'88 KACC 1988. 10. 21~22

Design and implementation of VSS controller on personal computers

Tetsuo Shiotsuki and Shigeyasu Kawaji

Department of Electrical Engineering and Computer Science
Faculty of engineering
Kumamoto University
Kumamoto, 860 Japan

Abstract: A simple procedure is proposed for the design of VSS controller to stabilize the single inverted pendulum. The controller is implemented by using a 16bits personal computer.

1. Introduction

We consider a single input system

$$\dot{x}(t)=Ax(t)+bu(t) \;,\quad x(0)=x_0 \qquad (1)$$

where x is n-dimensional real state vector, u scalar input, and A,b are real constant matrices of appropriate sizes. x_0 is an initial condition and the system is defined in $t\geq 0$. We assume that the equation(1) describes just the inverted pendulum or its augmented system with internal model of disturbances.

VSS(variable structure system) is well known as a robust control scheme [1]. The control strategy of the relay-type VSS is described in programing language style as follws.

```
repeat for every clock period Δ
{
   sample x;
   evaluate s = f^T x;
   if s > 0 then let u = -M           (2)
            else let u = M;
   output u;
}
```

where s is a switching variable and M is a magnitude parameter. The surface defined by s=0 in state space is called switching surface.Under certain condition the behavior of (1) with (2) is described in the following two motions

$$\begin{aligned} &x1:\ s \rightarrow 0 \\ &x2:\ x \rightarrow 0 \text{ on } s = 0 \end{aligned} \qquad (3)$$

where trajectory x1 corresponds to fast motion of VSS and its behavior depends mainly on the parameter M. Trajectory x2 is called as sliding motion and the behavior depends on the parameter f. The control scheme which causes such a motion is called a sliding mode control.

The problem is the design of control law and it is reduced to determination of the parameters {f,M} satisfying the two conditions:

$$\begin{aligned} &(\text{I})\ \ s(t)\dot{s}(t)<0 \\ &(\text{II})\ x(t)\rightarrow 0 \ \text{ on } s(t)=0. \end{aligned} \qquad (4)$$

Another problem is the synthesis of a controller to realize the control strategy (2). In this paper, we utilize a commercially available personal computer to solve it.

2. Determination of parameters

To determine the control parameters {f,M} it is convienent to use the 'cheap control technique', which is formulated as follows.

Consider a performance index

$$J_\varepsilon = \int_{t=0}^{\infty} (|x|_Q^2+(1/\varepsilon^2)|u|^2)dt \qquad (5)$$

where $Q = D^TD$ is nonnegative matrix, ε positive real number. It is well known that the value of the index is evaluated as

$$J_\varepsilon = x_0^T P_\varepsilon x_0 \qquad (6)$$

where P is the positive definite solution of the Riccati equation

$$A^T P_\varepsilon + P_\varepsilon A + Q - (1/\varepsilon^2)P_\varepsilon bb^T P_\varepsilon = 0 \qquad (7)$$

If $\varepsilon\rightarrow 0$ implies $J\rightarrow 0$, the control strategy is called the cheap control and the feedback control law is determined as

$$u \doteq -(1/\varepsilon^2)b^T P_\varepsilon x \qquad (8)$$

It results the closed loop system such that

$$\dot{x}(t) = (A + (1/\varepsilon^2)bb^TP_\varepsilon) x$$

or in descriptor form

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A & b \\ f & -\varepsilon \end{bmatrix}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \quad (9)$$

where

$$f = -(1/\varepsilon)b^TP_\varepsilon \quad (10)$$

One of the poles of resulting closed loop system tend to infinite and remainders are finitely located such that they cansel the all zeros of the open loop system. Since the zeros which is characterized as the zeros of the pencil

$$\begin{bmatrix} A-sI & b \\ D & 0 \end{bmatrix} \quad (11)$$

must be stable; i.e. the system must be minimum phase.

The complete characterization of cheap control is presented in reference [2][3].We will be utilize a sufficient condition.

Lemma 1: if (D,A,b) is completely controllable, observable and minimum phase, then

$$P_\varepsilon \rightarrow 0$$

$$\varepsilon P_\varepsilon \rightarrow L_0 < \infty \quad (12)$$

as $\varepsilon \rightarrow 0+$. □

proof : see [3].

It follows from lemma 1 that f defined in (4) converges to a finite value and is able to be numerically calculated. The algorithm is illustrated as follows.

ALGORITHM

step1: Solve the generalized eigen problem with sufficiently small positive number ε

$$\left(\lambda \begin{bmatrix} 0 & I & 0 \\ -I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & A & b \\ A^T & Q & 0 \\ b^T & 0 & \varepsilon^2 \end{bmatrix} \right) \begin{bmatrix} z \\ x \\ u \end{bmatrix} = 0 \quad (13)$$

and select n-eigenvectors corresponding to stable eigen-values.

step2: Evaluate f as

$$f = (1/\varepsilon)[u_1,u_2,\ldots,u_n][x_1,x_2,\ldots,x_n]^+ \quad (14)$$

These algorithm is easily developed by using the subroutine packages for eigen-problems,EISPACK [4]. It is well known that the square matrix $P = (1/\varepsilon)[z_1,z_2,\ldots,z_n][x_1,x_2,\ldots,x_n]^+$ converges to positive semidefinete matrix P^0.

3. Analysis of closed loop system

In this chapter we investigate the behavir of the closed loop system. At first the condition (I) is checked.

If s(t)>0 according to the control law (2), the closed loop system is described as

$$\dot{x}(t) = A\,x(t) - bM \quad (15)$$

and then

$$\begin{aligned} \dot{s}(t) &= f^T\dot{x}(t) \\ &= f^TAx(t) - (1/\varepsilon)b^TPbM \\ &= s_1(t)+s_2 \end{aligned} \quad (16)$$

When |x(t)| and ε is sufficiently small and M is sufficiently large , the positiveness of P implise |s1|<<|s2| or

$$\dot{s}(t) < 0 \quad (17)$$

or

$$s(t)\dot{s}(t)<0. \quad (18)$$

In the case of s(t)<0 the same discussions hold.

It follows that if x(0) is in a certain region s(t) tends to the surface s=0 and constrained on it. And the magnitudeparameter M should be determined to satisfy the condition (II) sufficiently.

After the state x has reached to the surface s=0 it might drift on the surface. Next the stability of the drifting behavior or the sliding motion must be checked. This constrained motion is approximated in the descriptor form.

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A & b \\ f & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \quad (19)$$

which is obtained by putting $\varepsilon=0$ into eq.(9). The poles of the system is identical to the zeros of the open loop system with output

$$y(t) = f^Tx(t). \quad (20)$$

Since we select only the stable eigenvectors in the algorithm, the resulting system is stable. Thus, the sliding motion is stable and condition (II) will be satisfied.

4. Real-time control program

To realize the control strategy (2) we use a 16-bits personal computer (8086 CPU). The basic functions of a personal computer are

(1) It has an environment to develop programs in high-level language.
(2) Sensor I/O systems, e.g. A/D,D/A,etc. are able to be adapted and directly controlled from user's program.
(3) Timer interrupt service is utilized for user's program.

Based on these functions we can determine the specification of the program package as
(1) C language is used to develope the real-time control task
(2) Each task is composed of 4-processes;
1- pre-process
2- main-process
3- post-process
4- emergency-process

and they are connected as depicted in Fig.1.
(3) Pre-process is used to initialize the control variables in computer memory.
(4) Main-process is used to realize a repitative process in a given period Δ. The control strategy (2) is programed in this process.
(5) Post-process is used to the normal termination of the control program, and emergency process for abnormal termination.

These specification is realized by using the timer interrupt service of computer hardware system and task scheduling system. The procces-ing rule of task scheduling is schematically illustraited in Fig.2. One example of program list of main-process is shown in Fig.3, which is for the control strategy (2).

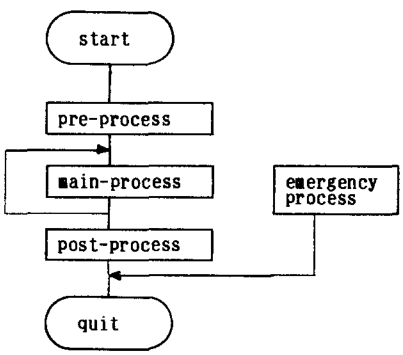

Fig.1: Structure of a task.

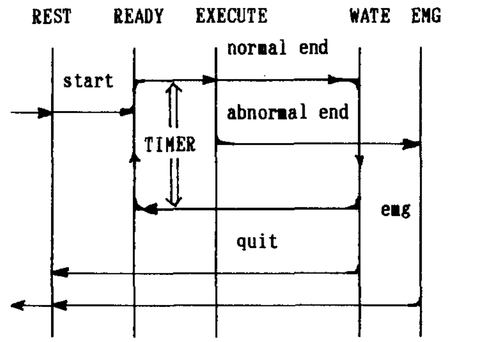

Fig.2 : Fence diagram of task scheduler.

```
main_process()
{
    x1 = get_data1();
    x2 = get_data2();
    x3 = get_data3();
    x4 = get_data4();
    s  = f1*x1+f2*x2+f3*x3+f4*x4;
    if(s>0.0) u = -M
      else    u = M;
    output_to_DA(u);
}
```

Fig.3 : A sample of program list(main process)

5. Experiment

The inverted pendulum used in our study is shown in Fig.4. The parameters in the state space eqation (1) is as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.28 & -4.1 & 0.028 \\ 0 & 19 & 7.7 & -1.9 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 0.98 \\ -1.8 \end{bmatrix}$$

where the elements of the state x=[x1,x2,x3,x4] are corresponding to
x1 : position of the cart [m],
x2 : angle of the pendulum [theta],
x3 : derivative of x1 [m/sec],
x4 : derivative of x2 [theta/sec]

respectively. x1,x2,x3 are measured directly by potentio-meters and tacho-meter. And x4 is approximated by the difference formula

$$x4 = (x2_k - x2_{k-1})/\Delta . \qquad (21)$$

One should note that the parameters describe only a nominal model and there are a lot of ambiguity, e.g. nonlinearity, measurement error, parameter drift, etc. So we must consider some robustness of control system.

The design procedure is executed with fixed values

$$\varepsilon = 1.0 \times 10^{-10},$$
$$M = 4.0 \text{ [V]}.$$

Fig.5 shows the response of the cart(x1) and the pendulum(x2) with control parameter

$$f^T = [10.0\ 42.9\ 9.88\ 8.85]$$

which is derived from

$$Q = \mathrm{diag}[100\ 10\ 10\ 10].$$

These experimental data implies an effectiveness of the VSS control strategy.

6. Conclusion

In this paper we present a design procedure and experimental result of a simple VSS controller of relay type. The design procedure is very simple and easily implemented to personal computer.

(1) VSS is fairly robust control strategy.
(2) Cheap control is useful to VSS control.
(3) Descriptor system representation is useful for the analysisof VSS controller.

So our controller is an effective robust con troller.

Reference

[1] V.I.Utkin : 'Sliding modes and their application in variable structure systems' ,Mir Moscow,1978

[1]B.A.Francis : ' The optimal linear-quadratic time-invariant regulator with cheap control' ; IEEE Trans. on Automatic Control,Vol.AC-24,No.4, pp.616/621(1979)

[2]H.Kwakernaak,R.Sivan : 'The maximally achievable accuracy of linear optimal regulators and linear optimal filters';IEEE Trans.on Automatic Control,Vol.AC-17,pp.79/86(1972)

[3]B.T.Smith,et al:'Matrix eigensystem routines EISPACK Guide',Springer-Verlag,1976;B.S.Garbow, et al:'Matrix eigensystem routines EISPACK Guide extension',Springer-Verlag,1977

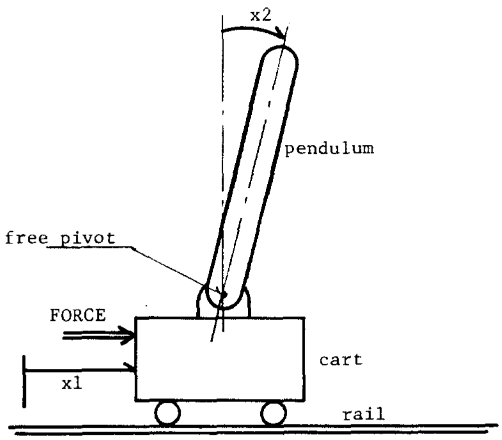

Fig.4 : Inverted pendulum

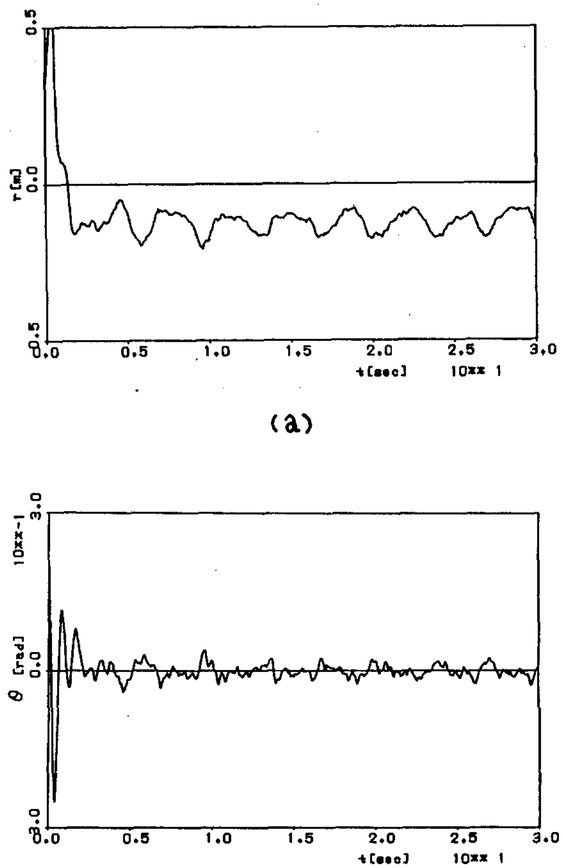

Fig.5 : Experimental result(a) x1 (b) x2