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Position Control of D.C. Motor under the Disturbances by New Sliding Mode Control

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Abstract

A new control method for position control of D.C. servo motor based on the variable structure control is presented. The desired trajectory satisfying the given performance requirement is used as the sliding curve. And the control input forcing the system to follow the desired model system is applied. As a result the method is robust to disturbance.

The performance of the proposed controller is compared with that of the conventional state feedback controller through digital computer simulation.

I. Introduction

In many motor control applications, the control engineer is required to design a system which is insensitive to disturbances. This can be a difficult task especially in cases where the system is under the influence of a wide variation of distrubances.

In this system, required system performance specifications can not be obtained by conventional control method such as proportional-integral-derivative (PID) and/or state feedback control method which is generally used.

A method of control, sliding mode control, which has been a subject of extensive theoretical study in the past,[1), 2), 3)] has recently reattracted attention and is suggested to enable the designer to prescribe the shape of the transient response.[4), 5), 6)] In this method, the representative point of the system is constrained to move along a predetermined hyperplane. In order to achieve such a sliding mode, the control law is required to have a discontinuous nature, resulting in a variable structure system (VSS).[7)] If the deviations from the sliding plane (hyperplane) are small, the motion of the system is completely determined by the chosen plane and consequently the changes in the distrubances can not affect the behavior.

A number of papers to position control of D.C. motor using sliding mode control have been reported. The representative approach can be devided into three part. First approach is a typical sliding mode control; sliding line is one straight line through origin point.[8)] This method does not guarantee the robustness to disturbances through whole interval. Second approach is adaptive sliding mode control; sliding lines consist of a number of straight lines through origin point.[9)] This method improves the robustness to disturbances a little. Third approach is new approach; sliding lines are consisted into three (or four) straight lines and cover whole interval.[10)] This approach gurantees the robustness to disturbances through whole interval but does not propose criterion how to choose sliding lines.

In this paper, a new sliding mode control which guarantees robustness to disturbances through whole interval and can propose criterion how to choose sliding curve is described.

II. Description of the system

In this paper, a fixed field D.C. motor, controlled by the armature voltage is considered. The overall block diagram of the system is shown on Fig. 1. The D.C. motor is modelled as a first order system neglecting the electrical time constant.

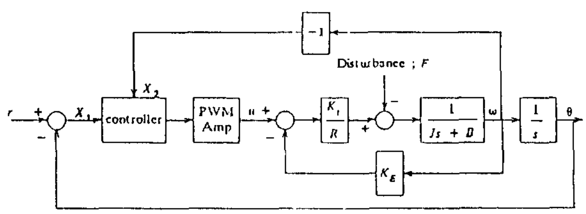

Fig. 1 Overall block diagram of the system

From Fig. 1, the phase variable state representation of the system can be written as follows;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -b\phi \end{bmatrix} u + \begin{bmatrix} 0 \\ c \end{bmatrix} F$$

where
x_1 = position error = $r - \theta$
$x_2 = \dot{x}_1$ = error velocity
ϕ = PWM Amp gain
$a = (K_T \cdot K_E = BR)/JR$
$b = K_T / JR$
$c = 1/J$

III. Description of new sliding mode controller

In linear system, state feedback control method can allocate system poles to desired poles. Also, by the use of optimal control theory, the control engineer can obtain the desired state feedback gains which satisfy the required system performance. Thus, the optimal state feedback controller has been used to position control of D.C. motor. But the system performance is lowed with variation of disturbances. The new sliding mode controller points out to maintain the advantages of optimal state feedback under the influence of distrubances.

State equation of the system without disturbances can be rewritten as follows;

$$\dot{X} = AX + Bu \qquad (2)$$

where X : 2×1, A : 2×2, B : 2×1 are matrix.

Let quadratic performance index be as follows;

$$J = \frac{1}{2}X^T(t_f)HX(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[X^T(t)QX(t)+u^T(t)Ru]dt \quad (3)$$

where

H, Q : real symmetric positive semidefinite matrix.
R : real symmetric positive definite matrix.
Q, R : weighting matrix.

The control engineer can obtain optimal input which minimizes the above performance index and satisfies system performance specifications by choosing proper Q, R matirx. The optimal input which minimizes the above performance index is as follows;

$$u^*(t) = -R^{-1}B^TK(t)X(t) = F(t)X(t) \quad (4)$$

where $\dot{K}(t) = -K(t)A - A^TK(t) - Q + K(t)BR^{-1}B^TK(t)$

$K(t_f) = H$

Applying the optimal input to the system equation (2), system equation (2) can be rewritten as follows;

$$\dot{X}^* = (A + BF)X^* \quad (5)$$

In equation (5), X^* represents state variable of the system with optimal feedback control. From equation (5), the trajectory pairs of $x_1^*(t)$, $x_2^*(t)$ with respect to time t can be obtained.

Under the distrubances, the system equation is represented as follows;

$$\dot{X} = AX + Bu + Df \quad (6)$$

where X : 2×1, A : 2×2, B : 2×1 are matrix.
f : disturbance

In this paper, in spite of disturbances, the state trajectorys of the system equation (6) are guaranteed to follow the state trajectorys of the system equation (5). Let σ be sliding curve, the σ is represented as follows;

$$\sigma = x_2^* - x_2 \text{ for } x_1^* = x_1 \quad (7)$$

And control input u is given as follows;

$$u = u^* + \Delta u \quad (8)$$

$$\Delta u = \begin{cases} \Delta u^+, & \sigma > 0 \\ \Delta u^-, & \sigma < 0 \end{cases}$$

where $u^* = F_1x_1^* + F_2x_2^*$: optimal input without disturbances.

The magnitude of Δu depends on the maximum magnitude of disturbances. If the magnitude of Δu is large, the magnitude of switching ripple is large. Fig. 2 is the phase trajectory of the proposed sliding mode control.

For example, consider following second order system.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - u + f \quad (9)$$

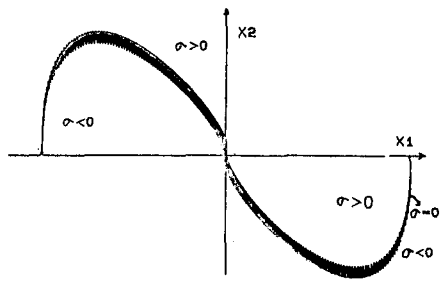

Fig. 2 Phase Plane trajectory

The eigenvalues of the above system are located at −1, 0. By state feedback, the eigenvalues of the system (9) are located -1, -2 by state feedback input u, where $u = 2x_1 + 2x_2$. Thus, the desired system equation without disturbance f is represented as follows;

$$\dot{x}_1^* = x_2^*$$
$$\dot{x}_2^* = -2x_1^* - 3x_2^* \quad (10)$$

Let σ be sliding curve, and u be control input, then

$$\sigma = x_2^* - x_2, \text{ for } x_1^* = x_1$$
$$u = 2x_1^* + 2x_2^* + \Delta u \quad (11)$$

To show existence condition, the derivative of σ must be calculated.

$$\dot{\sigma} = \dot{x}_2^* - \dot{x}_2 = -2x_1^* - 3x_2^* + x_2 + 2x_1^* + 2x_2^* + \Delta u - f$$
$$= -(x_2^* - x_2) + (\Delta u - f)$$
$$= -\sigma + (\Delta u - f) \quad (12)$$

$$\sigma\dot{\sigma} = \sigma(-\sigma + (\Delta u - f))$$
$$= -\sigma^2 + \sigma(\Delta u - f) \quad (13)$$

If the following condition (14) is satisfied, then $\sigma\dot{\sigma}$ is always negative.

$$\begin{cases} \Delta u < |f|_{max}, & \sigma > 0 \\ \Delta u > |f|_{max}, & \sigma < 0 \end{cases} \quad (14)$$

Thus, the existence condition is satisfied. Summerizing the above results, the following algorithm can be obtained.

Algorithm ; *i*) with initial value $x_1(0)$, $x_2(0)$, calculate the trajectory pairs of $x_1^*(t)$, $x_2^*(t)$

ii) for $x_1(t)$, $x_2(t)$, find $x_1^*(t)$, $x_2^*(t)$ trajectory pairs such that $x_1(t) = x_1^*(t)$

iii) calculate $\sigma = x_2^*(t) - x_2(t)$

iv) determine $u = u^* + \Delta u$
($u^* = F_1x_1^* + F_2x_2^*$; F_1, F_2 : optimal feedback gain)
(Δu can be obtained by equation (14))

IV. Computer Simulation

In this section, the performance of the proposed scheme is shown by means of simulation results, and compared with that of conventional optimal state feedback control scheme. The parameters of D.C. motor are as follows;

0.75HP, 100V, 2500 rpm PMDC MOTOR

$R = 0.67\Omega$

$J = 0.24\ kg.m^2$

$B = 0.7\ N.m.sec/rad$

$K_T = 0.4519\ N.m/A$

$K_E = 0.4519\ V.sec/rad$

Thus, the pahse variable state equation is as follows;

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -4.19 \end{bmatrix} x + \begin{bmatrix} 0 \\ -3.09 \end{bmatrix} u + \begin{bmatrix} 0 \\ 4.17 \end{bmatrix} F \qquad (15)$$

Let weighting matrix Q, R be as follows;

$$Q = \begin{bmatrix} 30 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = 0.5$$

Then, the optimal input u^* is as follows;

$$u^*(t) = [\ 7.75 \quad 1.62\ ]x(t)$$

For optimal state feedback control without disturbances, the following state equation is obtained.

$$\dot{x}_1^* = x_2^*$$
$$\dot{x}_2^* = -23.95x_1^* - 9.2x_2^* \qquad (16)$$

The sliding curve σ is represented as follows:

$$\sigma = x_2^* - x_2 \quad \text{for } x_1^* = x_1$$
$$\dot{\sigma} = \dot{x}_2^* - \dot{x}_2 = -23.95x_1^* - 9.2x_2^* + 4.19x_2$$
$$+3.09(7.75x_1^* + 1.62x_2^* + \Delta u) + 4.17f$$
$$= -4.19(x_2^* - x_2) + (3.09\Delta u + 4.17f)$$
$$= -4.19\sigma + (3.09\Delta u + 4.17f)$$
$$\sigma\dot{\sigma} = -4.19\sigma^2 + (3.09\Delta u + 4.17f)\sigma \qquad (17)$$

Thus, equation (17) is always negative, if the following condition is satisfied ;

$$\begin{cases} \Delta u < 1.35|f|_{max}, & \sigma > 0 \\ \Delta u > 1.35|f|_{max}, & \sigma < 0 \end{cases} \qquad (18)$$

Computer simulation results are shown in Fi.g 3 - Fig. 14. In case of Fig. 3 - Fig. 6, the $2N.m$ step disturbance is applied when $x_1(t)$ is smaller than the half of $x_1(0)$, and the initial values of $x_1(t)$, $x_2(t)$ are 5, 0 respectively. These results show that the steady-state error of x_1 exist in case of state feedback control, and that the desired method guarantees the state trajectorys of the system to follow the desired trajectorys in spite of disturbances. In case of Fig. 7 - Fig. 10, the $2N.m$ sinusoidal disturbance with $2Hz$ is applied and the initial values of $x_1(t)$, $x_2(t)$ are 5, 0 respectively.

From these results, it is shown that the proposed controller guarantees the rejection of disturbances, but state feedback controller does not.

In case of Fig. 11 - Fig. 14, the $2N.m$ step disturbance is applied when $x_1(t)$ is smaller than the halt of $x_1(0)$, and the initial values of $x_1(t)$, $x_2(t)$ are 5, 2 respectively.

These results are similiar to the resu lts of Fig. 3 - Fig. 6. From the simulation results, it is shown that the performance of the optimal state feedback controller is lowed by the influence of disturbances, but that of the proposed controller is relatively good in spite of disturbances.

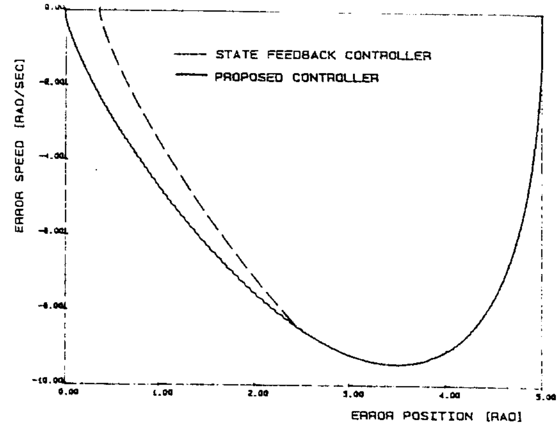

FIG 3. PHASE PLANE TRAJECTORY

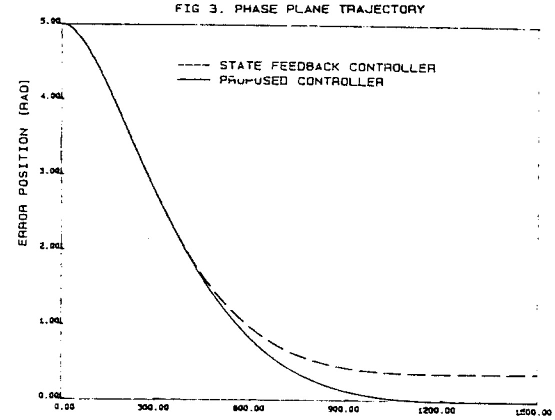

FIG 4. POSITION ERROR TIME [msec]

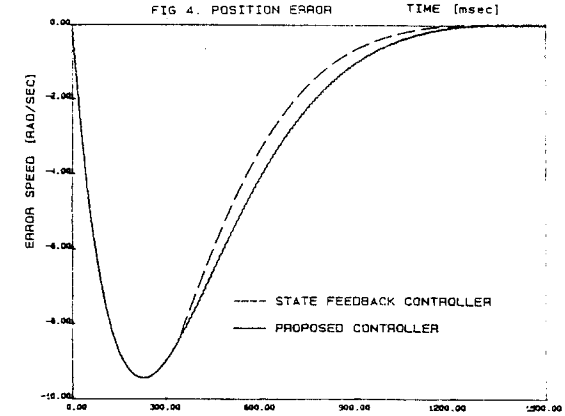

FIG 5. SPEED ERROR TIME [msec]

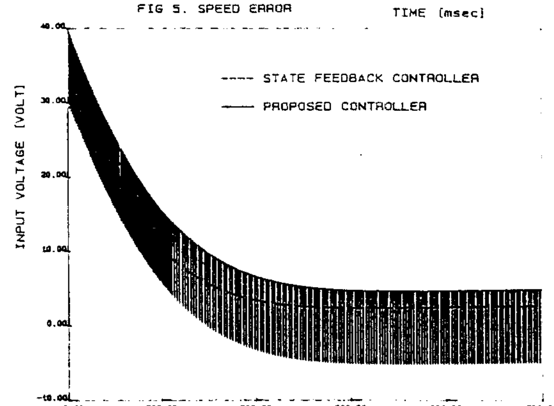

FIG 6. INPUT WAVEFORM TIME [msec]

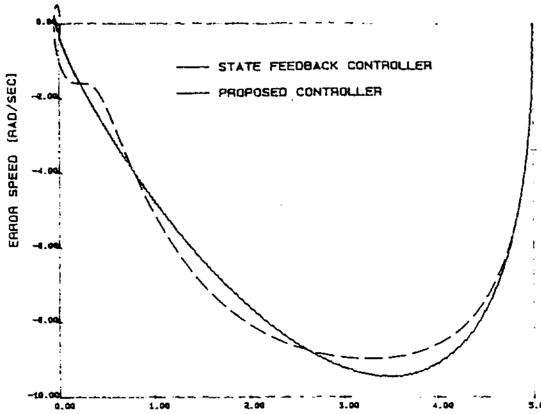

FIG 7. PHASE PLANE TRAJECTORY ERROR POSITION [RAD]

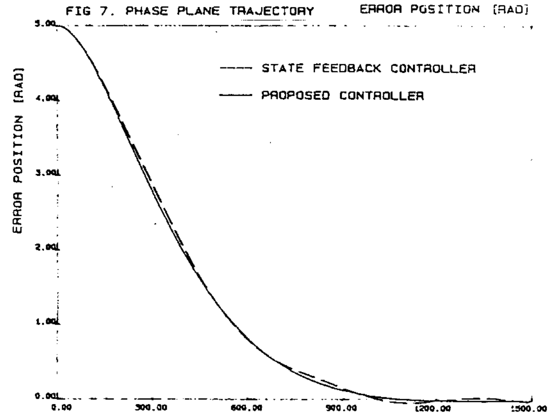

FIG 8. POSITION ERROR TIME [msec]

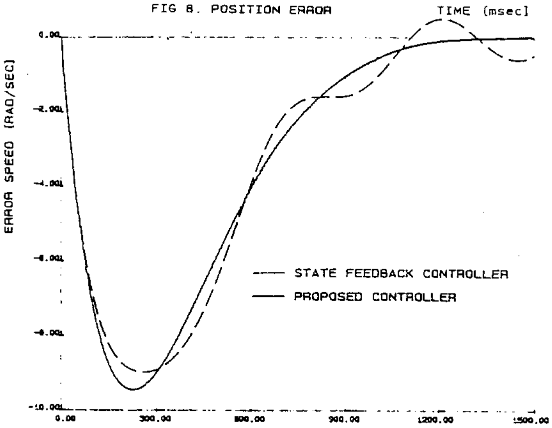

FIG 9. SPEED ERROR TIME [msec]

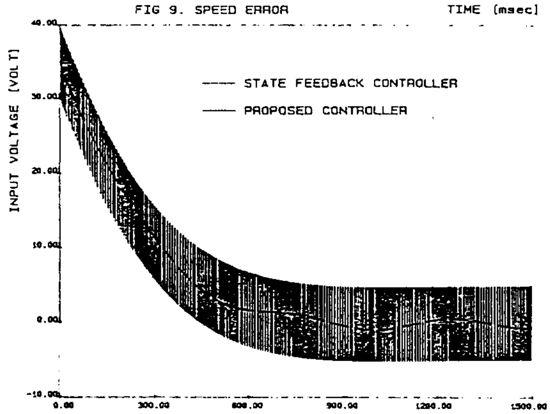

FIG 10. INPUT WAVEFORM TIME [msec]

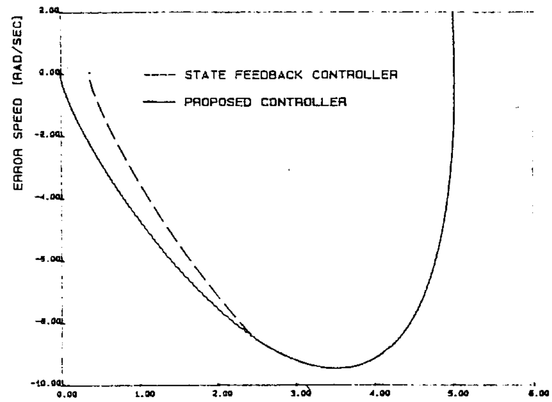

FIG 11. PHASE PLANE TRAJECTORY ERROR POSITION [RAD]

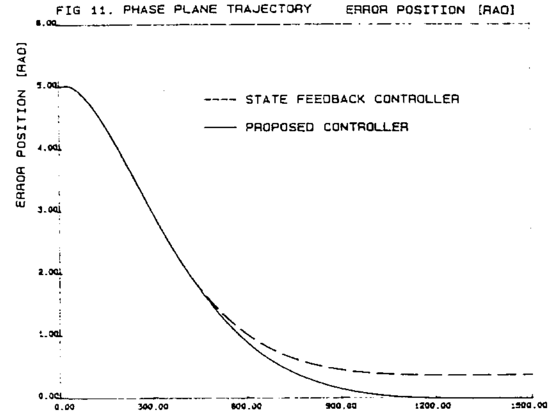

FIG 12. POSITION ERROR TIME [msec]

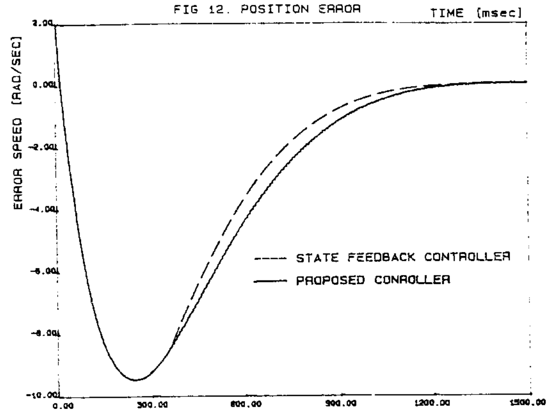

FIG 13. SPEED ERROR TIME [msec]

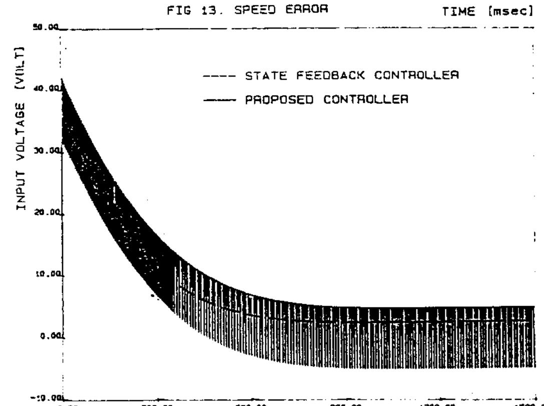

FIG 14. INPUT WAVEFORM TIME [msec]

V. Conclusions

In this paper, it is shown that the system is made insensitive to disturbances by the proposed controller. And, using the proposed controller, the control engineer can obtain easily the desired trajectory from optimal state feedback control theory neglecting the influence of disturbances.

The proposed controller guarantee to follow the desired trajectory in spite of the influence of disturbances.

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