E is the young's modulus, θ is rotating angle and c is the internal damping coefficient. Boundary conditions of the arm and the equlibrium equation between the torque and the moment on the shaft are given by

$$W(0,t)=0,\quad \partial W(0,t)/\partial r=0, \qquad (2,3)$$

$$J_P\frac{\partial}{\partial t^2}\{\theta+\frac{\partial W(l,t)}{\partial r}\}=-E(1+c\frac{\partial}{\partial t})I\frac{\partial^2 W(l,t)}{\partial r^2} \qquad (4)$$

$$M_P\frac{\partial}{\partial t^2}\{l\theta+W(l,t)\}=E(1+c\frac{\partial}{\partial t})I\frac{\partial^3 W(l,t)}{\partial r^3} \qquad (5)$$

$$JN\frac{\partial^2\theta}{\partial t^2}+N(\varepsilon+\frac{K\tau K_b}{R})\frac{\partial\theta}{\partial t}$$

$$=\frac{E}{N}(1+c\frac{\partial}{\partial t})I\frac{\partial^2 W(0,t)}{\partial r^2}+\frac{K\tau}{R}u(t), \qquad (6)$$

where Mp is the payload mass, J is the moment of inertia of the motor and shaft, ε is the viscous friction coefficient, Jp is the payload mass moment of inertia, N is the gear ratio, u(t) is the armature input voltage, R is the resistance of the armature, and $K\tau$ and Kb are constants related to the motor torque and the back electro magnetive force, respectively. Equation (1) and (6) can be solved by applying the method of the Laplace transform with respect to t, defined by

$$L[W(r,t),\ \theta(t),\ u(t)]$$

$$=\int_0^\infty [W(r,t),\ \theta(t),\ u(t)]\exp(-st)dt$$

$$=[W(r,s),\ \Theta(s),\ \bar{u}(s)] \qquad (7)$$

$$L^{-1}[W(r,s),\ \Theta(s),\ \bar{u}(s)]$$

$$=(2\pi i)^{-1}\int_{\zeta-i\infty}^{\zeta+i\infty}[W(r,s),\ \Theta(s),\ \bar{u}(s)]\exp(st)ds$$

$$=[W(r,t),\ \theta(t),\ u(t)] \qquad (8)$$

Transforming equations (1)-(6) with respect to t gives

$$E(1+cs)Id^4W/dr^4+\rho As^2(W+r\Theta)=0, \qquad (9)$$

$$W(0)=0,\quad dW(0)/dr=0, \qquad (10,11)$$

$$J_Ps^2\{\Theta+dW(l)/dr\}+E(1+cs)Id^2W(l)/dr^2=0, \qquad (12)$$

$$M_Ps^2\{W(l)+l\Theta\}-E(1+cs)Id^3W(l)/dr^3=0, \qquad (13)$$

$$JNs^2\Theta+(\varepsilon+K\tau K_b/R)Ns\Theta$$

$$=E(1+cs)I/Nd^2W(0)/dr^2+(K\tau/R)\bar{u}(s). \qquad (14)$$

Hereinafter, the following dimensional and non-dimensional quantities are introduced:

$$\beta^4=-\alpha^2/(1+\eta_c\alpha)=-\rho As^2l^4/EI(1+cs),\quad \bar{W}=W/l,$$

$$\bar{W}_s=W_s/l,\quad \bar{r}=r/l,\quad \bar{J}_P=J_P/\rho Al^3,\quad \bar{M}_P=M_P/\rho Al,$$

$$\bar{J}=JN/\rho Al^3,\quad \alpha^2=\rho Al^4s^2/EI,$$

$$\eta_c=c(\rho Al^4/EI)^{-1/2},\quad K_1=K\tau l/EIR,$$

$$\bar{\varepsilon}=(\varepsilon+K\tau K_b/R)N(l/\rho Al^2EI)^{1/2}. \qquad (15)$$

A general solution to equation (1) is

$$\bar{W}(\bar{r})=A\cos\beta\bar{r}+B\sin\beta\bar{r}+C\cosh\beta\bar{r}+D\sinh\beta\bar{r}-\bar{r}\Theta \qquad (16)$$

A, B, C and D are unknown constants determined from the boundary conditions. Substitution of eq.(16) into equations (10)-(14) leads to

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix}\begin{vmatrix} A \\ B \\ \Theta\end{vmatrix}=\begin{vmatrix} 0 \\ 0 \\ K_1\bar{u}(s)\end{vmatrix} \qquad (17)$$

where

$$a_{11}=-\bar{J}_P\alpha^2\beta(\sin\beta-\sinh\beta)$$
$$-(1+\eta_c\alpha)\beta^2(\cos\beta+\cosh\beta),$$

$$a_{12}=\bar{J}_P\alpha^2\beta(\cos\beta-\cosh\beta)$$
$$-(1+\eta_c\alpha)\beta^2(\sin\beta+\sinh\beta),$$

$$a_{13}=\bar{J}_P\alpha^2\cosh\beta+(1+\eta_c\alpha)\beta\sinh\beta,$$

$$a_{21}=\bar{M}_P\alpha^2(\cos\beta-\cosh\beta)$$
$$-(1+\eta_c\alpha)\beta^3(\sin\beta-\sinh\beta),$$

$$a_{22}=\bar{M}_P\alpha^2(\sin\beta-\sinh\beta)$$
$$+(1+\eta_c\alpha)\beta^3(\cos\beta+\cosh\beta),$$

$$a_{23}=\bar{M}_P\alpha^2\sinh\beta/\beta-(1+\eta_c\alpha)\beta^2\cosh\beta,$$

$$a_{31}=2(1+\eta_c\alpha)\beta^2,\quad a_{32}=0,\quad a_{33}=\bar{J}\alpha^2+\bar{\varepsilon}\alpha.$$

Here we put $a_{31}=0$, $a_{32}=0$, $a_{33}=1$ in eq.(17) when input is applied as a rotating angle. From eq.(16) one has

$$\bar{u}'(s)=\Delta\times\bar{W}(\bar{r},s)/K_1\{\Delta_A(\cos\beta\bar{r}-\cosh\beta\bar{r})$$
$$+\Delta_B(\sin\beta\bar{r}-\sinh\beta\bar{r})$$
$$+\Delta_C(\sinh\beta\bar{r}/\beta-\bar{r})\} \quad (18)$$

where

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix},\quad \Delta_A=\begin{vmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 1 & a_{32} & a_{33} \end{vmatrix},$$

$$\Delta_B=\begin{vmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 1 & a_{33} \end{vmatrix},\quad \Delta_C=\begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{vmatrix}$$

Here we consider the desired tip trajectory $\bar{W}_s(1,s)=e^{-0.5s}(1-e^{-0.5s})^2/0.25s^3$ as shown in Figure 2. This desired tip trajectory is based on bang-bang torque, which is required for time optimal control in the case of a single link rigid robot arm. The final results in the time domain may be obtained through the application of the numerical inversion of Laplace transform [9-10].

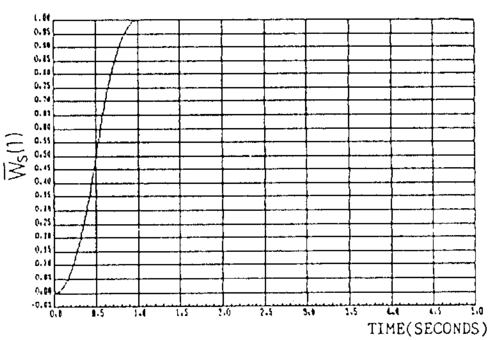

Figure 2 Desired tip trajectory.

3.The Calculation of Inversion of Laplace Transform

The Laplace inversion techniques are classified as (1)method using simple rules and table of transforms, (2)method using Bromwich integral and Cauchy integral theorem, and (3)numerical method. In order to solve these problems the present paper has been used a numerical method[9-10] for computer use with the following advantages: (1)Programming is easy. (2)Required memory size is small. (3)Estimation and control of the errors are easy. (4)Application range is wide. The essential point of this method consists in the approximation of the exponential function exp(s) by

$$E_{ec}(s,a)=\exp(a)/2\cosh(s-a)$$
$$=(e^a/2)\sum_{n=-\infty}^{\infty} j(-1)^n/[s-a-j(n-0.5)\pi]$$
$$=e^s-e^{-2a}e^{3s}+e^{-4a}e^{5s}-\cdots \quad (19)$$

We define a function for t>0

$$f_{ec}(t,a)=(2\pi i)^{-1}\int_{\varsigma-i\infty}^{\varsigma+i\infty} F(s)E_{ec}(st,a)ds \quad (20)$$

On substituting (19) into (20), we have

$$f_{ec}(t,a)=f(t)-e^{-2a}f(3t)+e^{-4a}f(5t)-\cdots \quad (21)$$

and

$$f_{ec}(t,a)=(e^a/t)(F_1+F_2+F_3+\cdots) \quad (22)$$

where

$$F_n=(-1)^n I_m F\{[a+j(n-0.5)\pi]/t\} \quad (23)$$

Equation (21) shows that the function fec(t,a) gives a good approximation when a>>1 and will be used in error estimation. On the other hand, (22) can be used to compute the numerical value of the inverse Laplace transform effectively. In practice, we must truncate the infinite series in (22) to some finite terms. Simple truncation, however, results in a relatively large error and is not realistic. In this respect, an effective method using Euler transformation has been developed. We transform (22) as follows

$$f_{ec}(t,a)=(e^a/t)\left(\sum_{n=1}^{k-1}F_n+\sum_{n=0}^{\infty}D^nF_k/2^{n+1}\right) \quad (24)$$

In practice, (24) is truncated to some finite terms, so that it is more convenient to use the expression

$$\sum_{n=0}^{m}D^nF_k/2^{n+1}=(1/2^{m+1})\sum_{n=0}^{m}A_{mn}F_{k+n} \quad (25)$$

where the Amn are defined recursively by

$$A_{mn}=1,\quad A_{mn-1}=A_{mn}+\binom{m+1}{n} \quad (26)$$

Thus we calculate the (l,m)th approximation by

$$f_{ec}^{lm}(t,a)=(e^a/t)\left(\sum_{n=1}^{l-1}F_n+2^{-m-1}\sum_{n=0}^{m}A_{mn}F_{l+n}\right) \quad (27)$$

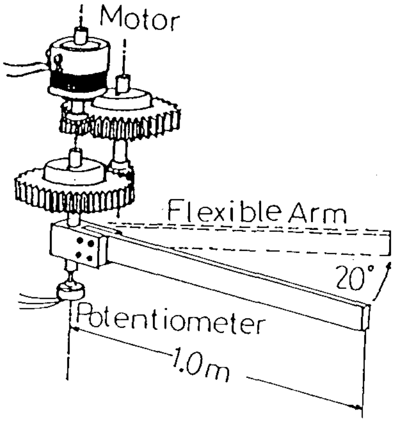

Figure 3 Experimental device.

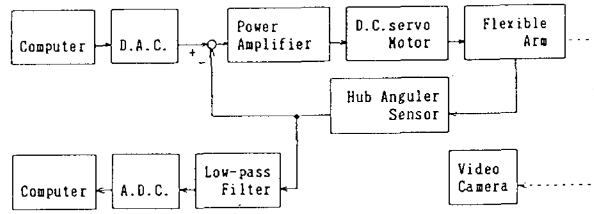

Figure 4 Block diagram.

4.Experimental device and experimental process

Figure 3 shows the experimental arm. It is a 1000(mm) long, 29.9(mm) thick, 2.0(mm) width, very flexible that can bend freely in the horizontal plane but not in the vertical plane. At the one end, the arm is clamped on a rigid hub mounted on the vertical gear shaft(gear ratio N=5) which is driven by a dc motor; thus, the arm can rotate in the horizontal plane. The reference input has two types. One is a rotation angle and second is a torque. The rotation angle input is more easy to control than torque input. Thus we used here the rotation angle input. Figure 4 is the block diagram of a trajectory control experiment. The dc motor(SANYO DENKI SM60) is driven by a current amplifier and its rotating angle is measured by a potentiometer(MIDORI PRECISIONS CP-2U). The reference input is caliculated by a computer(NEC PC-9801 VM21) and transformed into electrical signals which are transmitted to mechanical arm. This signal is analogized by 12 bits digital-analog(D/A) converter. The angular signal from the potentiometer is amplified and filtered by an analog low-pass filter. This filter's cut-off frequency is 100 (Hz). The signal is then digitized by 12 bits analog-digital(A/D) converter. And this signal is sent to a computer; these data are stored in its memory. The CCD video camera(NATIONAL MACLOAD) is used to measure the motion of the arm. In this case the sampling period used for the computer input is 5(ms) and it used for measurement of tip position is 1/30 (sec).

5.Experimental results and discussion

Fig.5 is the input rotation angle without payload mass. Fig.6 shows the end point trajectory. Fig.7 is the mode simulation. In this case the numerical results agree with the experimental results. The input rotation angle keeps on moving 0.1 sec before and after the tip moved its start and final position, during this time the input rotation angle is absorbing or compensating the vibrations.

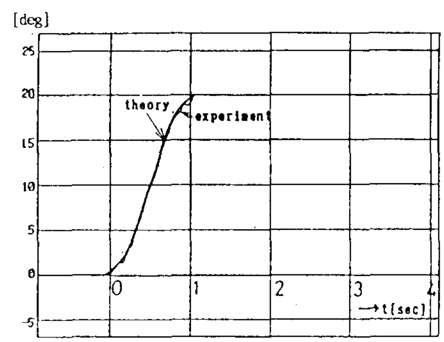

Figure 5 Base angle(without payload mass).

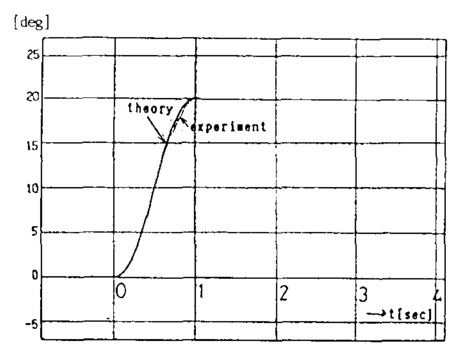

Figure 6 Tip trajectory(without payload mass).

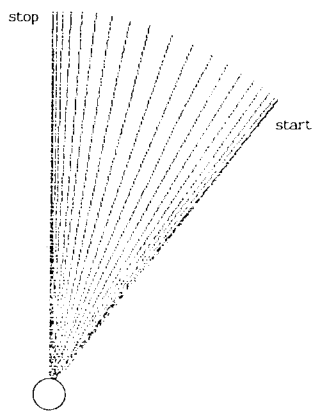

Figure 7 Mode simulation(without payload mass).

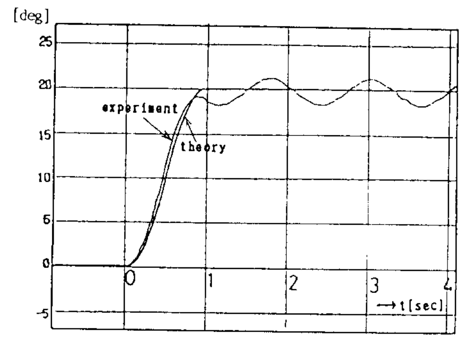

Figure 9 Tip trajectory(with payload mass).

Fig.8 shows the input rotation angle with payload mass. Fig.9 is the results for the tip trajectory. Fig.10 is the mode simulation. When the desired tip trajectory's period is smaller than the first natural frequency, the swing-back motion is necessary. The rotation angle is very difficult to follow the calculated reference input because of gears backlash, modeling error and any other nonlinearity. The results show that the residual vibrations remain. Compensating this residual vibrations we attached the additional sensor using strain gage. We feed back the flexural distortion at r=20(cm) and add reference input after the tip reached the commanded position. Fig.11 is the block diagram of a trajectory control experiment with strain gage feed back. This experimental results are shown in Fig.12-13. The residual vibrations slightly damped. This results make clear that this technique has the good potential for the control of tip trajectory of flexible arms.

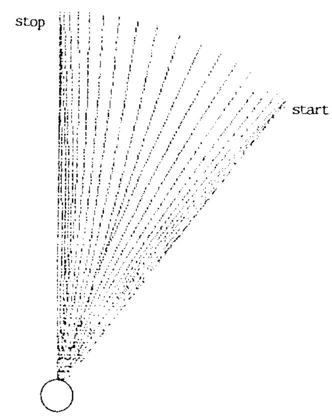

Figure 10 Mode simulation(with payload mass).

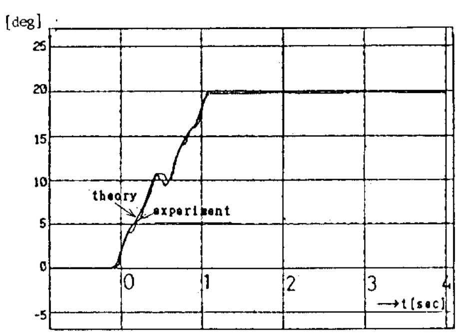

Figure 8 Base angle(with payload mass).

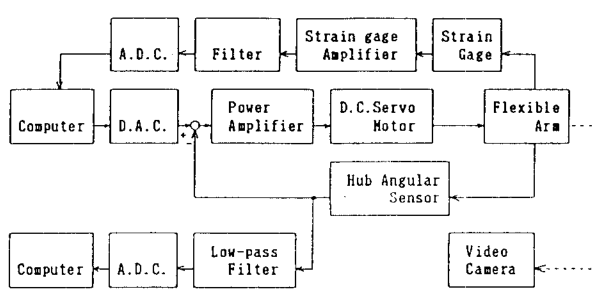

Figure 11 Block diagram(with strain gage).

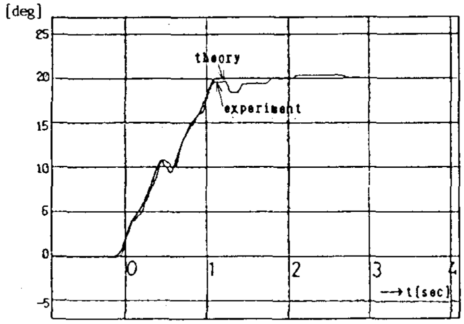

Figure 12 Base angle(with compensation).

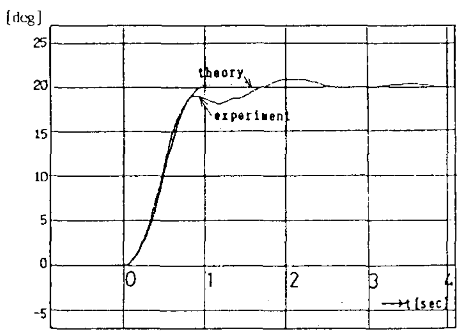

Figure 13 Tip trajectory(with compensation).

Conclusions

A new method has been developed for the inverse dynamics of a single link flexible robot arm with payload mass driven by a DC motor about an axis through the arm's fixed end. The inverse dynamics are important to flexible robot arm control and programming, since they allow one to find the appropriate input rotating angle necessary for producing the desired outputs trajectory without overshoot and inverse response. This proposed method is based on the numerical inversion of Laplace transform. Results obtained are summarized as follows.

(1) The first advantage of the numerical inversion Laplace transform method is to approximate the exponential function in the Bromwich integral and can easily be implemented in efficient subroutines, micro-programs and special hardware devices.

(2) The second advantage of this method is that we need not consider the mode numbers and the natural frequencies of the system.

(3) We compare the simulation results with the experimental results and can find good agreement.

(4) The proposed procedure has the good potential for the control of tip trajectory of flexible robot arms.

(5) When the arm has a payload mass, the natural frequency decreases; the influence of the flexibility of the arm increases and the residual vibration remains longer. In this case the input keeps on actuating after the tip has reached its final position because of compensating the residual vibration.

(6) When the desired tip trajectory's period is smaller than the first natural frequency the swing-back motion is necessary.

References

[1] Fukuda, T. "Flexibility Control of Elastic Robotic Arms," J. Robotics Syst. 2, 73-88(1985).

[2] Sakawa, Y., Matsuno, F. and Fukushima, S. "Modeling and Feedback Control of a Flexible Arm," J. Robotics Syst. 2, 453-472(1985).

[3] Cannon, R.H.Jr. and Schmitz, E. "Initial Experiments on the End-Point Control of a Flexible One-Link Robot," Int. J. Robot. Res. 3, 62-75(1984).

[4] Truckenbrodt, A. "Truncation Problems in the Dynamics and Control of Flexible Mechanical Systems," Proc. VIII. IFAC. World Congress Kyoto/Japan. 1909-1914(1981).

[5] Harashima, F. Ueshiba, T. and Hashimoto, H. "An Adaptive Control Method for a Single-Link Flexible Arm Using a CCD Camera as an End-Point Position Sensor," Proc. 4th Annual Conference of RSJ, 419-422(1986).

[6] Hanafusa, H. Nakamura, Y. and Kadokawa, Y. "Vibration Control of a Flexible Arm," JAACE Symposium on Flexible Automation, Osaka, Japan, 27-30(1985).

[7] Ma, Z.D. Asada, H. and Tokumaru, H. "Inverse Dynamics of Flexible Robot Arms for Trajectory Control," J. System and Control, **31**, 764-772(1987).

[8] Bayo, E. "A Finite-Element Approach to Control The End-Point Motion of a Single-Link Flexible Robot," J. Robotic Systems, **4**, 63-75(1987).

[9] Hosono, T. "Numerical inversion of Laplace transform," (in Japanese), J. Inst. Electr. Eng. Jpn., **494**, 54-A64(1979).

[10] Hosono, T. "Numerical inversion of Laplace transform and some applications to wave optics," Radio Science, **16**, 1015-1019(1981).